Computational Science:
**Computational Methods in Engineering**

# Naïve Gauss Elimination

## What is Naïve Gauss Elimination?

Naïve Gauss elimination (GE) is the simplest method for solving a system of equations.

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix}\begin{bmatrix} \\ x \\ \\ \end{bmatrix}=\begin{bmatrix} \\ b \\ \\ \end{bmatrix}$$

While simple, it can be unstable and "pivoting" is required to stabilize it. The Naïve algorithm ignores pivoting.

In fact, naïve Gauss elimination was sort of already implemented in this course when solving the first system of linear equations. It was just not done in matrix form.

# Step 1

Start with a matrix problem…

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

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# Step 2

Eliminate $x_1$ from rows 2 and 3.

$$(\text{New Row 2}) = (\text{Old Row 2}) - \frac{a_{21}}{a_{11}}(\text{Row 1})$$

$$(\text{New Row 3}) = (\text{Old Row 3}) - \frac{a_{31}}{a_{11}}(\text{Row 1})$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & a'_{32} & a'_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b'_2 \\ b'_3 \end{bmatrix}$$

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# Step 3

Eliminate $x_2$ from row 3.

$$(\text{New Row 3}) = (\text{Old Row 3}) - \frac{a'_{32}}{a'_{22}}(\text{Row 2})$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & a'_{32} & a'_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b'_2 \\ b'_3 \end{bmatrix} \rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & 0 & a''_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b'_2 \\ b''_3 \end{bmatrix}$$

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# Step 4

Now $x_3$ is known.

$$x_3 = \frac{b''_3}{a''_{33}}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & 0 & a''_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b'_2 \\ b''_3 \end{bmatrix}$$

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# Step 5

Back-substitute to find $x_1$ and $x_2$.

$$x_2 = \frac{b'_2}{a'_{22}} - \frac{a'_{23}}{a'_{22}} x_3 \qquad x_1 = \frac{b_1}{a_{11}} - \frac{a_{12}}{a_{11}} x_2 - \frac{a_{13}}{a_{11}} x_3$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & 0 & a''_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b'_2 \\ b''_3 \end{bmatrix}$$



# Triangular Matrices

Notice that an upper triangular matrix is formed from Gauss elimination.

Triangular matrices represent systems of equations that are “almost” solved.

It is usually a very quick and easy procedure to solve triangular matrices.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & 0 & a''_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b'_2 \\ b''_3 \end{bmatrix}$$

# Observation

Observe that three special factors were calculated.

$$l_{21} = \frac{a_{21}}{a_{11}} \qquad l_{31} = \frac{a_{31}}{a_{11}} \qquad l_{32} = \frac{a'_{32}}{a'_{22}}$$

These will arise again in *LU decomposition*.

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