



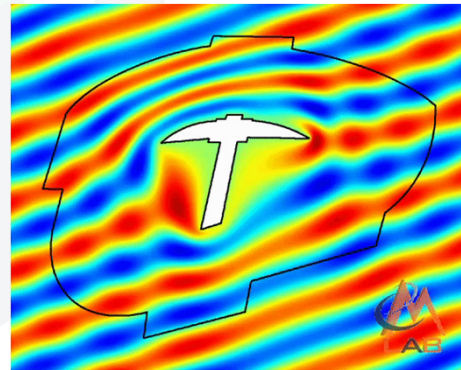
Advanced Electromagnetics:
21st Century Electromagnetics

Numerical Transformation Optics



Lecture Outline

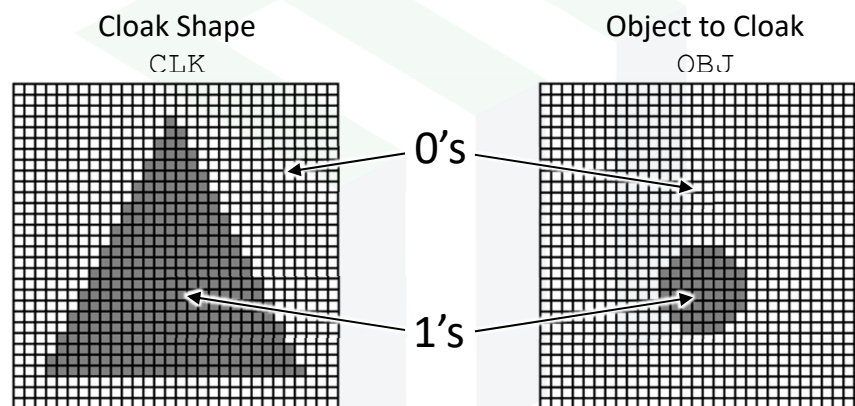
- Steps 1-5: Calculating coordinate transforms using Laplace's equation
- Steps 6-9: Calculating $[\mu]$ and $[\epsilon]$ from the coordinate transform



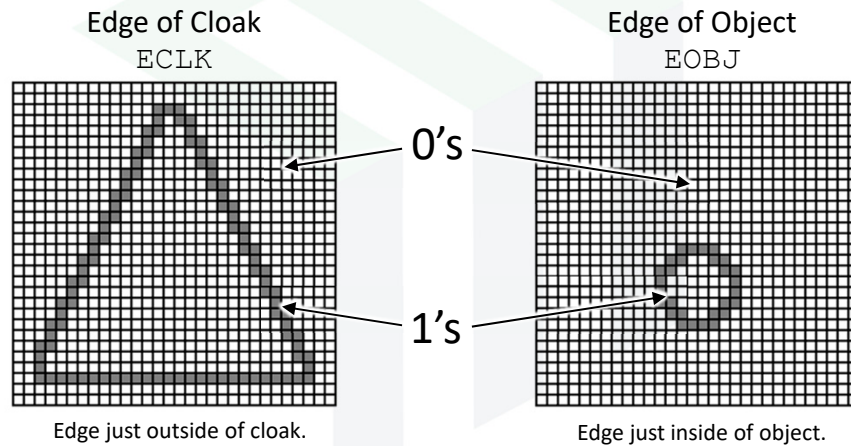
Steps 1 – 5: Calculating Coordinate Transforms Using Laplace's Equation

Slide 3

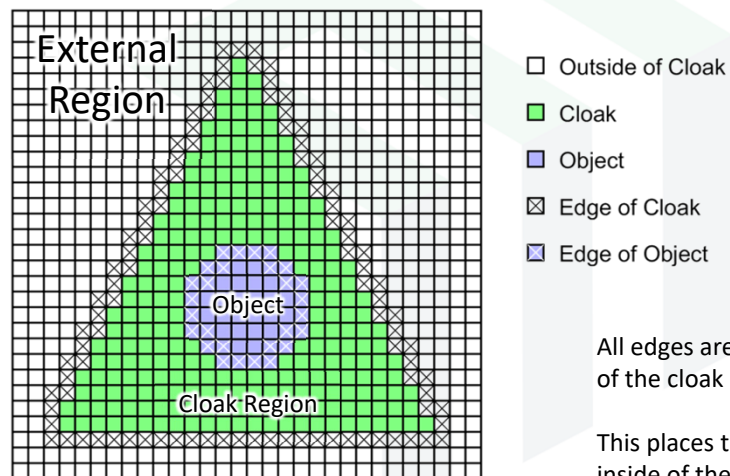
Step 1: Construct Object and Cloak



Step 2: Identify Edges of Cloak and Object (1 of 2)



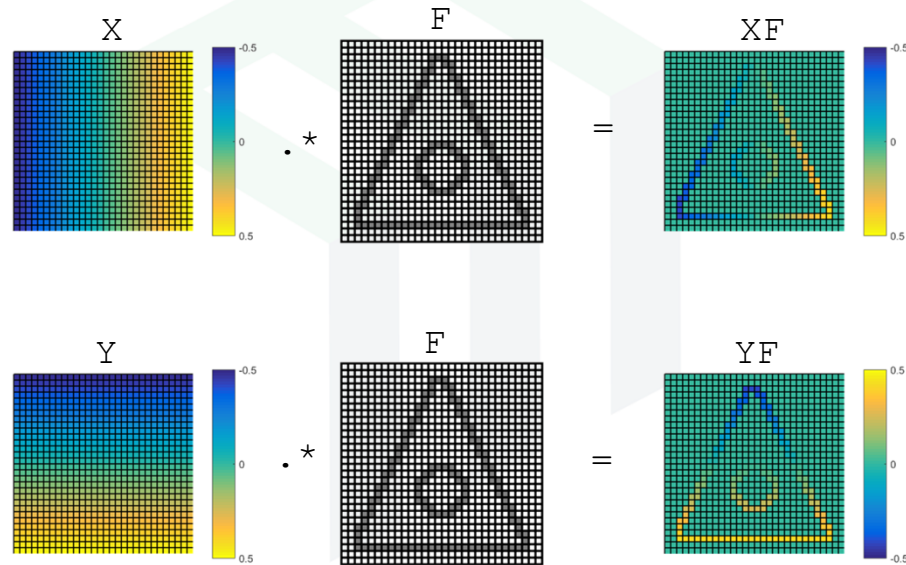
Step 2: Identify Edges of Cloak and Object (2 of 2)



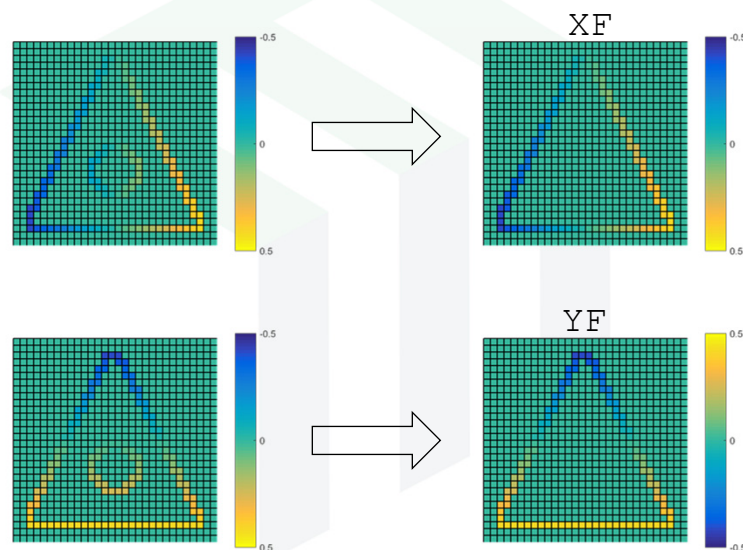
All edges are detected at positions outside of the cloak region.

This places the edge of the object to be inside of the object.

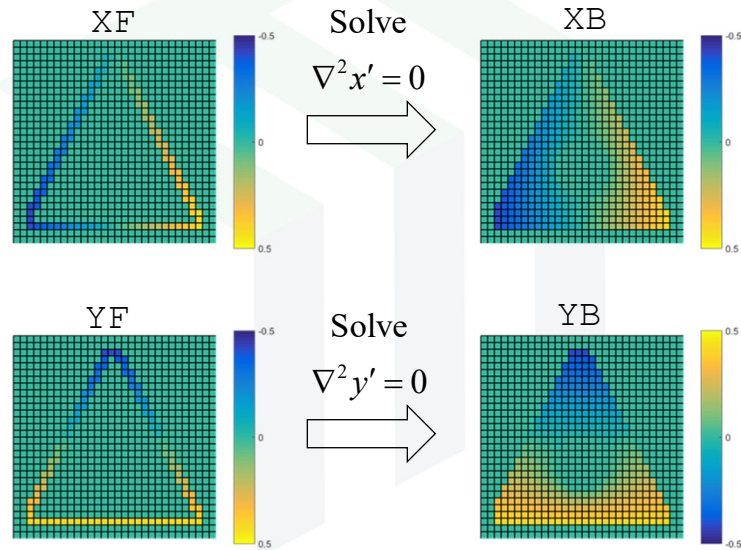
Step 3: Mask Meshgrid to Set Initial Boundary Values



Step 4: Force Coordinates on Object to Zero



Step 5: Fill In Missing Coordinates (1 of 4)



Step 5: Fill In Missing Coordinates (2 of 4)

The matrix equation is constructed in three steps:

Step 1/3 – Construct ordinary Laplacian matrix equation.

$$\mathbf{L} = \mathbf{D}_x^2 + \mathbf{D}_y^2$$

$\mathbf{b} \equiv$ column vector of forced values $b(x, y)$

Step 2/3 – Enforce physical boundary conditions.

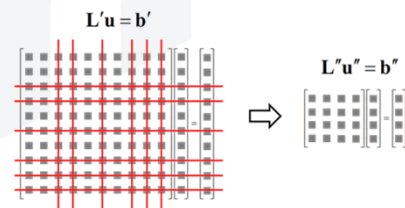
$$\mathbf{L}' = \mathbf{F} + (\mathbf{I} - \mathbf{F})\mathbf{L}$$

$$\mathbf{b}' = \mathbf{F}\mathbf{b}$$

Step 3/3 – Eliminate don't care points.

$$\mathbf{L}'' \leftarrow \mathbf{L}'$$

$$\mathbf{b}'' \leftarrow \mathbf{b}'$$



Step 5: Fill In Missing Coordinates (3 of 4)

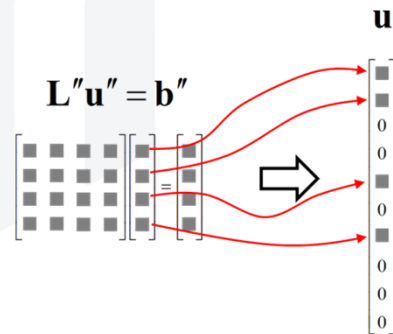
The matrix equation is solved in two steps:

Step 1/2 – Solve for \mathbf{u}'' .

$$\mathbf{u}'' = (\mathbf{L}'')^{-1} \mathbf{b}''$$

Step 2/2 – Insert points in \mathbf{u}'' back into full grid.

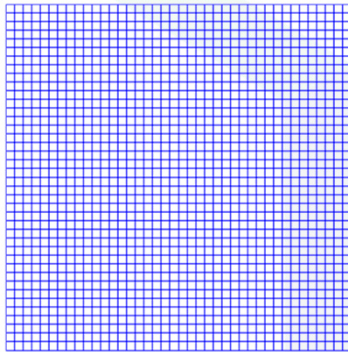
$$\mathbf{u} \leftarrow \mathbf{u}''$$



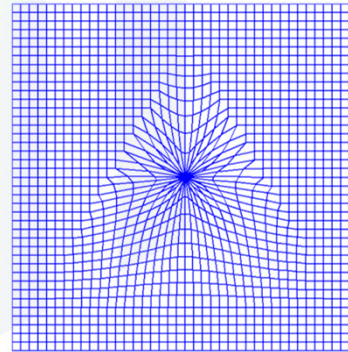
Step 5: Fill In Missing Coordinates (4 of 4)

Alternate view of coordinate transform:

Grid Before Transform



Grid After Transform

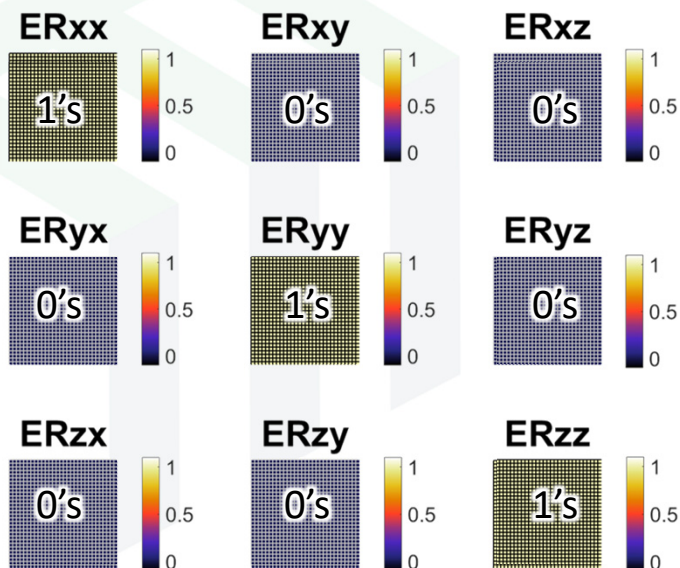


Steps 6 - 9: Calculating $[\mu]$ and $[\varepsilon]$ from the Coordinate Transform

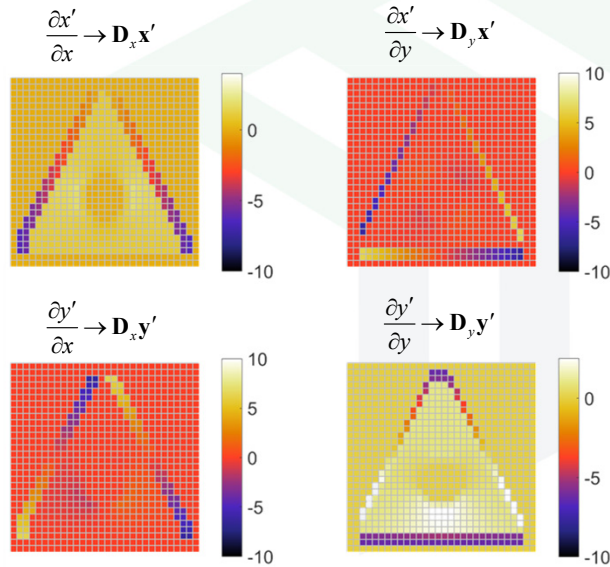
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Step 6: Initialize Background Permittivity and Permeability

$$[\mu_r] = [\varepsilon_r] =$$



Step 7: Calculate Derivatives of Transformed Grid

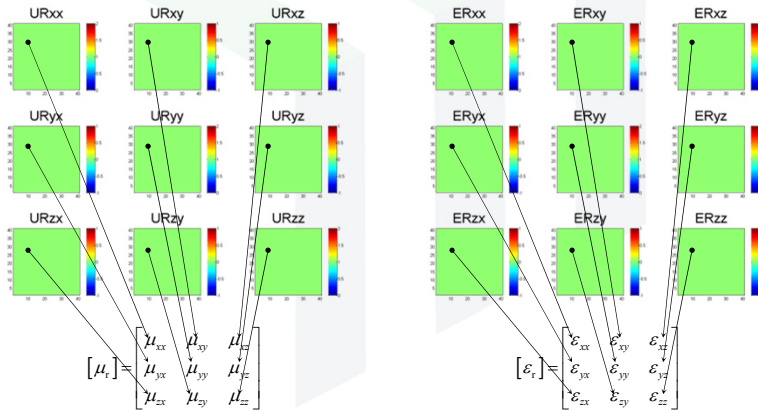


These arrays will be needed to calculate the Jacobian transformation matrix at each point on the grid.

Step 8: Build UR and ER (1 of 4)

This step loops through each point on the grid. **For each point in the cloak region...**

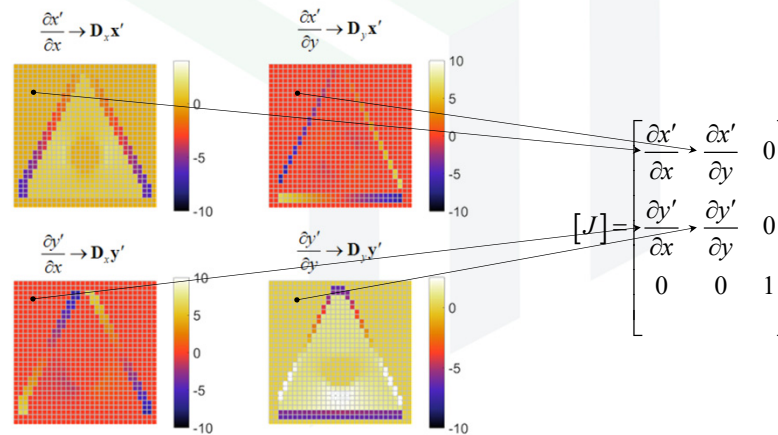
a. Build $[\mu_r]$ and $[\epsilon_r]$ background tensors.



Step 8: Build UR and ER (2 of 4)

This step loops through each point on the grid. **For each point in the cloak region...**

b. Build the Jacobian transformation matrix $[J]$.



Step 8: Build UR and ER (3 of 4)

This step loops through each point on the grid. **For each point in the cloak region...**

c. Transform $[\mu_r]$ and $[\varepsilon_r]$ using the inverse of the Jacobian.

$$\mathbf{G} = \mathbf{J}^{-1}$$

$$[\mu_r'] = \frac{\mathbf{G}[\mu_r]\mathbf{G}^T}{\det \mathbf{G}}$$

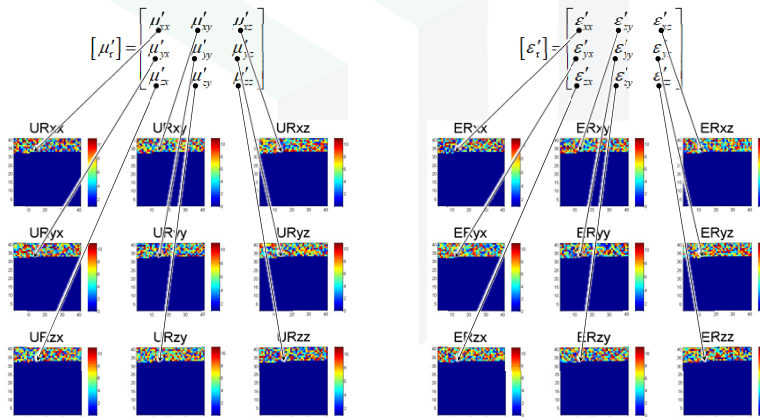
$$[\varepsilon_r'] = \frac{\mathbf{G}[\varepsilon_r]\mathbf{G}^T}{\det \mathbf{G}}$$

```
% Transform UR and ER
J = inv(J);
UR = J*UR*J.'/det(J);
ER = J*ER*J.'/det(J);
```

Step 8: Build UR and ER (4 of 4)

This step loops through each point on the grid. For each point in the cloak region...

d. Populate grid with transformed values of $[\mu_r]$ and $[\epsilon_r]$.



Step 9: Done!

Numerical TO is done! The material tensors can be imported into a CEM code for simulation.

