Computational Science:
Computational Methods in Engineering

Simpson's Rules for Numerical Integration

Outline

- Simpson's 1/3 Rule
- Simpson's 3/8 Rule

2

Simpson's 1/3 Rule

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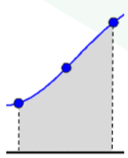
3

Simpson's 1/3 Rule

Suppose three discrete points of a function are known and these are fit them to a second-order polynomial.



$$f(x) \cong a_0 + a_1 x + a_2 x^2$$

Now integrate the polynomial under the curve.

$$\int_{x_1}^{x_3} f(x)\,dx \approx \int_{x_1}^{x_3} \left(a_0 + a_1 x + a_2 x^2\right) dx$$

$$\approx \frac{1}{3}\Delta x\left(f_1 + 4f_2 + f_3\right)$$

To implement Simpson's 1/3 rule, simply apply this to $f(x)$ in groups of 3 points.

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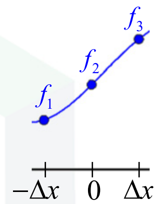
Slide 4

Derivation of Simpson's 1/3 Rule

First, fit the three points to a polynomial.



$$f(x) \cong a_0 + a_1 x + a_2 x^2$$

$$a_0 = f_2 \qquad a_1 = \frac{f_3 - f_1}{2\Delta x} \qquad a_2 = \frac{f_3 - 2f_2 + f_1}{2(\Delta x)^2}$$

Second, integrate the polynomial from $-\Delta x$ to Δx.

$$\int_{-\Delta x}^{\Delta x} \left(a_0 + a_1 x + a_2 x^2\right) dx = \left(a_0 x + \frac{1}{2} a_1 x^2 + \frac{1}{3} a_2 x^3\right)\Bigg|_{-\Delta x}^{\Delta x} = 2a_0 \Delta x + \frac{2}{3} a_2 (\Delta x)^3$$

Substitute in the expressions for a_0, a_1, and a_2.

$$2a_0 \Delta x + \frac{2}{3} a_2 (\Delta x)^3 = 2 f_2 \Delta x + \frac{2}{3} \frac{f_3 - 2f_2 + f_1}{2(\Delta x)^2} (\Delta x)^3 = \frac{1}{3} \Delta x \left(f_1 + 4f_2 + f_3\right)$$

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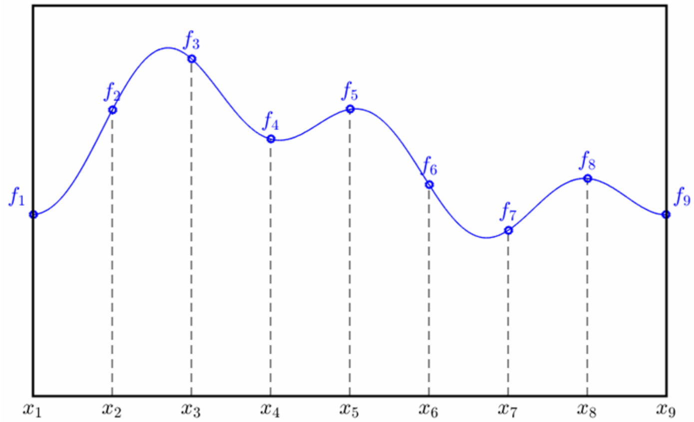
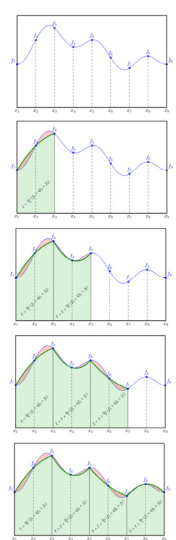
Implementation of Simpson's 1/3 Rule

Animation of Numerical Integration Using Simpson's 1/3 Rule





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Simpson's 3/8 Rule

Simpson's 3/8 Rule

This is similar to Simpson's 1/3 rule, except $f(x)$ is fit to a polynomial in groups of 4 points.

$$\int_{x_1}^{x_4} f(x)\,dx \approx \frac{3}{8}\Delta x\left(f_1 + 3f_2 + 3f_3 + f_4\right)$$