



Computational Science:
Computational Methods in Engineering

Simpson's Rules for Numerical Integration



1

Outline

- Simpson's 1/3 Rule
- Simpson's 3/8 Rule



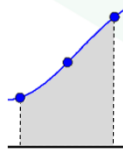
2

Simpson's 1/3 Rule

3

Simpson's 1/3 Rule

Suppose three discrete points of a function are known and these are fit them to a second-order polynomial.



$$f(x) \cong a_0 + a_1x + a_2x^2$$

Now integrate the polynomial under the curve.

$$\int_{x_1}^{x_3} f(x) dx \approx \int_{x_1}^{x_3} (a_0 + a_1x + a_2x^2) dx$$

$$\approx \frac{1}{3} \Delta x (f_1 + 4f_2 + f_3)$$

To implement Simpson's 1/3 rule, simply apply this to $f(x)$ in groups of 3 points.

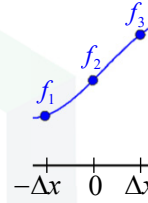
4

Derivation of Simpson's 1/3 Rule

First, fit the three points to a polynomial.

$$f(x) \cong a_0 + a_1x + a_2x^2$$

$$a_0 = f_2 \quad a_1 = \frac{f_3 - f_1}{2\Delta x} \quad a_2 = \frac{f_3 - 2f_2 + f_1}{2(\Delta x)^2}$$



Second, integrate the polynomial from $-\Delta x$ to Δx .

$$\int_{-\Delta x}^{\Delta x} (a_0 + a_1x + a_2x^2) dx = \left(a_0x + \frac{1}{2}a_1x^2 + \frac{1}{3}a_2x^3 \right) \Big|_{-\Delta x}^{\Delta x} = 2a_0\Delta x + \frac{2}{3}a_2(\Delta x)^3$$

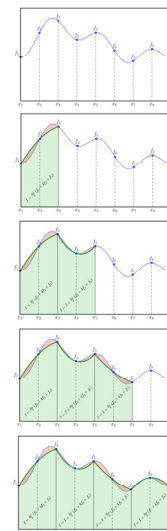
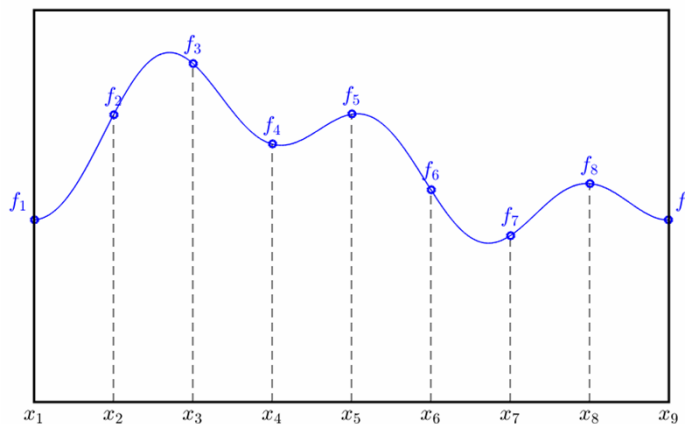
Substitute in the expressions for a_0 , a_1 , and a_2 .

$$2a_0\Delta x + \frac{2}{3}a_2(\Delta x)^3 = 2f_2\Delta x + \frac{2}{3} \frac{f_3 - 2f_2 + f_1}{2(\Delta x)^2} (\Delta x)^3 = \frac{1}{3} \Delta x (f_1 + 4f_2 + f_3)$$

5

Implementation of Simpson's 1/3 Rule

Animation of Numerical Integration Using Simpson's 1/3 Rule



6

Simpson's 3/8 Rule

7

Simpson's 3/8 Rule

This is similar to Simpson's 1/3 rule, except $f(x)$ is fit to a polynomial in groups of 4 points.

$$\int_{x_1}^{x_4} f(x) dx \approx \frac{3}{8} \Delta x (f_1 + 3f_2 + 3f_3 + f_4)$$

8



9