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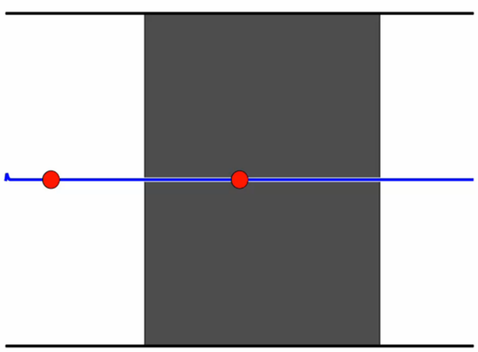
Computational Science:

Computational Methods in Engineering

Slab Waveguides

Refractive Index n

Light travels at different speeds when it is inside different materials.



Frequency f is constant.
Speed v changes.
Wavelength λ changes.

$$n = \frac{c_0}{v}$$

The factor by which a wave slows down is called the *refractive index* n.

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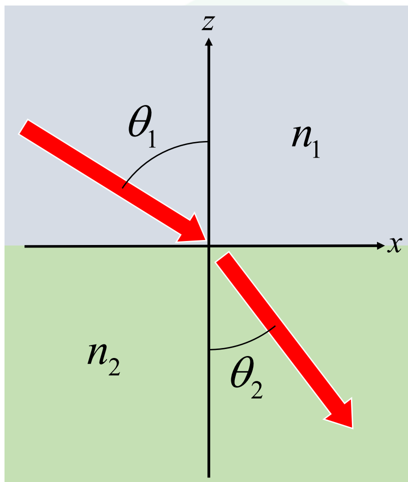
Snell's Law



Snell's law quantifies the angles of light rays at an interface.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

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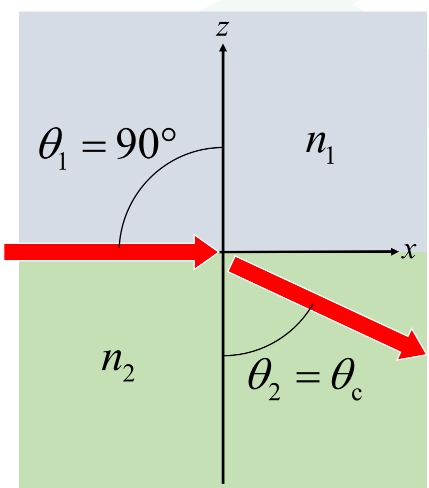
Critical Angle θ_c



There exists a special angle, the *critical angle*, where the ray in the low-index medium is at 90°.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$n_1 \sin 90^\circ = n_2 \sin\theta_c$$
$$n_1 = n_2 \sin\theta_c$$
$$\sin\theta_c = n_1/n_2$$
$$\theta_c = \sin^{-1}\left(n_1/n_2\right)$$

$$\theta_c = \sin^{-1}\left(n_1/n_2\right)$$
$$\text{where } n_2 > n_1$$

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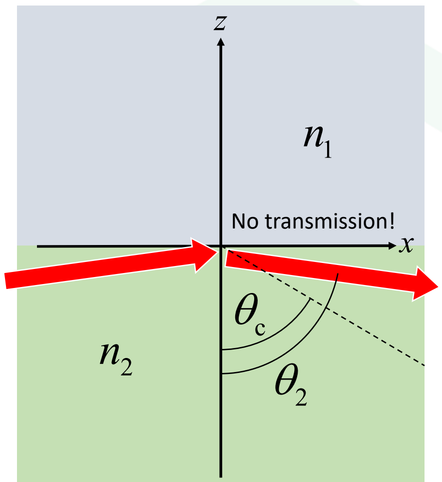
Total Internal Reflection (TIR)



When a light ray is incident onto an interface at an angle greater than the critical angle, the light completely reflects and no light is transmitted.

This is called *total internal reflection* (TIR).

$$\theta_2 > \theta_c$$

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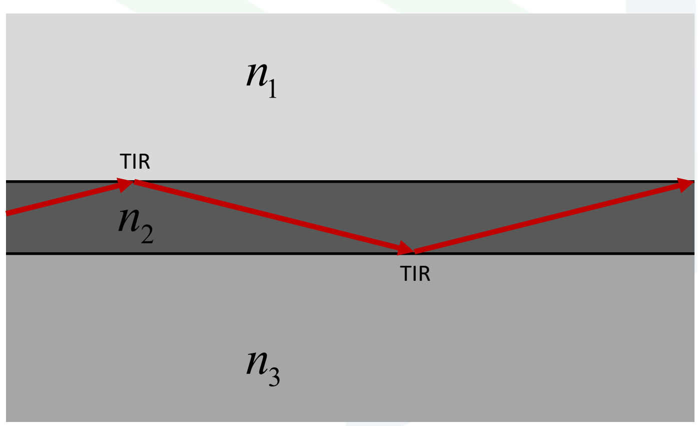
The Slab Waveguide

If a slab of high-index material is placed between two materials with lower refractive index, a slab waveguide is formed. The wave is trapped due to total internal reflection.

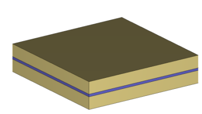


<u>Conditions</u>

$n_2 > n_1$

and

$n_2 > n_3$



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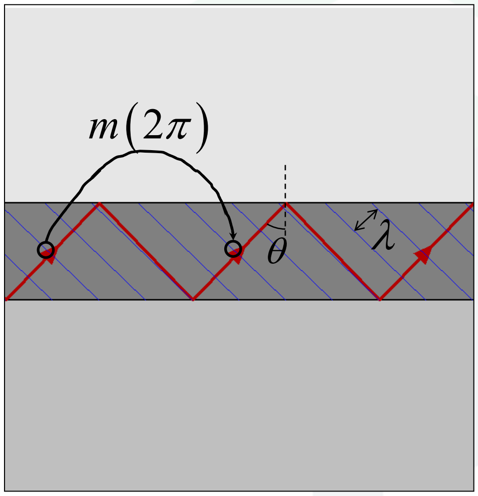
Ray Tracing Picture



The round trip phase of a ray must be an integer multiple of 2π. Otherwise the wave will interfere with itself and escape from the slab.

Because of this, only certain angles are allowed to propagate in the waveguide.

This is the origin of discrete modes in a waveguide.

$$\beta = k_0 n_{\text{eff}} = k_0 n \sin\theta$$

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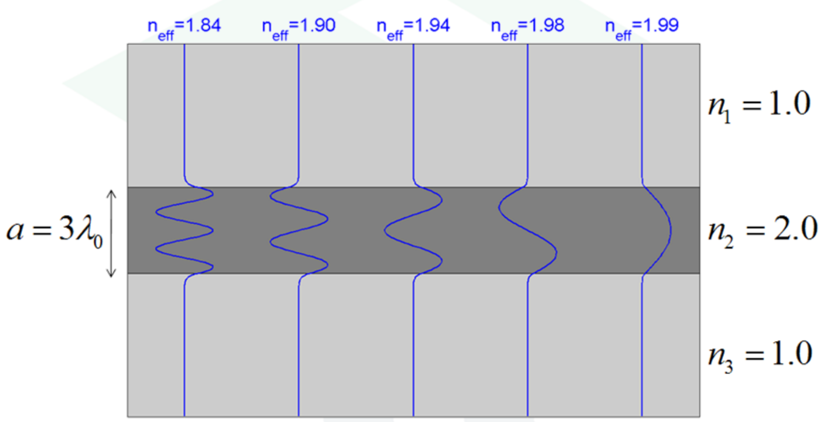
Rigorous Analysis



$$\beta = k_0 n_{\text{eff}} = k_0 n \sin\theta$$

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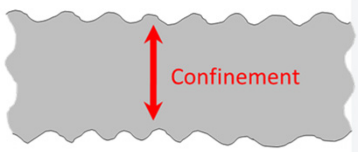
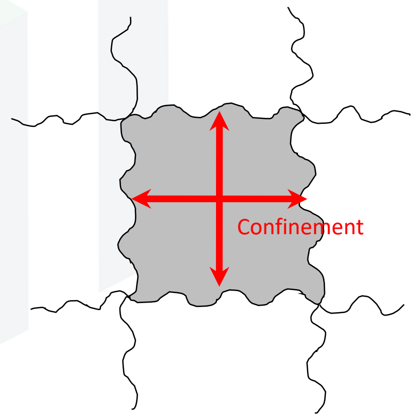
Slide 8

Slab Vs. Channel Waveguides

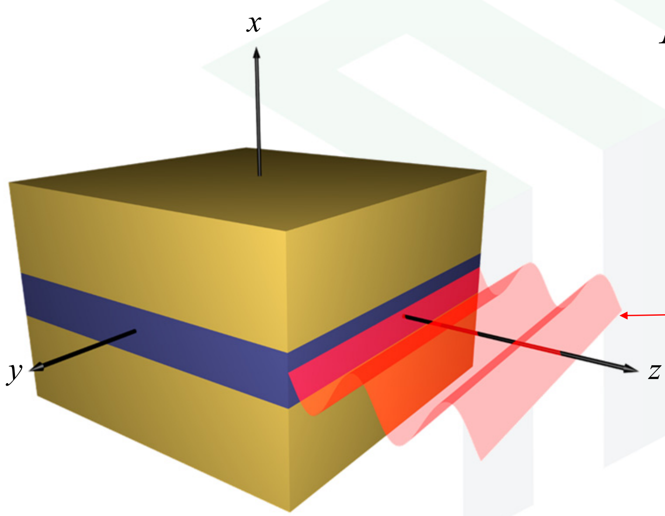
Slab waveguides confine energy in only one transverse direction.

Channel waveguides confine energy in both transverse directions.



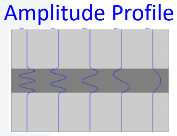


Mathematical Form of Solution of Guided Wave



$$\vec{E}(x,y,z) = \vec{A}(x)e^{-j\beta z}$$

Amplitude Profile



Wave oscillations

$\beta \equiv$ phase constant