Computational Science:
Computational Methods in Engineering

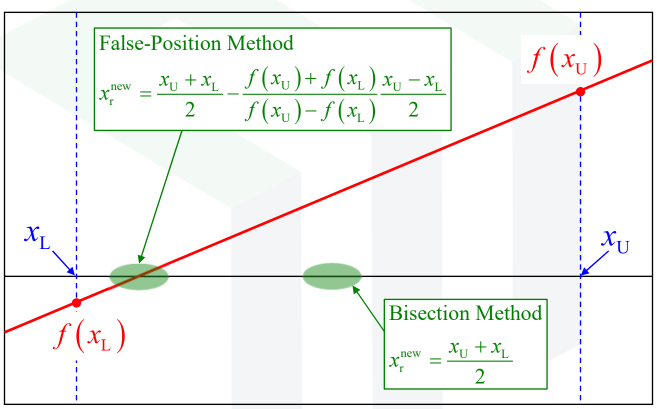
The False Position Method

Outline

- Description of the Method
- Derivation of the Root Estimate
- Notes on Implementation

Description of the Method

More Intelligent "Midpoint"



Since $|f(x_L)| << |f(x_U)|$, it is a good assumption that the root is closer to x_L than it is to x_U.

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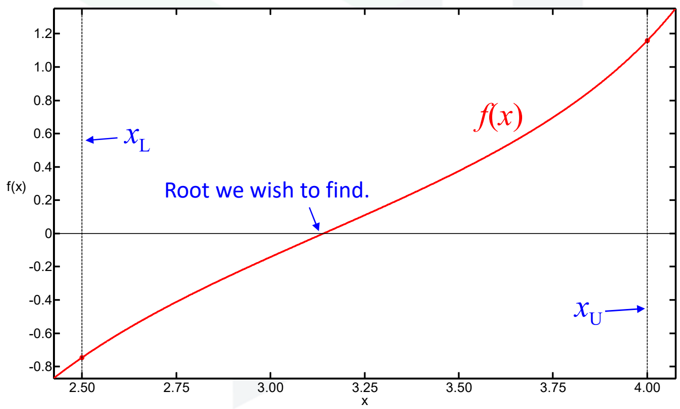
Step 1

Pick a lower and upper bound, x_L and x_U that is known to contain a single root.



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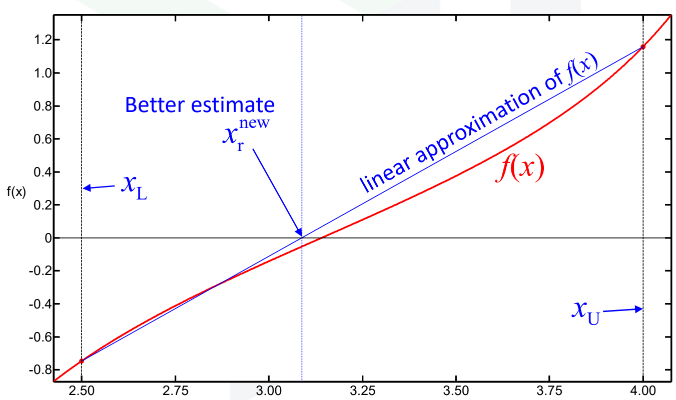
Step 2

Calculate a better estimate of the position of the root using a linear approximation.



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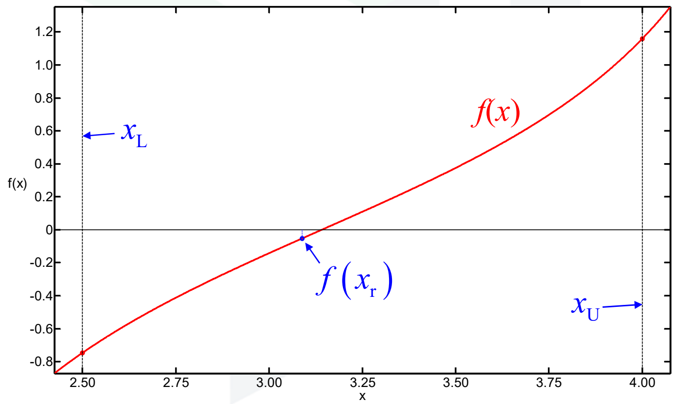
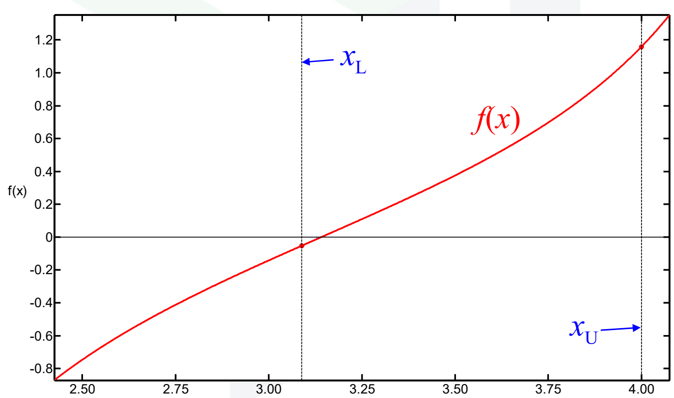
Step 3

Calculate the function at the new estimate for x_r.



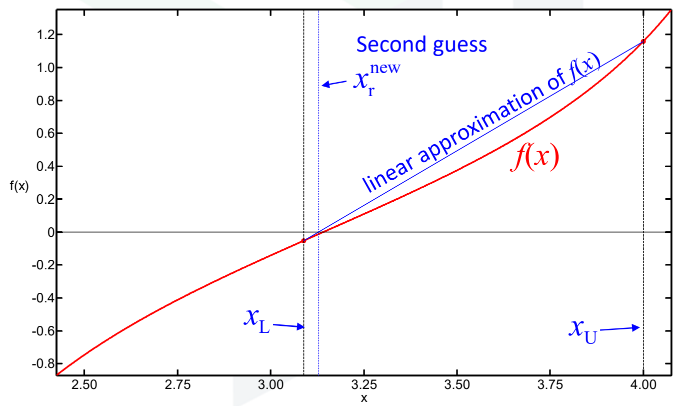
Step 4

Adjust the bounds.



Step 5

Estimate the position of the root by linear approximation.



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Derivation of the Root Estimate

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Derivation of Estimate (1 of 3)

The equation of a line given a point (x_0, y_0) and slope m is

$$(y - y_0) = m(x - x_0)$$

Assuming the function connecting the bounds is close to linear, the slope is approximately

$$m \approx \frac{f(x_\mathrm{U}) - f(x_\mathrm{L})}{x_\mathrm{U} - x_\mathrm{L}}$$

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Derivation of Estimate (2 of 3)

Choose the lower bound to be the point (x_0, y_0) in the line equation.

$$x_0 = x_\mathrm{L} \qquad y_0 = f(x_\mathrm{L})$$

$$y - f(x_\mathrm{L}) = \underbrace{\frac{f(x_\mathrm{U}) - f(x_\mathrm{L})}{x_\mathrm{U} - x_\mathrm{L}}}_{m}(x - x_\mathrm{L})$$

Now estimate the position of the root by setting $y = 0$ and solving for x.

$$0 - f(x_\mathrm{L}) = \frac{f(x_\mathrm{U}) - f(x_\mathrm{L})}{x_\mathrm{U} - x_\mathrm{L}}\left(x_\mathrm{r}^\mathrm{new} - x_\mathrm{L}\right)$$

$$\boxed{x_\mathrm{r}^\mathrm{new} = x_\mathrm{L} - \frac{f(x_\mathrm{L})}{f(x_\mathrm{U}) - f(x_\mathrm{L})}(x_\mathrm{U} - x_\mathrm{L})}$$

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Derivation of Estimate (3 of 3)

Choose the upper bound to be the point (x_0, y_0) in the line equation.

$$x_0 = x_U \qquad y_0 = f(x_U)$$

$$y - f(x_U) = \underbrace{\frac{f(x_U) - f(x_L)}{x_U - x_L}}_{m}(x - x_U)$$

Now estimate the position of the root by setting $y = 0$ and solving for x.

$$0 - f(x_U) = \frac{f(x_U) - f(x_L)}{x_U - x_L}\left(x_r^{\text{new}} - x_U\right)$$

$$\boxed{x_r^{\text{new}} = x_U - \frac{f(x_U)}{f(x_U) - f(x_L)}(x_U - x_L)}$$

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Interpretation of Estimate (1 of 2)

There are now two possible equations to estimate the position of the root.

$$x_r^{\text{new}} = x_L - \frac{f(x_L)}{f(x_U) - f(x_L)}(x_U - x_L) \qquad x_r^{\text{new}} = x_U - \frac{f(x_U)}{f(x_U) - f(x_L)}(x_U - x_L)$$

Average these equations.

$$x_r^{\text{new}} = \frac{\left[x_L - \frac{f(x_L)}{f(x_U) - f(x_L)}(x_U - x_L)\right] + \left[x_U - \frac{f(x_U)}{f(x_U) - f(x_L)}(x_U - x_L)\right]}{2}$$

$$= \frac{x_U + x_L}{2} - \frac{f(x_U) + f(x_L)}{f(x_U) - f(x_L)}\frac{x_U - x_L}{2}$$

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Interpretation of Estimate (2 of 2)

$$x_{\mathrm{r}}^{\mathrm{new}} = \underbrace{\frac{x_{\mathrm{U}} + x_{\mathrm{L}}}{2}}_{\text{Typical bisection method equation}} - \underbrace{\frac{f(x_{\mathrm{U}}) + f(x_{\mathrm{L}})}{f(x_{\mathrm{U}}) - f(x_{\mathrm{L}})} \frac{x_{\mathrm{U}} - x_{\mathrm{L}}}{2}}_{\text{A correction term to give a better estimate.}}$$

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Notes on Implementation

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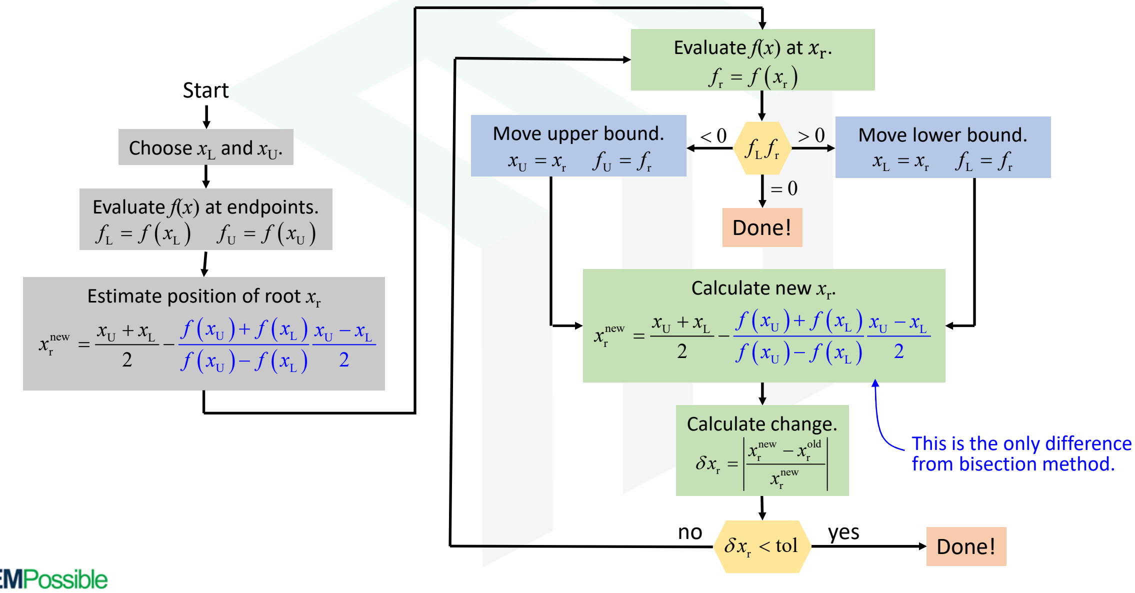
Block Diagram of False-Position Method



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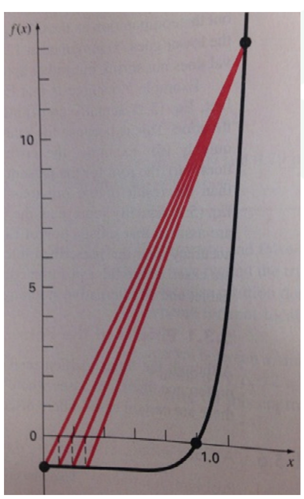
When False-Position Fails



The false-position method can fail or exhibit extremely slow convergence when the function is highly nonlinear between the bounds.

This happens because the estimated root is a linear fit and a very poor estimate of a nonlinear function.

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Notes on False-Position Method

- Very similar to bisection method
- Calculates a more intelligent "midpoint"
- Converges much faster for near linear functions
- Converges slower for highly nonlinear functions

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