Computational Science:
Computational Methods in Engineering

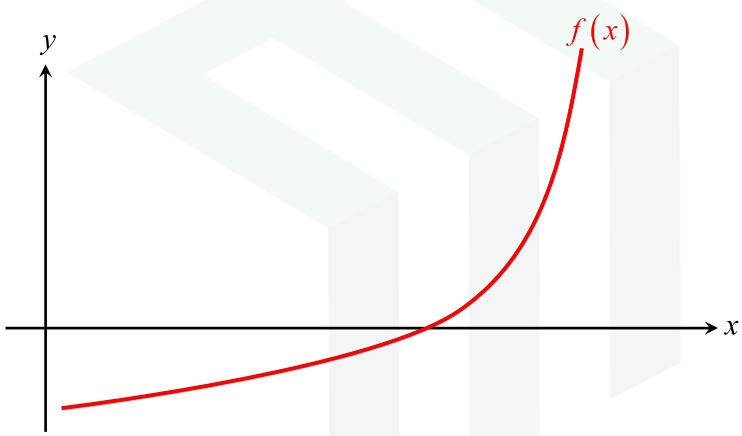
The Newton-Raphson Method

Outline

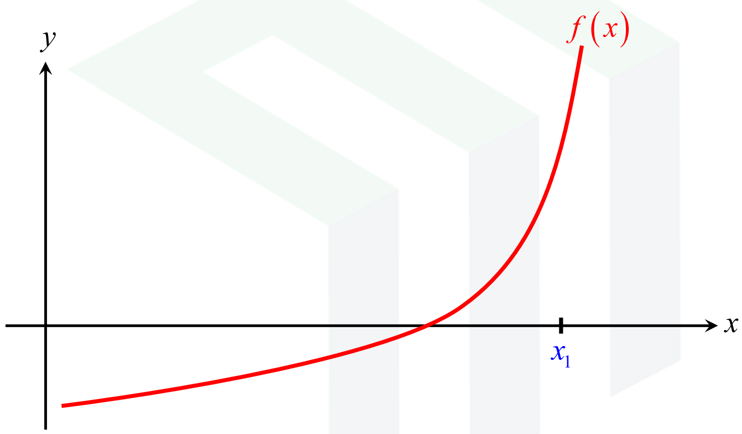
- Description of the Method
- Notes on Implementation
- Example

Description of the Method

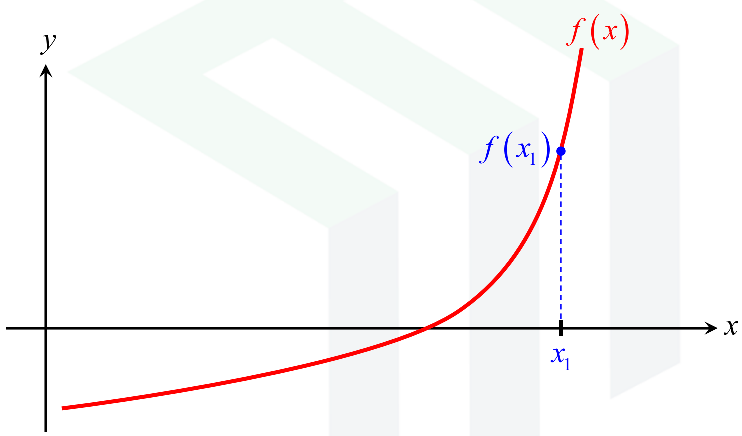
Start with Any Function



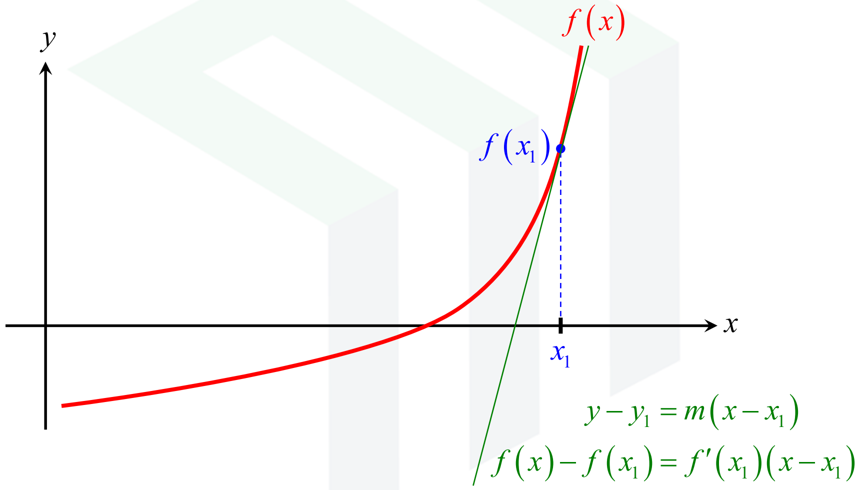
Make an Initial Guess for the Position of the Root x_1



Evaluate the Function at x_1



Calculate the Equation of the Line Tangential to the Point on $f(x)$



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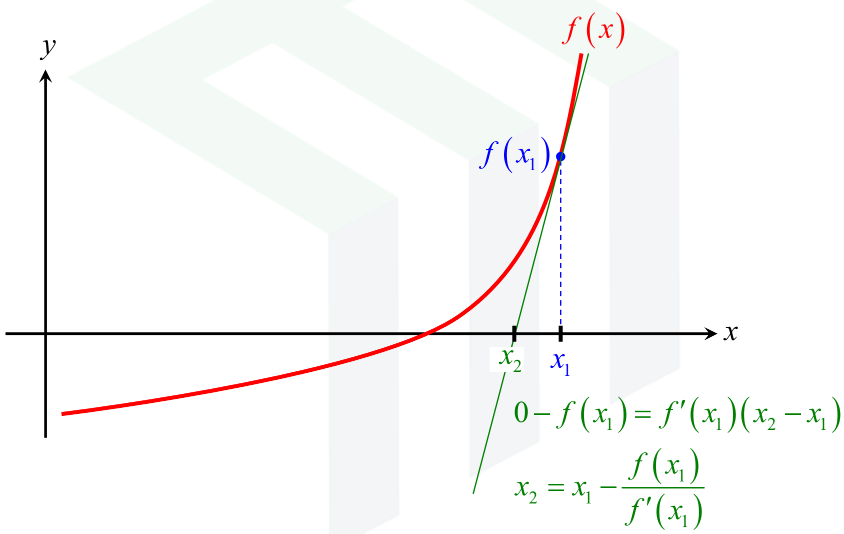
Calculate Where the Line Crosses the x-Axis



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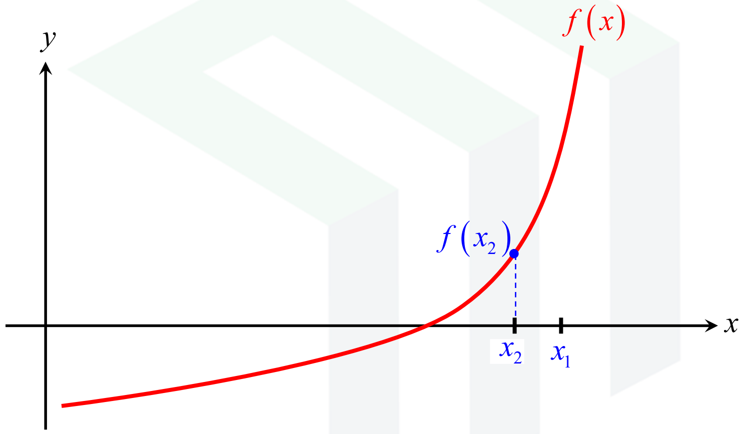
Evaluate the Function at x_2



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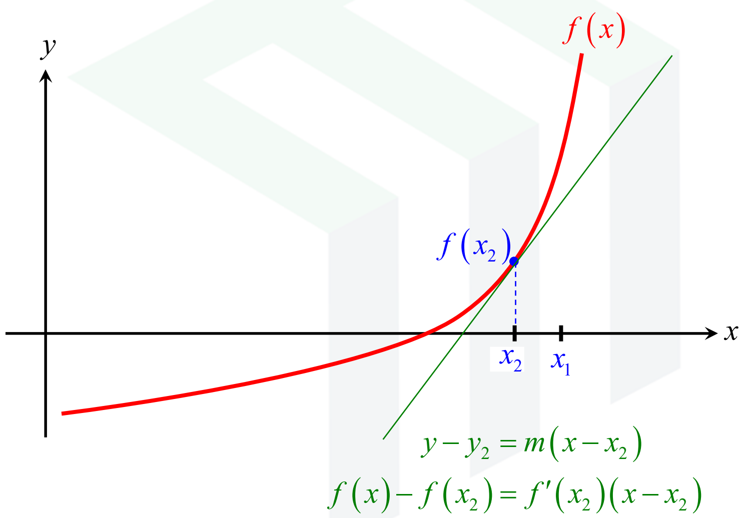
Calculate the Equation of the Line Tangential to the Point on $f(x)$



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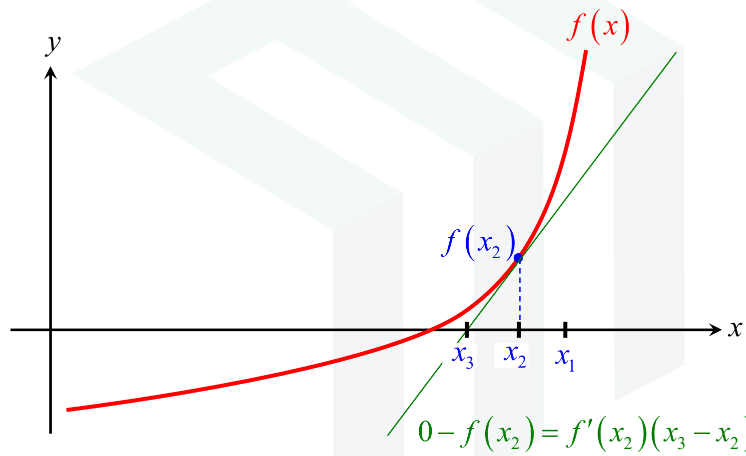
Calculate Where the Line Crosses the x-Axis



$$0 - f(x_2) = f'(x_2)(x_3 - x_2)$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

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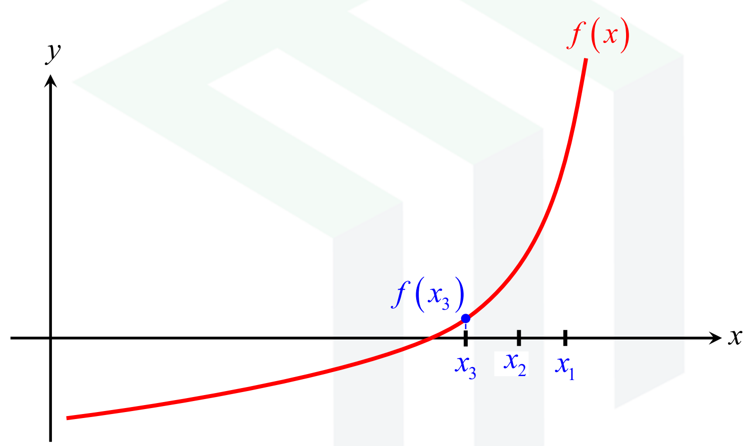
Evaluate the Function at x_3



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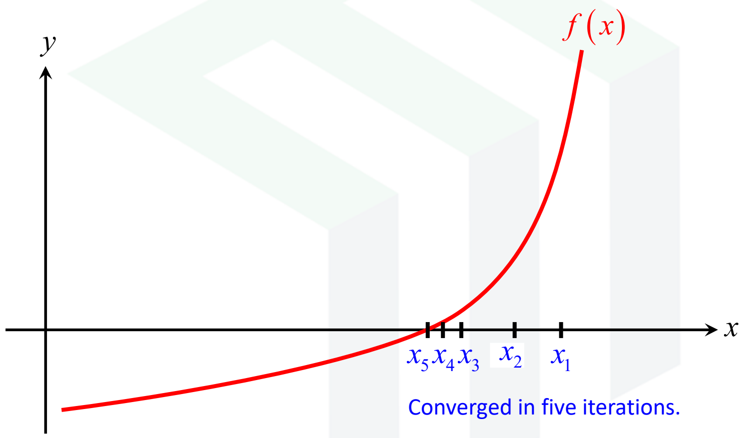
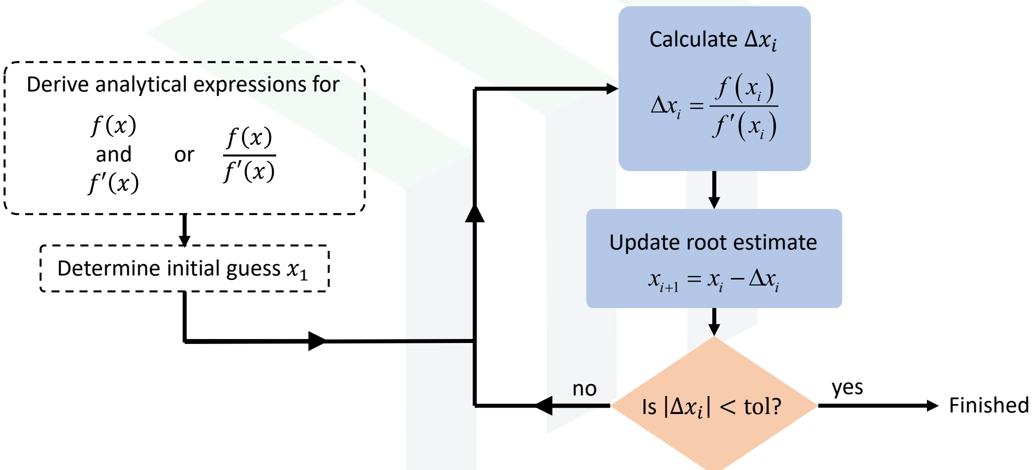
And so on…



Converged in five iterations.

Notes on Implementation

Block Diagram of Algorithm



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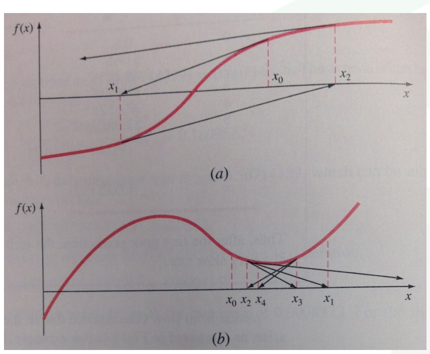
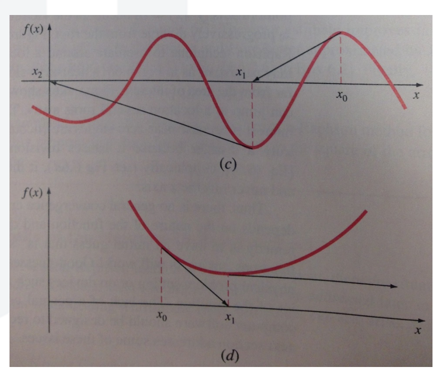
Poor or Unstable Convergence





Steven C. Chapra, *Numerical Methods for Engineers*, 7th Ed., McGraw Hill, 2015.

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Handling Multiple Roots

If $f(x)$ has multiple roots, perform root finding on the auxiliary function $u(x)$ instead.

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad \rightarrow \quad x_{i+1} = x_i - \frac{u(x_i)}{u'(x_i)}$$

The new update equation can be written completely in terms of $f(x)$ and its derivatives as follows.

$$\boxed{x_{i+1} = x_i - \frac{f(x_i) f'(x_i)}{\left[f'(x_i)\right]^2 - f(x_i) f''(x_i)}}$$

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Notes on Newton-Raphson Method

- Does not require bounds.
- Requires a “good” initial guess.
- Requires $f(x)$ and $f'(x)$ to be analytical.
- Converges extremely fast for functions that are near linear.
- Algorithm vulnerable to instability
- Can converge to the wrong root if multiple roots exist.
- Newton-Raphson Method ≠ Newton’s Method
  - While closely related, NRM is for root finding whereas NM is for optimization.

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Example

Example #1

Let $f(x) = \sin x$. What is the root of $f(x)$ in the proximity of $x = 4$?

Step 1 -- Derive analytical expression for $f(x)/f'(x)$

$$f(x) = \sin x$$

$$\Delta x = \frac{f(x)}{f'(x)} = \frac{\sin x}{\frac{d}{dx}\sin x} = \frac{\sin x}{\cos x} = \tan x$$

This means the update equation is

$$x_{i+1} = x_i - \Delta x_i = x_i - \tan x_i$$

Step 2 -- MATLAB code

```
xr  = 4;
tol = 1e-6;
dx  = inf;
while abs(dx) > tol
    dx = tan(xr);
    xr = xr - dx;
end
```

Converges to 3.1416 after 4 iterations.

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