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Computational Science:
Computational Methods in Engineering

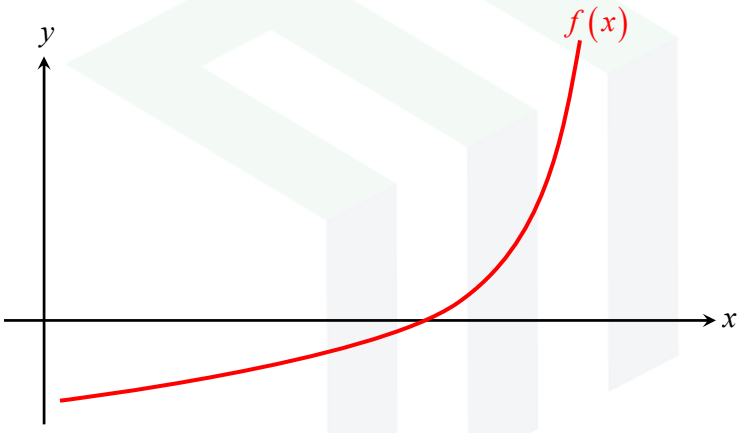
The Secant Method

Outline

- Description of the Method
- Notes on Implementation
- Example

Description of the Method

Start with Some Function



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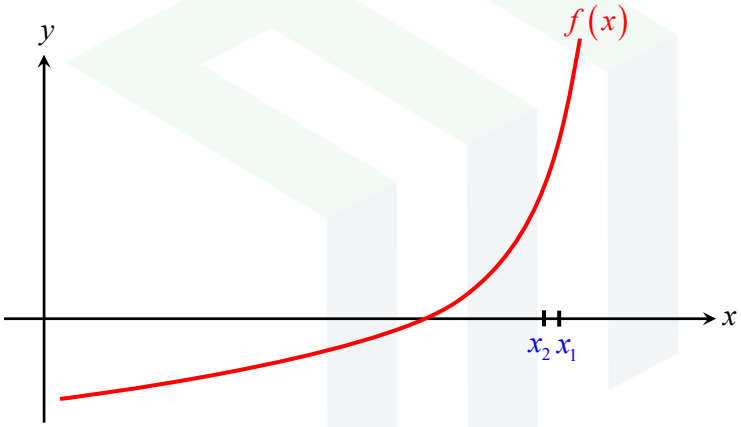
Make Two Initial Guesses for the Position of the Root, x_1 and x_2



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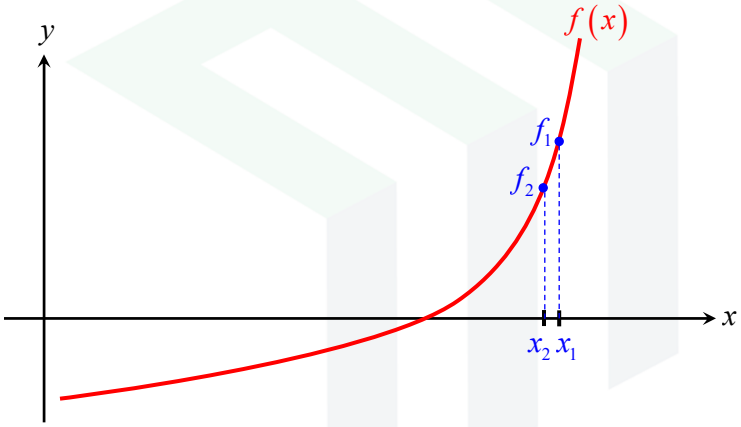
Evaluate the Function at x_1 and x_2



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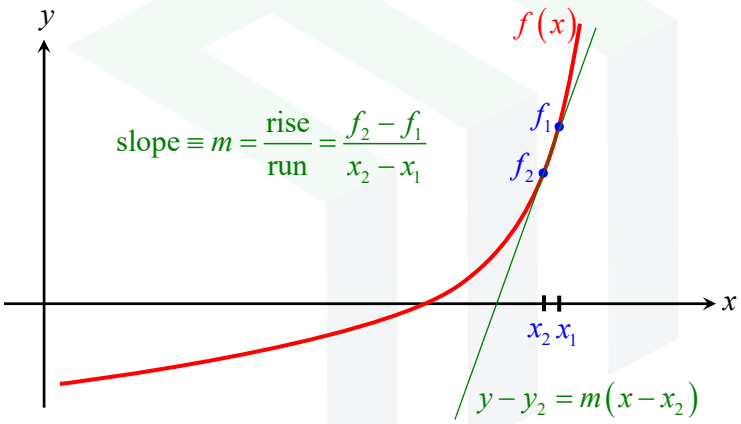
Calculate the Equation of the Line Connecting (x_1, f_1) to (x_2, f_2)



$$\text{slope} \equiv m = \frac{\text{rise}}{\text{run}} = \frac{f_2 - f_1}{x_2 - x_1}$$

$$y - y_2 = m(x - x_2)$$

$$f(x) - f_2 = \frac{f_2 - f_1}{x_2 - x_1}(x - x_2)$$

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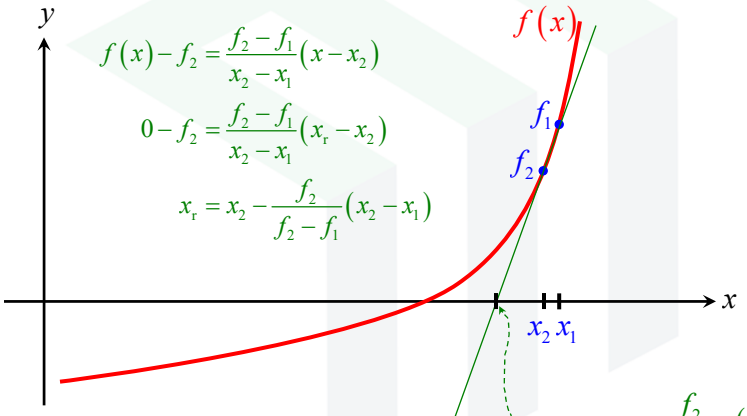
Calculate Where the Line Crosses the x-Axis



$$f(x) - f_2 = \frac{f_2 - f_1}{x_2 - x_1}(x - x_2)$$

$$0 - f_2 = \frac{f_2 - f_1}{x_2 - x_1}(x_r - x_2)$$

$$x_r = x_2 - \frac{f_2}{f_2 - f_1}(x_2 - x_1)$$

$$x_r = x_2 - \frac{f_2}{f_2 - f_1}(x_2 - x_1)$$

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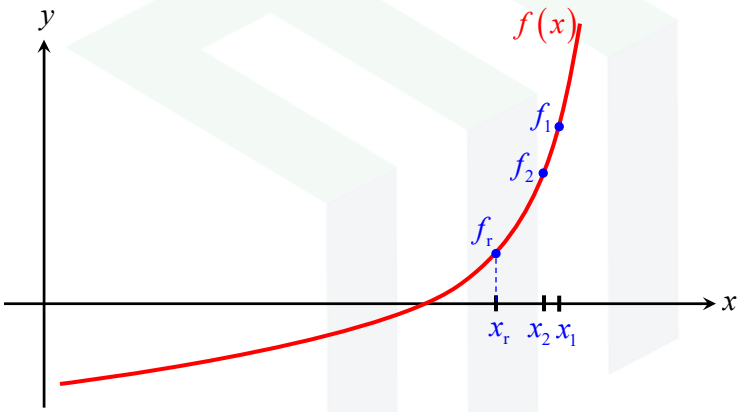
Evaluate the Function at the New Point x_r



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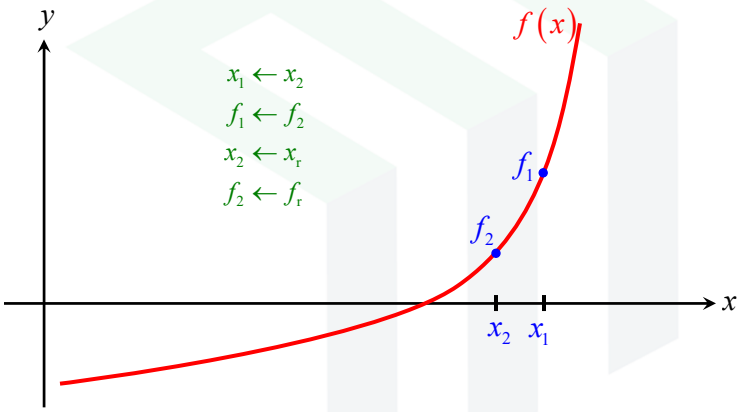
Adjust Points to x_1 and x_2



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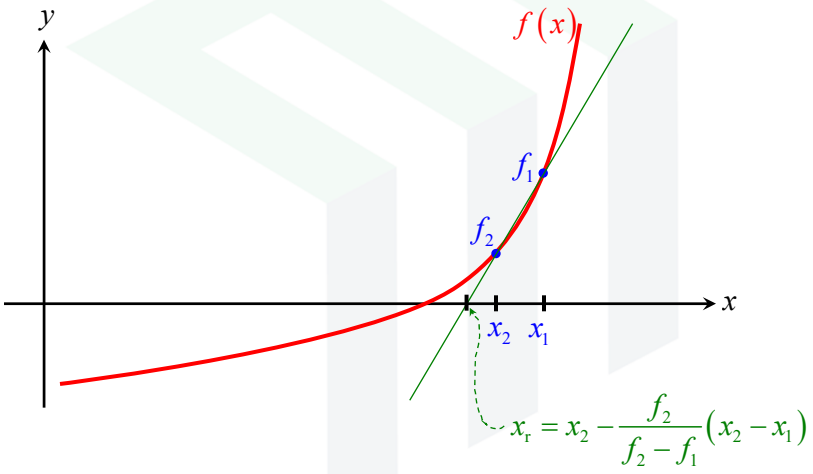
Calculate Where the New Line Crosses the x-Axis



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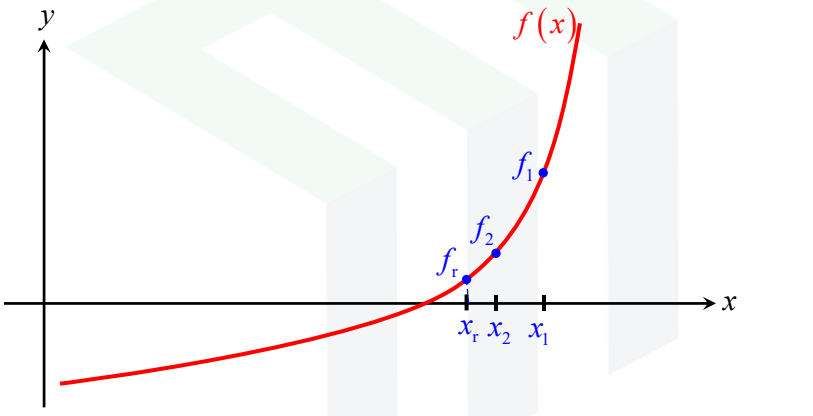
Evaluate the Function at the New Point x_r



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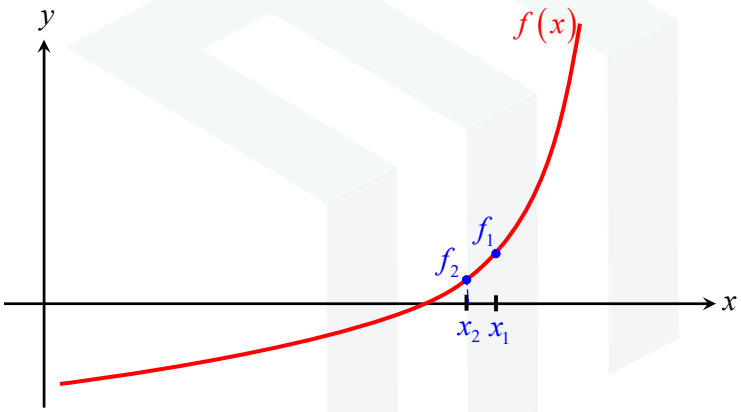
Adjust Points to x_1 and x_2



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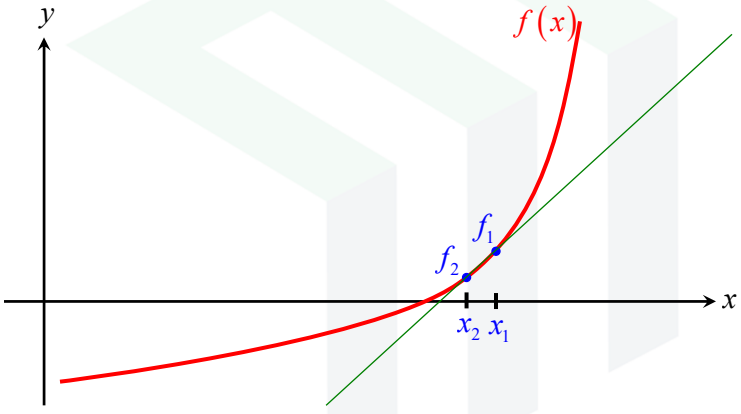
And so on…



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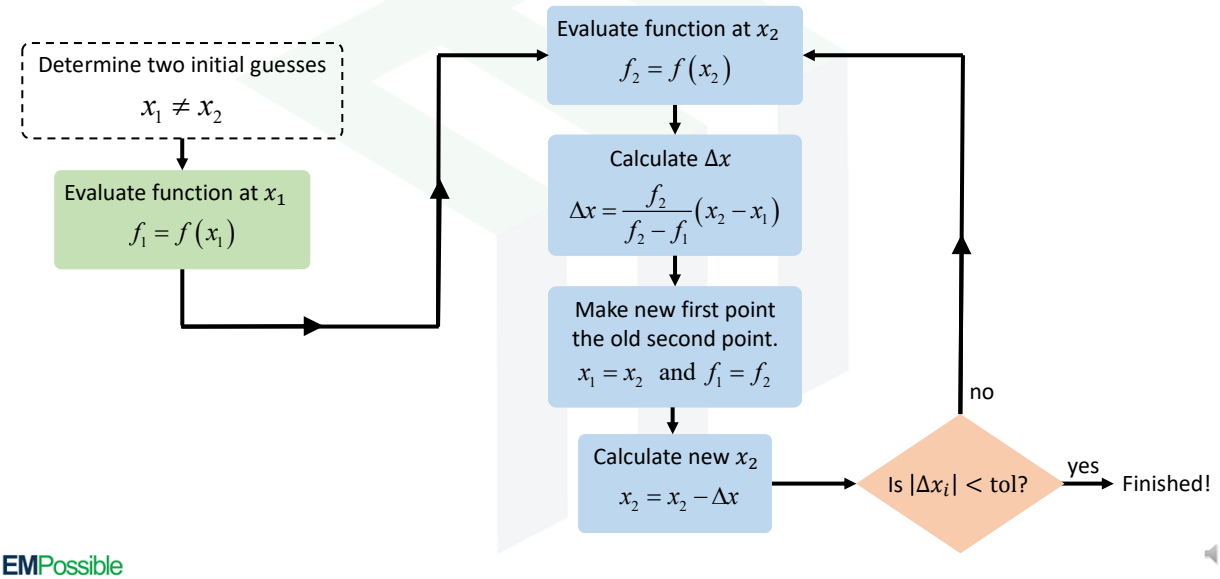
Notes on Implementation

Block Diagram of Algorithm



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Notes on Secant Method

- Does not require bounds
- Requires two "good" initial guesses
- Does not require $f(x)$ or $f'(x)$ to be analytical
- Fully numerical version of Newton-Raphson's method
- Same weaknesses as Newton-Raphson method
  - Algorithm vulnerable to instability
  - Can converge to the wrong root

Example

Example #1

Let $f(x) = \sin x$. What is the root of $f(x)$ in the proximity of $x = 4$?

1. Determine two good initial guesses: x_1=4.0 and x_2=3.9.
2. Evaluate the function at x_1.
   $f_1 = f(x_1)$
3. Iterate until converged
   a. Evaluate function at x_2.
      $f_2 = f(x_2)$
   b. Calculate Δx.
      $$\Delta x = \frac{f_2}{f_2 - f_1}(x_2 - x_1)$$
   c. Make new first point the old second point.
      $x_1 = x_2$ and $f_1 = f_2$
   d. Calculate new x_2.
      $x_2 = x_2 - \Delta x$
   e. If $|\Delta x| <$ tolerance, Done!

```
% DASHBOARD
func = @sin;
x1   = 4.0;
x2   = 3.9;
tol  = 1e-6;

% IMPLEMENT SECANT METHOD
f1 = func(x1);
dx = inf;
while abs(dx) > tol
    f2 = func(x2);
    dx = (x2 - x1)*f2/(f2 - f1);
    x1 = x2;
    f1 = f2;
    x2 = x2 - dx;
end
xr = x2;
```

Converges to 3.1416 after 5 iterations.

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