EMPossible

Electromagnetics:

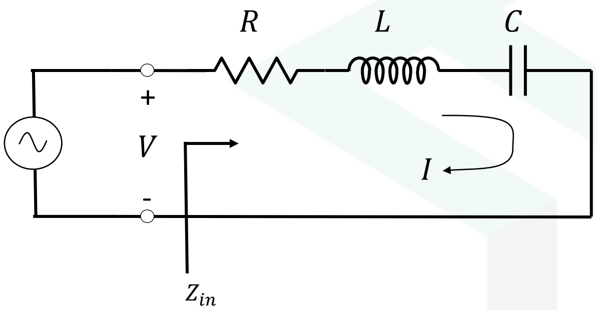
Microwave Engineering

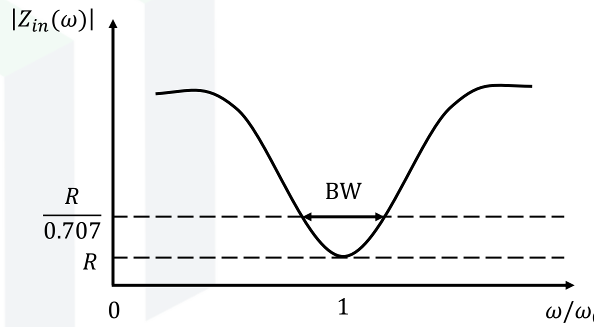
Series and Parallel Microwave Resonators

Lecture Outline

- Series and Parallel Resonant Circuits
- Quality Factor Q
- Loaded and Unloaded Q

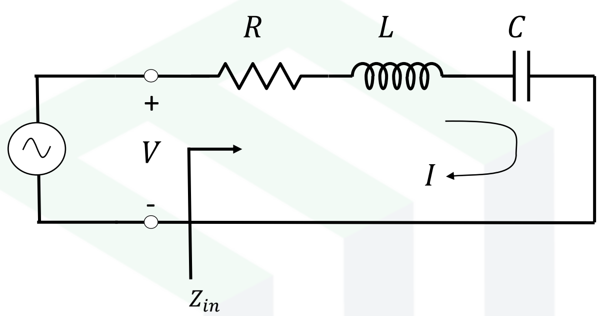
Series Resonant Circuit - Overview





$$Z_{in} = R + j\omega L + \frac{1}{j\omega C}$$

Series Resonant Circuit –Parameters



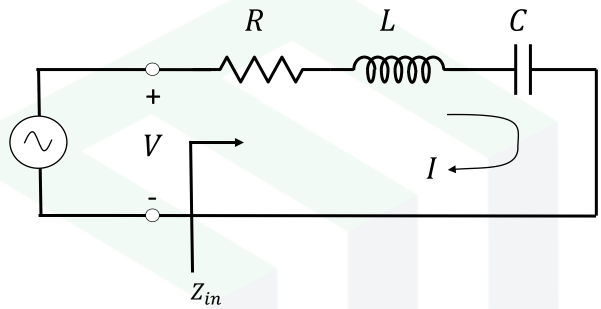
Complex power delivered to the resonator

$$P_{in} = \frac{1}{2}VI^* = \frac{1}{2}Z_{in}|I|^2 = \frac{1}{2}|I|^2\left(R + j\omega L - j\frac{1}{\omega C}\right)$$

Power dissipated by the resistor R

$$P_{loss} = \frac{1}{2}|I|^2 R$$

Series Resonant Circuit –Parameters



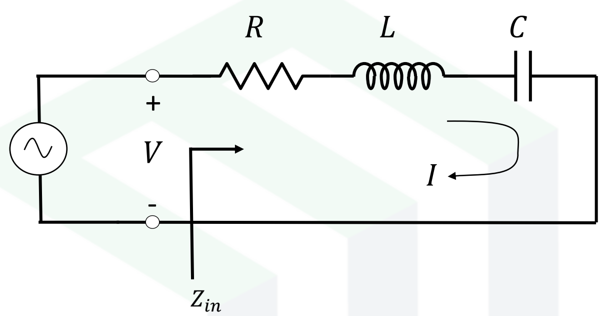
Average magnetic energy stored in the inductor

$$W_m = \frac{1}{2}|I|^2 L$$

Average electric energy stored in the capacitor

$$W_e = \frac{1}{4}|V_c|^2 C = \frac{1}{4}|I|^2 \frac{1}{\omega^2 C}$$

Series Resonant Circuit –Parameters



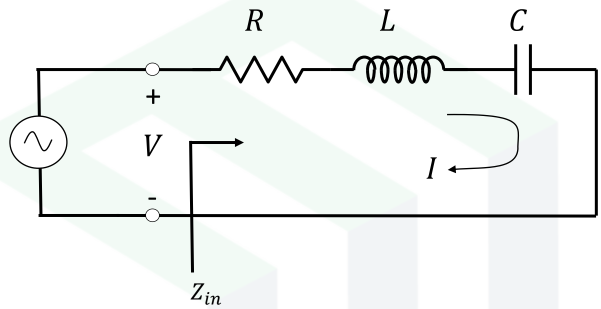
Complex power rewritten in terms of energy stored and power dissipated

$$P_{in} = P_{loss} + 2j\omega(W_m - W_e)$$

Input impedance rewritten in terms of energy stored and power dissipated

$$Z_{in} = \frac{2P_{in}}{|I|^2} = \frac{P_{loss} + 2j\omega(W_m - W_e)}{\frac{1}{2}|I|^2}$$

Series Resonant Circuit –Parameters



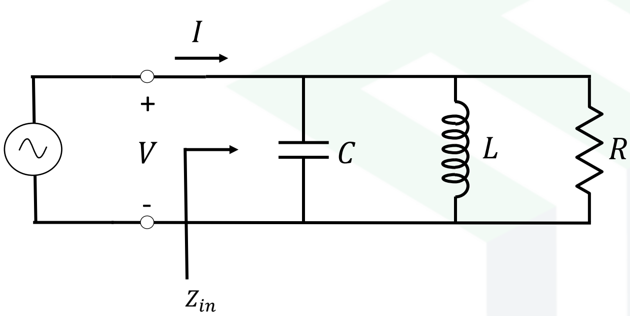
At resonance, we have that $W_m = W_e$, so Z_{in} becomes

$$Z_{in} = \frac{P_{loss}}{\frac{1}{2}|I|^2} = R$$

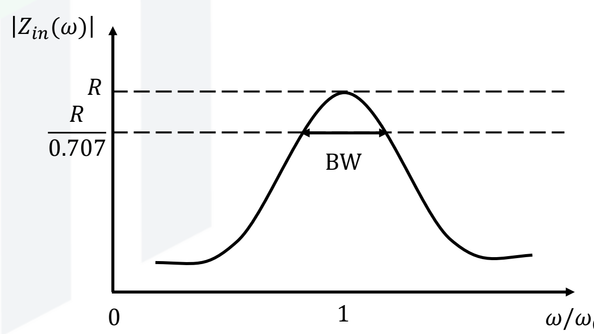
and the resonant frequency is $$\omega_0 = \frac{1}{\sqrt{LC}}$$

EMPossible 7

Parallel Resonant Circuit - Overview

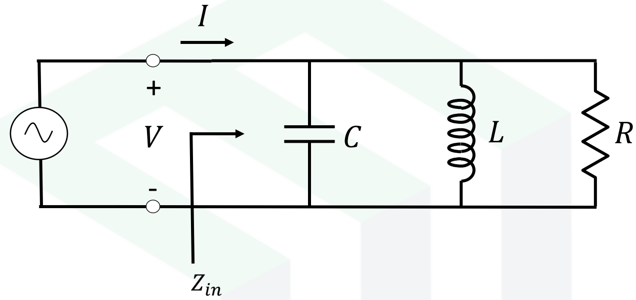


$$Z_{in} = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right)^{-1}$$



EMPossible 8

Parallel Resonant Circuit –Parameters



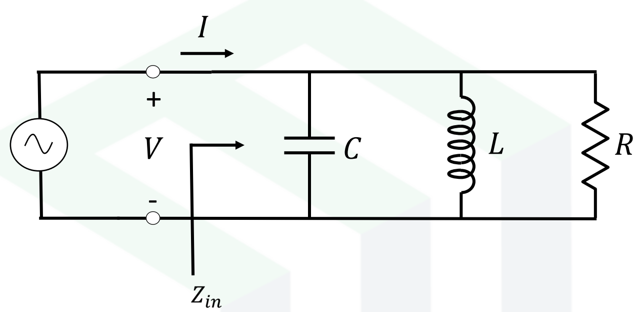
Complex power delivered to the resonator

$$P_{in} = \frac{1}{2} V I^* = \frac{1}{2} \frac{|V|^2}{Z_{in}^*} = \frac{1}{2} |V|^2 \left(\frac{1}{R} + \frac{j}{\omega L} - j\omega C \right)$$

Power dissipated by the resistor R

$$P_{loss} = \frac{1}{2} \frac{|V|^2}{R}$$

Parallel Resonant Circuit –Parameters



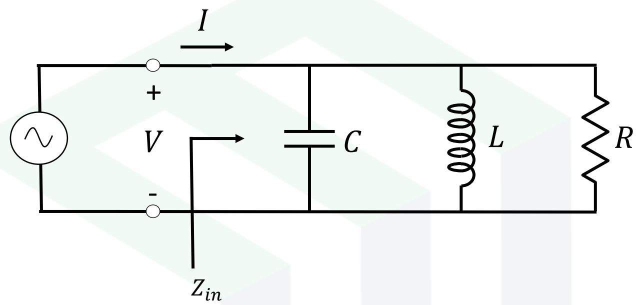
Average magnetic energy stored in the inductor

$$W_m = \frac{1}{4} |I_L|^2 L = \frac{1}{4} |V|^2 \frac{1}{\omega^2 L}$$

Average electric energy stored in the capacitor

$$W_e = \frac{1}{4} |V|^2 C$$

Parallel Resonant Circuit –Parameters



Complex power rewritten in terms of energy stored and power dissipated

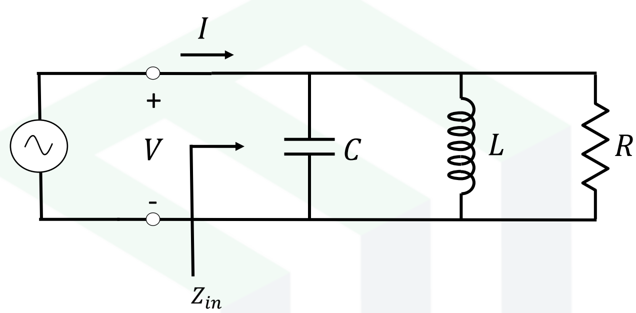
$$P_{in} = P_{loss} + 2j\omega(W_m - W_e)$$

Input impedance rewritten in terms of energy stored and power dissipated

$$Z_{in} = \frac{2P_{in}}{|I|^2} = \frac{P_{loss} + 2j\omega(W_m - W_e)}{\frac{1}{2}|I|^2}$$

EMPossible

Parallel Resonant Circuit –Parameters



At resonance, we have that $W_m = W_e$, so Z_{in} becomes

$$Z_{in} = \frac{P_{loss}}{\frac{1}{2}|I|^2} = R$$

and the resonant frequency is $$\omega_0 = \frac{1}{\sqrt{LC}}$$

Note: resonance in a parallel RLC circuit is sometimes referred to as an *antiresonance*.

EMPossible

Definition of Quality Factor Q

The quality factor Q is the ratio of energy stored in the resonator to the energy lost per cycle.

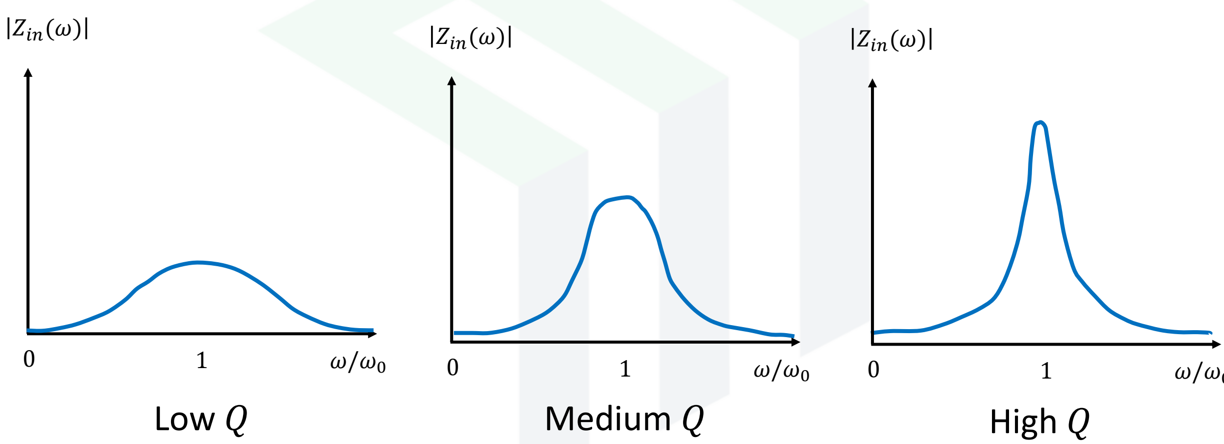
$$Q = \omega \frac{\text{average energy stored}}{\text{energy loss/second}} = \omega \frac{W_m + W_e}{P_{loss}}$$

- Losses in the resonant circuit are attributed to conductor loss, dielectric loss, radiation loss, or external network connections. Loss is represented by R.
- Lower R means lower loss -> Higher Q
- The Q of a circuit without any external loading effects is called Q_0 (unloaded Q)
- The higher the Q, the lower the bandwidth BW $\quad \text{BW} = \frac{1}{Q_0}$

EMPossible

13

Definition of Quality Factor Q



EMPossible

14

Q of Series Resonant Circuit

At resonance, we have that $W_m = W_e$, so Q_0 in a series resonant circuit becomes

$$Q_0 = \omega_0 \frac{2W_m}{P_{loss}} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$$

Q increases as R decreases

Let $\omega = \omega_0 + \Delta\omega$, where $\Delta\omega$ is small. Z_{in} now becomes

$$Z_{in} = R + j\omega L\left(1 - \frac{1}{\omega^2 LC}\right) = R + j\omega L\left(\frac{\omega^2 - \omega_0^2}{\omega^2}\right)$$

We also have that $\omega^2 - \omega_0^2 \cong 2\omega\Delta\omega$ when $\Delta\omega$ is small.

$$Z_{in} \cong R + j2L\Delta\omega \cong R + j\frac{2RQ_0\Delta\omega}{\omega_0}$$

Q of Series Resonant Circuit

A series resonator with loss can also be modeled as a lossless resonator whose resonant frequency ω_0 is replaced with a complex effective resonant frequency

$$\omega_0 \leftarrow \omega_0\left(1 + \frac{j}{2Q_0}\right)$$

This is useful because for lossy resonators, we can begin the solution for the lossless case, then replace ω_0 with the complex resonant frequency.

Q of Parallel Resonant Circuit

At resonance, we have that $W_m = W_e$, so Q_0 in a parallel resonant circuit becomes

$$Q_0 = \omega_0 \frac{2W_m}{P_{loss}} = \frac{\omega_0 R}{L} = \omega_0 RC$$

Q increases as R increases

Let $\omega = \omega_0 + \Delta\omega$, where $\Delta\omega$ is small. Z_{in} now becomes

$$Z_{in} \cong \left(\frac{1}{R} + \frac{1 - \frac{\Delta\omega}{\omega_0}}{j\omega_0 L} + jC(\omega_0 + \Delta\omega) \right)^{-1} \cong \frac{R}{1 + 2j\Delta\omega RC} \cong \frac{R}{1 + 2jQ_0 \Delta\omega/\omega_0}$$

When $R = \infty$, Z_{in} now becomes

$$Z_{in} = \frac{1}{j2C(\omega - \omega_0)}$$

EMPossible 17

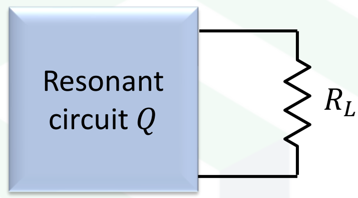
Q of Parallel Resonant Circuit

As in the series case, a parallel resonator with loss can also be modeled as a lossless resonator whose resonant frequency ω_0 is replaced with a complex effective resonant frequency

$$\omega_0 \leftarrow \omega_0 \left(1 + \frac{j}{2Q_0}\right)$$

EMPossible 18

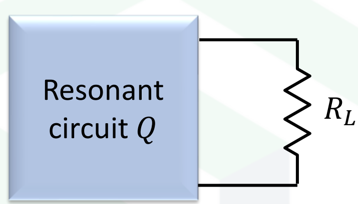
Loaded and Unloaded Q



- If the load resistance $R_L = \infty$, then $Q = Q_0$
- The series RLC resonator has Z_L connected in series $(R + R_L)$
- The parallel RLC resonator has Z_L connected in parallel $(RR_L)/(R + R_L)$

EMPossible 19

Loaded and Unloaded Q



We define an external Q, Q_e

$$Q_e = \begin{cases} \dfrac{\omega_0 L}{R_L} & \text{For series circuits} \\ \dfrac{R_L}{\omega_0 L} & \text{For parallel circuits} \end{cases}$$

And the loaded Q (Q_L) is expressed as

$$\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q_0}$$

EMPossible 20