Electromagnetics:

Microwave Engineering

Transmission Line Resonators

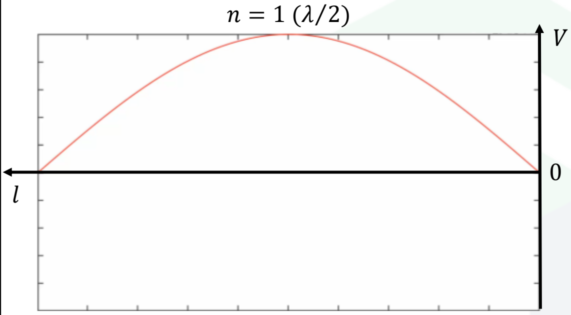
Lecture Outline

- Short-circuited $\lambda/2$ resonator
- Short-circuited $\lambda/4$ resonator
- Open-Circuited $\lambda/2$ resonator

Short-Circuited $\lambda/2$ Line Resonator

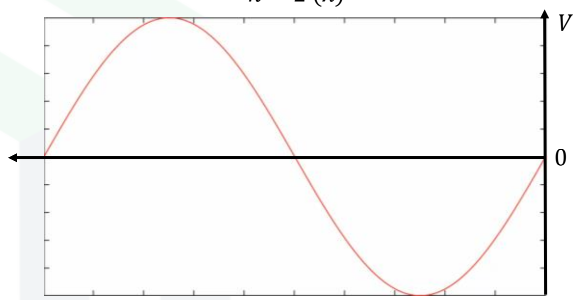
Resonant frequency $\omega = \omega_0$, the length is $l = \lambda/2$

$n = 1\ (\lambda/2)$



$$l = \frac{n\lambda}{2}$$

$n = 2\ (\lambda)$



$$Z_{in} = Z_0 \tanh[(\alpha + j\beta)l]$$

$$Z_{in} = Z_0 \frac{\tanh(\alpha l) + j\tan(\beta l)}{1 + j\tan(\beta l)\tanh(\alpha l)}$$

For a low-loss transmission line, $\alpha l \ll 1 \;\rightarrow\; \tan(\alpha l) \cong \alpha l$

Short-Circuited $\lambda/2$ Line Resonator

For frequencies near resonance, $\omega = \omega_0 + \Delta\omega$

$$\beta l = \frac{\omega l}{v_p} = \frac{\omega_0 l}{v_p} + \frac{\Delta\omega l}{v_p}$$

At the resonant frequency $\omega = \omega_0$, we have

$$l = \frac{\lambda}{2} = \pi \frac{v_p}{\omega_0}$$

$$\beta l = \pi + \frac{\Delta\omega\pi}{\omega_0}$$

$$\tan(\beta l) = \tan\left(\pi + \frac{\Delta\omega\pi}{\omega_0}\right) \cong \frac{\Delta\omega\pi}{\omega_0}$$

Short-Circuited $\lambda/2$ Line Resonator

Using this result, the input impedance Z_{in} is

$$Z_{in} \cong Z_0 \frac{\alpha l + j\left(\frac{\Delta\omega\pi}{\omega_0}\right)}{1 + j\left(\frac{\Delta\omega\pi}{\omega_0}\right)\alpha l} \cong Z_0\left(\alpha l + j\frac{\Delta\omega\pi}{\omega_0}\right)$$

This has the form of the input impedance of a series RLC resonant circuit

$$Z_{in} = R + 2jL\Delta\omega$$

Short-Circuited $\lambda/2$ Line Resonator

The RLC parameters are

$$R = Z_0 \alpha l$$

$$L = \frac{Z_0 \pi}{2\omega_0}$$

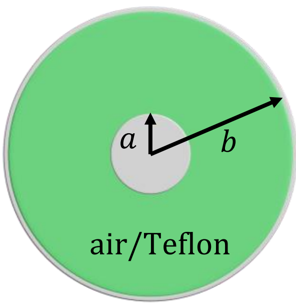
$$C = \frac{1}{\omega_0^2 L}$$

And the unloaded Q factor Q_0 is

$$Q_0 = \frac{\omega_0 L}{R} = \frac{\pi}{2\alpha l} = \frac{\beta}{2\alpha}$$

Q of $\lambda/2$ Coaxial Line Resonator

Assume a $\lambda/2$ resonator is made from a piece of copper coaxial line having an inner conductor radius of 1 mm and an outer conductor radius of 4 mm. If the resonant frequency is 5 GHz, compare the Q_0 of an air-filled coaxial line to that of a Teflon-filled coaxial line resonator.



$$a = 1\text{ mm}$$
$$b = 4\text{ mm}$$

For air:
$$\epsilon_r = 1$$
$$\tan\delta = 0$$

For Teflon:
$$\epsilon_r = 2.08$$
$$\tan\delta = 0.0004$$

<u>Answer:</u> First we calculate the attenuation of the coaxial line.

$$\sigma_{copper} = 5.813 \times 10^{7}\text{ S/m}$$

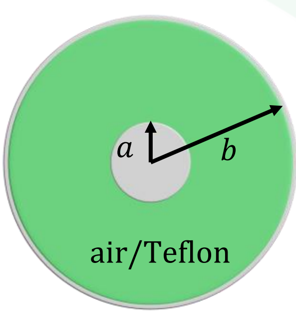
The surface resistivity at 5 GHz is

$$R_s = \sqrt{\frac{\omega\mu_0}{2\sigma_{copper}}} = 1.84 \times 10^{-2}\ \Omega$$

Q of $\lambda/2$ Coaxial Line Resonator

The attenuation due to conductor loss is given by

$$\alpha_c = \frac{R_s}{2\eta\ln(b/a)}\left(\frac{1}{a}+\frac{1}{b}\right)$$



For air, the attenuation is

$$\alpha_{c,air} = \frac{R_s}{2(377)\ln(0.004/0.001)}\left(\frac{1}{0.001}+\frac{1}{0.004}\right) = 0.022\text{ Np/m}$$

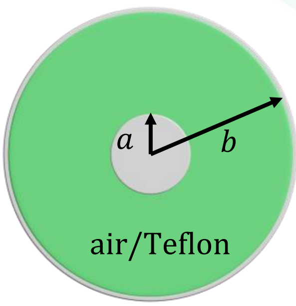
For Teflon, the attenuation is

$$\alpha_{c,Teflon} = \frac{R_s\left(\sqrt{2.08}\right)}{2(377)\ln(0.004/0.001)}\left(\frac{1}{0.001}+\frac{1}{0.004}\right) = 0.032\text{ Np/m}$$

Q of $\lambda/2$ Coaxial Line Resonator

Now we will calculate the attenuation due to dielectric loss

$$\alpha_d = k_0 \frac{\sqrt{\epsilon_r}}{2} \tan\delta$$



For air, since $\tan\delta = 0$, we have $\alpha_d = 0$

For Teflon, the attenuation due to dielectric loss is

$$\alpha_d = \frac{2\pi}{\lambda_0}\frac{\sqrt{\epsilon_r}}{2}\tan\delta = \frac{(104.7)\sqrt{2.08}\,(0.0004)}{2} = 0.030\ \text{Np/m}$$

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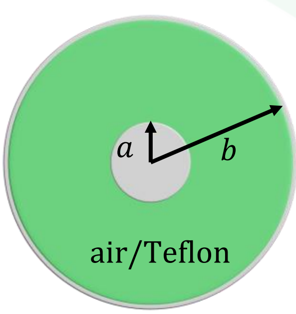
Q of $\lambda/2$ Coaxial Line Resonator

We can calculate the unloaded Qs



$$Q_{air} = \frac{\beta}{2\alpha} = \frac{104.7}{2(0.022)} = 2380$$

$$Q_{Teflon} = \frac{\beta}{2\alpha} = \frac{104.7}{2(0.032 + 0.030)} = 1218$$

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Short-Circuited $\lambda/4$ Line Resonator

Creates a parallel resonance (antiresonance). The input impedance is

$$Z_{in} = Z_o \tanh[(\alpha + j\beta)l] = Z_0 \frac{\tanh(\alpha l) + j\tan(\beta l)}{1 + j\tan(\beta l)\tan(\alpha l)}$$

$$= Z_0 \frac{1 - j\tanh(\alpha l)\cot(\beta l)}{\tanh(\alpha l) - j\cot(\beta l)}$$

For the $\lambda/4$ resonator, and at frequencies near resonance, we have

$$\beta l = \frac{\omega_0 l}{v_p} + \frac{\Delta\omega l}{v_p} = \frac{\pi}{2} + \frac{\pi\Delta\omega}{2\omega_0}$$

$$\cot(\beta l) \cong -\tan\left(\frac{\pi\Delta\omega}{2\omega_0}\right) \cong -\frac{\pi\Delta\omega}{2\omega_0}$$

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11

Short-Circuited $\lambda/4$ Line Resonator

Assuming small loss, $\tanh(\alpha l) \cong \alpha l$, so Z_{in} is

$$Z_{in} = Z_0 \frac{1 + j\alpha l\pi\Delta\omega/2\omega_0}{\alpha l + j\pi\Delta\omega/2\omega_0} = \frac{Z_0}{\alpha l + j\pi\Delta\omega/2\omega_0}$$

Which has the same form as the impedance of a parallel RLC circuit

$$Z_{in} = \frac{1}{(1/R) + 2j\Delta\omega C}$$

And the RLC parameters are

$$R = \frac{Z_0}{\alpha l} \qquad L = \frac{1}{\omega_0^2 C} \qquad C = \frac{\pi}{4\omega_0 Z_0}$$

And unloaded Q is given by $$Q_0 = \omega_0 RC = \frac{\pi}{4\alpha l} = \frac{\beta}{2\alpha}$$

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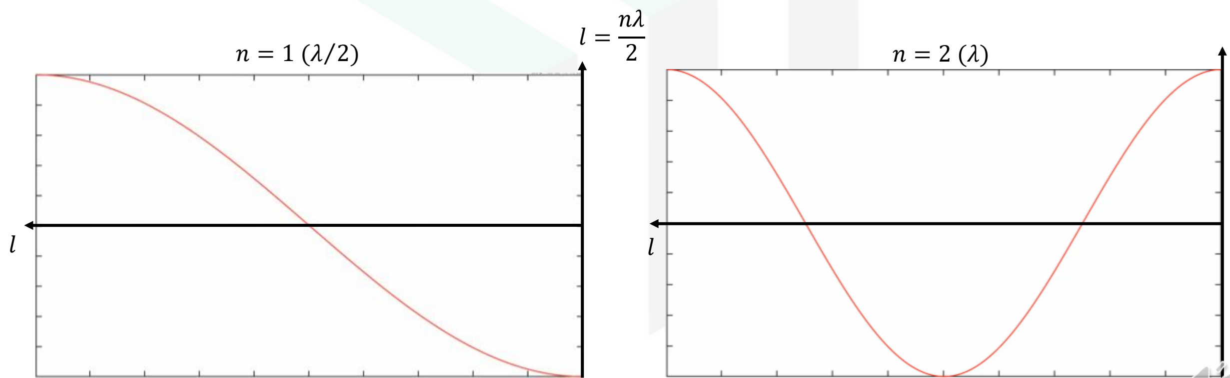
12

Open-Circuited $\lambda/2$ Line Resonator

This resonator behaves as a parallel resonant circuit at multiples of $\lambda/2$

$$Z_{in} = Z_0 \coth[(\alpha + j\beta)l] = Z_0 \frac{1 + j\tan(\beta l)\tanh(\alpha l)}{\tanh(\alpha l) + j\tan(\beta l)}$$



Open-Circuited $\lambda/2$ Line Resonator

Assume $l = \lambda/2$ at $\omega = \omega_0$, and let $\omega = \omega_0 + \Delta\omega$

$$\beta l = \pi + \frac{\pi\Delta\omega}{\omega_0} \quad \rightarrow \quad \tan(\beta l) = \tan\left(\frac{\Delta\omega\pi}{\omega}\right) \cong \frac{\Delta\omega\pi}{\omega_0}, \quad \tan(\alpha l) \cong \alpha l$$

So the input impedance Z_{in} has the same form as a parallel resonant circuit

$$Z_{in} = \frac{Z_0}{\alpha l + j\,(\Delta\omega\pi/\omega_0)}$$

Open-Circuited $\lambda/2$ Line Resonator

And the RLC parameters are

$$R = \frac{Z_0}{\alpha l} \qquad L = \frac{1}{\omega_0^2 C} \qquad C = \frac{\pi}{2\omega_0 Z_0}$$

And unloaded Q is given by

$$Q_0 = \frac{\pi}{2\alpha l} = \frac{\beta}{2\alpha}$$

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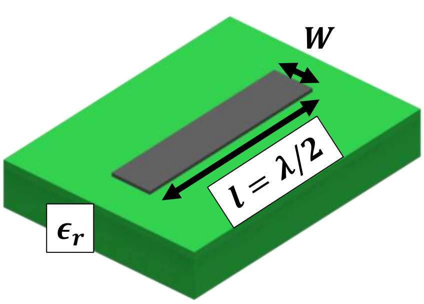
15

Q_0 of Open-Circuited $\lambda/2$ Line Resonator

A microstrip resonator is constructed from a $\lambda/2$ length of $50\ \Omega$ open-circuited mictrostrip line made of copper. The substrate is Teflon ($\epsilon_r = 2.08, \tan\delta = 0.0004$), with a thickness of 0.159 cm. Calculate the required length for resonance at 5 GHz and the unloaded Q of the resonator. Ignore fringing fields at the edge of the line.



Solution:
From Lecture 2c, "Transmission Line Examples", the width is

$$W = 0.508 \text{ cm}$$

And the effective permittivity is

$$\epsilon_{r,eff} = 1.80$$

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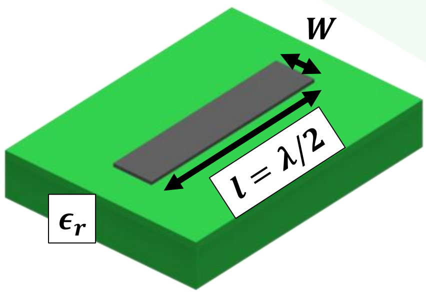
16

Q_0 of Open-Circuited $\lambda/2$ Line Resonator



The resonant length can now be calculated:

$$l = \frac{\lambda}{2} = \frac{c_0}{2f\sqrt{\epsilon_{r,eff}}} = 2.24 \text{ cm}$$

The propagation constant β is

$$\beta = \frac{2\pi f}{v_p} = \frac{2\pi f\sqrt{\epsilon_{r,eff}}}{c_0} = 151.0 \text{ rad/m}$$

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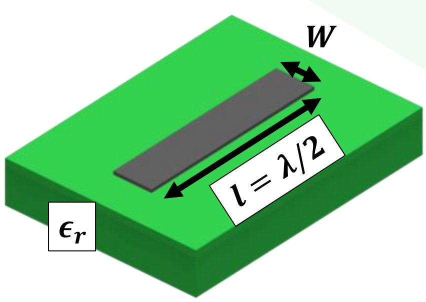
17

Q_0 of Open-Circuited $\lambda/2$ Line Resonator



The attenuation due to conductor loss in a microstrip line is

$$\alpha_c = \frac{R_s}{Z_0 W} = \frac{1.84 \times 10^{-2}}{50(0.00508)} = 0.0724 \text{ Np/m}$$

The attenuation due to the dielectric loss in a microstrip is

$$\alpha_d = \frac{k_0\epsilon_r\left(\epsilon_{r,eff} - 1\right)\tan\delta}{2\sqrt{\epsilon_{r,eff}}(\epsilon_r - 1)} = 0.024 \text{ Np/m}$$

And the unloaded Q is

$$Q = \frac{\beta}{2\alpha} = \frac{151.0}{2(0.0724 + 0.024)} = 783$$

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18