Electromagnetics:

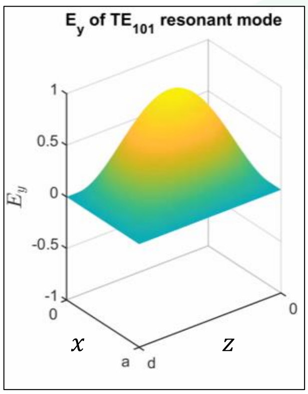
Microwave Engineering

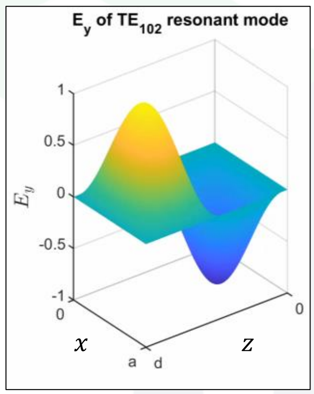
Rectangular Waveguide Cavity Resonators

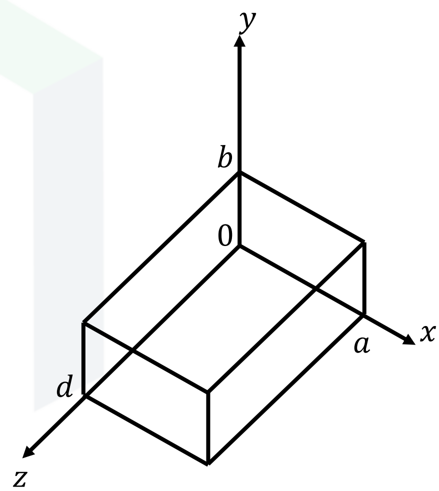
Lecture Outline

- Resonant Frequencies of Rectangular Cavity
- Design of a Rectangular Cavity Resonator

Resonant Frequencies of Rectangular Cavity







Resonant Frequencies of Rectangular Cavity

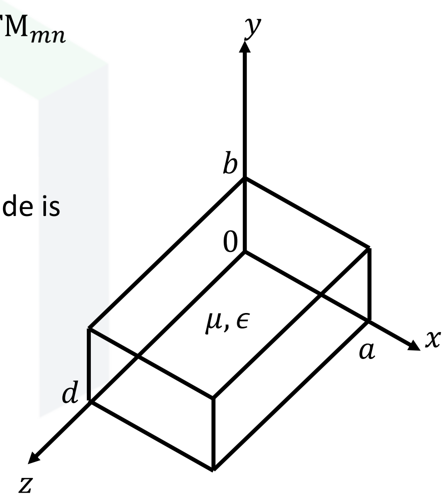
The transverse electric fields E_x, E_y of the TE_{mn} and TM_{mn} rectangular waveguide mode are

$$\vec{E}_t(x,y,z) = \vec{e}(x,y)\left(A^+ e^{-j\beta_{mn}z} + A^- e^{j\beta_{mn}z}\right)$$

The propagation constant of the m,nth TE or TM mode is

$$\beta_{mn} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$$k = \omega\sqrt{\mu\epsilon}$$



Resonant Frequencies of Rectangular Cavity

Applying the conditions for perfectly conducting walls at $z = 0$

$$\vec{E}_t = 0$$

$$A^+ = -A^-$$

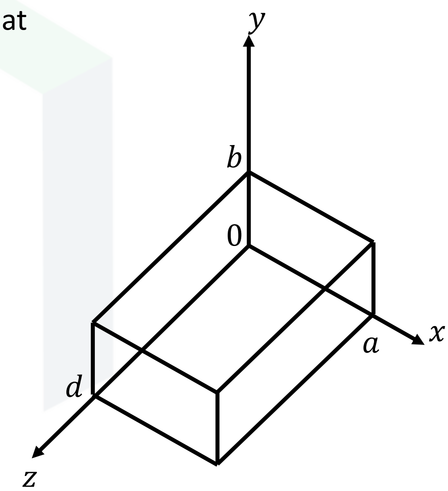
and at $z = d$,

$$\vec{E}_t(x, y, d) = -\vec{e}(x, y)A^+ 2j\sin(\beta_{mn}d) = 0$$

And the only nontrivial solution is when

$$\beta_{mn}d = l\pi, \qquad l = 1,2,3\ldots$$

the cavity must be an integer multiple of half-wavelength guide at resonant frequency.



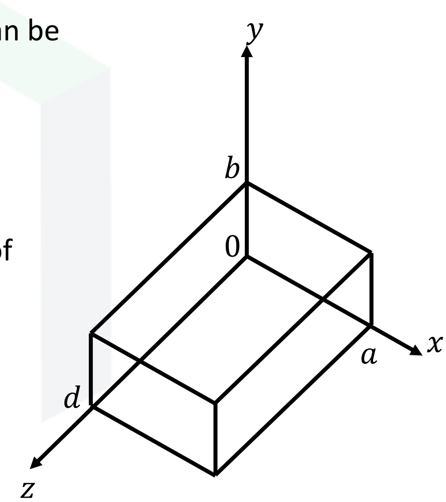
Resonant Frequencies of Rectangular Cavity

A resonance wavenumber for the rectangular cavity can be defined as

$$k_{mnl} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

We can refer to the TE_{mnl} or TM_{mnl} resonant mode of the cavity, where the standing wave pattern variations correspond to

$$m \rightarrow x - \text{direction}$$
$$n \rightarrow y - \text{direction}$$
$$l \rightarrow z - \text{direction}$$

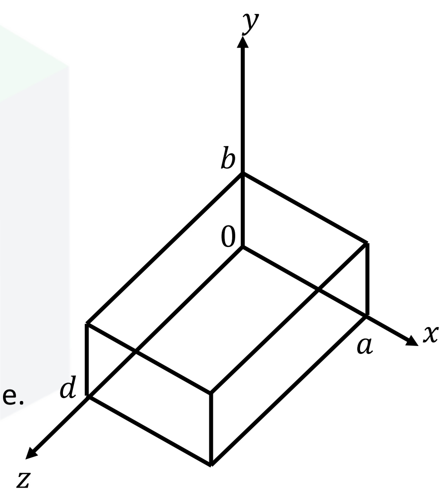


Resonant Frequencies of Rectangular Cavity

The resonant frequency is given by

$$f_{mnl} = \frac{ck_{mnl}}{2\pi\sqrt{\mu_r\epsilon_r}} = \frac{c}{2\pi\sqrt{\mu_r\epsilon_r}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

For $b < a < d$, the dominant resonant mode is the TE_{101} mode. The dominant TM resonant mode is the TM_{110} mode.



Unloaded Q of the TE_{10l} mode

The unloaded Q with lossy dielectric filling but perfectly conductor walls is

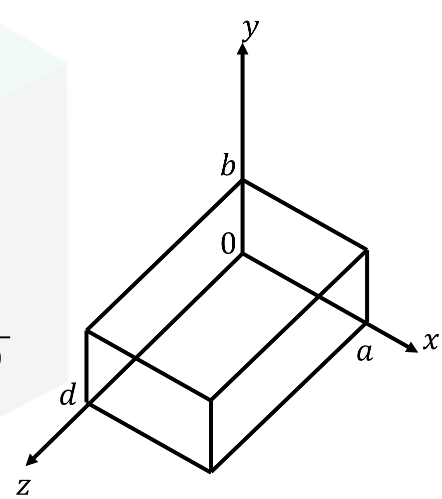
$$Q_d = \frac{2\omega W_e}{P_d} = \frac{\epsilon'}{\epsilon''} = \frac{1}{\tan\delta}$$

The unloaded Q with lossy conducting walls but lossless dielectric is

$$Q_c = \frac{2\omega_0 W_e}{P_c} = \frac{(kad)^3 b\eta}{2\pi^2 R_s}\frac{1}{(2l^2a^3b + 2bd^3 + l^2a^3d + ad^3)}$$

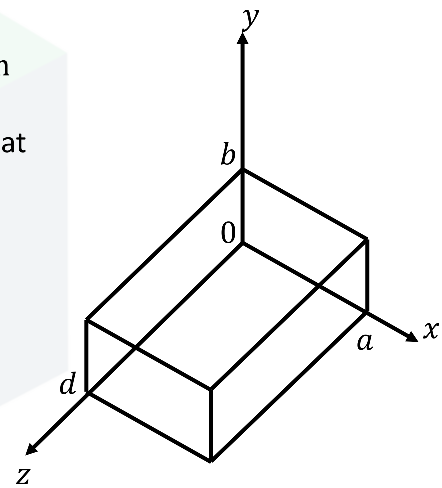
And the total unloaded Q is

$$Q_0 = \left(\frac{1}{Q_c} + \frac{1}{Q_d}\right)^{-1}$$



Design of a Rectangular Cavity Resonator

A rectangular waveguide cavity is made from a piece of copper $\mathrm{WR}-187$ H-band waveguide, with $a = 4.755$ cm and $b = 2.215$ cm. The cavity is filled with polyethylene ($\epsilon_r = 2.25, \tan\delta = 0.0004$). If the resonance is to occur at $f = 5$ GHz, find the required length d, and the resulting unloaded Q for the $l = 1$ and $l = 2$ resonant modes.



Design of a Rectangular Cavity Resonator

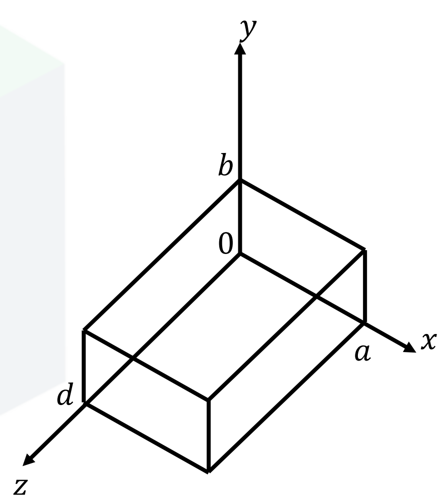
<u>Solution:</u>

We first calculate the wavenumber k at 5 GHz

$$k = \frac{2\pi f\sqrt{\epsilon_r}}{c} = 157.08/m$$

The dominant mode is TE_{101} so $m = 1, n = 0$. Then we can find the resonance for $l = 1$ and 2

$$d = \frac{l\pi}{\sqrt{k^2 - \left(\frac{\pi}{a}\right)^\wedge 2}}$$



Design of a Rectangular Cavity Resonator

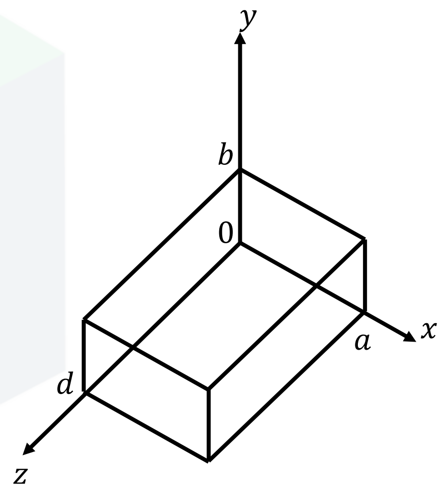
Solution:

For $l = 1$,

$$d = 2.20 \text{ cm}$$

For $l = 2$,

$$d = 4.40 \text{ cm}$$



EMPossible

Design of a Rectangular Cavity Resonator

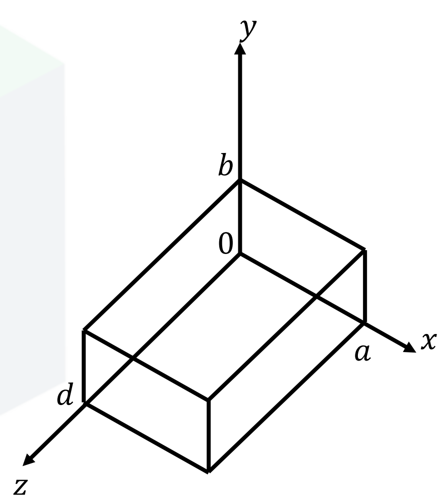
Solution:

Now we can calculate the unloaded Q. The Q due to conductor loss is given by

$$Q_c = \frac{(kad)^3 b\eta}{2\pi^2 R_s} \frac{1}{(2l^2a^3b + 2bd^3 + l^2a^3d + ad^3)}$$

Where $\eta = \frac{377}{\sqrt{\epsilon_r}} = 251.3\ \Omega$ for polyethylene.

For $l = 1$, $Q_c = 8{,}403$
For $l = 2$, $Q_c = 11{,}898$



EMPossible

Design of a Rectangular Cavity Resonator

Solution:

The Q due to dielectric loss is given by

$$Q_d = \frac{1}{\tan\delta} = 2{,}500$$

For both $l = 1$ and 2. Then the total unloaded Qs are

$$\text{For } l = 1, \quad Q_0 = \left(\frac{1}{8{,}403} + \frac{1}{2{,}500}\right)^{-1} = 1927$$

$$\text{For } l = 2, \quad Q_0 = \left(\frac{1}{11{,}898} + \frac{1}{2{,}500}\right)^{-1} = 2065$$