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Electromagnetics:

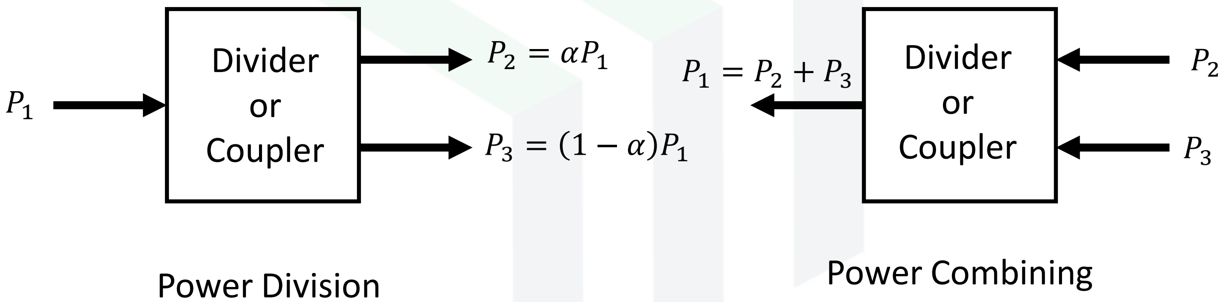
Microwave Engineering

Power Dividers and Directional Couplers

Lecture Outline

- Basic Properties of Dividers and Couplers
- Three-Port Networks
- Four-Port Networks
- Characterization of Directional Couplers
- Hybrid Couplers

Basic Properties of Dividers and Couplers



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Three-Port Networks

The simplest type of power divider is the $T - junction$, a three-port network with two inputs and one output

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

If the device is passive and isotropic, it must be reciprocal and symmetric $\left(S_{ij} = S_{ji}\right)$. It is desired to have a junction that is lossless and matched $(S_{ii} = 0)$, If the network is reciprocal and matched, we obtain

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

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Three-Port Networks

For a network to be lossless, we have the following conditions:

$$|S_{12}|^2 + |S_{13}|^2 = 1 \qquad S_{13}^* S_{23} = 0$$

$$|S_{12}|^2 + |S_{23}|^2 = 1 \qquad S_{23}^* S_{12} = 0$$

$$|S_{13}|^2 + |S_{23}|^2 = 1 \qquad S_{12}^* S_{13} = 0$$

It is impossible to satisfy all of these conditions, so a three-port network cannot be reciprocal, lossless, and matched at all ports. One of these conditions must be relaxed.

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Three-Port Networks

We can have a matched, lossless, non-reciprocal three-port network.

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

For a lossless network, $[S]$ is unitary, and the following conditions apply:

$$|S_{12}|^2 + |S_{13}|^2 = 1 \qquad S_{31}^* S_{32} = 0$$

$$|S_{21}|^2 + |S_{23}|^2 = 1 \qquad S_{21}^* S_{23} = 0$$

$$|S_{31}|^2 + |S_{32}|^2 = 1 \qquad S_{12}^* S_{13} = 0$$

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Three-Port Networks

The previous equations can be satisfied in two ways:

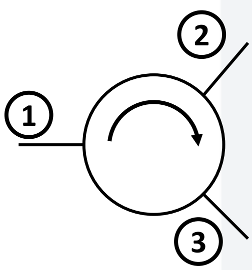
a. $S_{12} = S_{23} = S_{31} = 0, \qquad |S_{21}| = |S_{32}| = |S_{13}| = 1$

b. $S_{21} = S_{32} = S_{13} = 0, \qquad |S_{12}| = |S_{23}| = |S_{31}| = 1$

The two cases lead to two possible types of circulators:

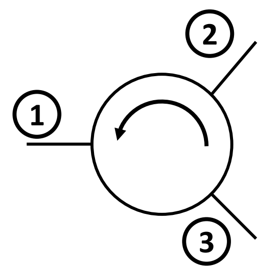
a.

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



b.

$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



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Four-Port Networks (Directional Couplers)

The scattering matrix of a reciprocal four-port network matched at all ports has the form:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

For a reciprocal and lossless network, the following conditions apply:

$$\sum_{k=1}^{N} S_{ki} S_{ki}^* = 1 \qquad \qquad \sum_{k=1}^{N} S_{ki} S_{kj}^* = 0\,, \qquad i \neq j$$

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Four-Port Networks (Directional Couplers)

Which leads to a series of 10 equations:

1 $|S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$

2 $|S_{12}|^2 + |S_{23}|^2 + |S_{24}|^2 = 1$

3 $|S_{13}|^2 + |S_{23}|^2 + |S_{34}|^2 = 1$

4 $|S_{14}|^2 + |S_{24}|^2 + |S_{34}|^2 = 1$

5 $S_{13}^* S_{23} + S_{14}^* S_{24} = 0$

6 $S_{12}^* S_{23} + S_{14}^* S_{34} = 0$

7 $S_{24}^* S_{12} + S_{34}^* S_{13} = 0$

8 $S_{13}^* S_{12} + S_{34}^* S_{24} = 0$

9 $S_{14}^* S_{12} + S_{34}^* S_{23} = 0$

10 $S_{14}^* S_{13} + S_{24}^* S_{23} = 0$

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Four-Port Networks (Directional Couplers)

We multiply eq. 5 by S_{24}^*, and eq. 10 by S_{13}^*

$$S_{24}^*(S_{13}S_{23}^* + S_{14}S_{24}^*) = 0$$

$$S_{13}^*(S_{13}S_{14}^* + S_{23}S_{24}^*) = 0$$

We subtract the equations to obtain

$$S_{14}^*(|S_{13}|^2 + |S_{24}|^2) = 0$$

Similarly, we multiply eq. 6 by S_{12} and eq. 9 by S_{34}

$$S_{12}(S_{12}^* S_{23} + S_{14}^* S_{34}) = 0$$

$$S_{34}(S_{14}^* S_{12} + S_{34}^* S_{23}) = 0$$

We subtract the equations to obtain

$$S_{23}(|S_{12}|^2 + |S_{34}|^2) = 0$$

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Four-Port Networks (Directional Couplers)

The pair of equations

$$S_{14}^*(|S_{13}|^2 + |S_{24}|^2) = 0$$

$$S_{23}(|S_{12}|^2 + |S_{34}|^2) = 0$$

can be satisfied if $S_{14} = S_{23} = 0$

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

These conditions imply that there is no coupling between ports 1 and 4 and between ports 2 and 3, which results in a directional coupler.

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Four-Port Networks (Directional Couplers)

Now the set of 10 equations reduce to

1 $|S_{12}|^2 + |S_{13}|^2 = 1$

2 $|S_{12}|^2 + |S_{24}|^2 = 1$

3 $|S_{13}|^2 + |S_{34}|^2 = 1$

4 $|S_{24}|^2 + |S_{34}|^2 = 1$

5 $S_{12}^* S_{24} + S_{13}^* S_{34} = 0$

6 $S_{12}^* S_{13} + S_{24}^* S_{34} = 0$

Eqs. 1 and 2 imply that $|S_{13}| = |S_{24}|$, and eqs. 2 and 4 imply that $|S_{12}| = |S_{34}|$

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Four-Port Networks (Directional Couplers)

We can simplify more by choosing the phase references on three of the four ports

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

$$S_{12} = S_{34} = \alpha$$
$$S_{13} = \beta e^{j\theta}$$
$$S_{24} = \beta e^{j\phi}$$

Where α and β are real, θ and ϕ are phase constants to be determined. Eq. 6 leads to

$$S_{12}^* S_{13} + S_{24}^* S_{34} = 0 \quad \Longrightarrow \quad \theta + \phi = \pi + 2n\pi$$

By letting $n = 0$, we have two choices of couplers.

Four-Port Networks (Directional Couplers)

1. *Symmetric Coupler* ($\theta = \phi = \pi/2$)

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

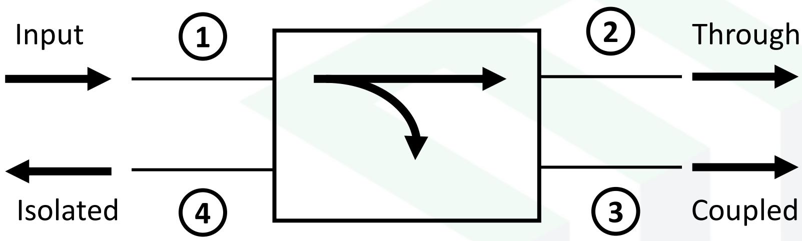
2. *Antisymmetric Coupler* ($\theta = 0, \phi = \pi$)

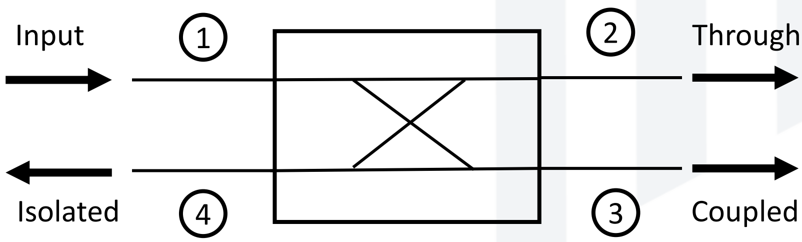
$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

These two couplers differ only in the choice of reference planes, and eq. 1 requires that

$$\alpha^2 + \beta^2 = 1$$

Four-Port Networks (Directional Couplers)





- Power supplied to Port 1 is coupled to Port 3
  $$|S_{13}|^2 = \beta^2$$
- The remainder of the input power is delivered to Port 2
  $$|S_{12}|^2 = \alpha^2 = 1 - \beta^2$$
- Ideally, no power is delivered to Port 4

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Characterization of Directional Couplers

- Coupling C
  $$C = 10\log\left(\frac{P_1}{P_3}\right) = -20\log(\beta) \quad \text{dB}$$

C —The fraction of the input power that is coupled to the output port

- Directivity D
  $$D = 10\log\left(\frac{P_3}{P_4}\right) = 20\log\left(\frac{\beta}{|S_{14}|}\right) \quad \text{dB}$$

D — Coupler's ability to isolate coupled and uncoupled ports

- Isolation I
  $$I = 10\log\left(\frac{P_1}{P_4}\right) = -20\log(|S_{14}|) \quad \text{dB}$$

I — Power delivered to the uncoupled port ($I = D + C$)

- Insertion Loss L
  $$L = 10\log\left(\frac{P_1}{P_2}\right) = -20\log(|S_{12}|) \quad \text{dB}$$

L —Input power delivered to the through port, diminished by power delivered to the coupled and isolated ports

Note: for ideal couplers, $S_{14} = 0, D = \infty$

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Hybrid Couplers

Hybrid Couplers are a special type of couplers where the coupling factor C is $3\ \text{dB}$

$$\left(\alpha = \beta = 1/\sqrt{2}\right)$$

- Quadrature Hybrid: A symmetric coupler that has a $90°$ phase shift between ports 2 and 3 $(\theta = \phi = \pi/2)$ when fed at port 1

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

- Magic-T and Rat-Race: An antisymmetric coupler that has a $180°$ phase shift between ports 2 and 3 when fed at port 4

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

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