Electromagnetics:

Microwave Engineering

The Wilkinson Power Divider

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Lecture Outline

- Basic Properties of Wilkinson Power Dividers
- Normalized and Symmetric Wilkinson Power Divider
- Even-Mode Analysis
- Odd-Mode Analysis
- Scattering Parameters for Wilkinson Power Divider
- Unequal Power Division in Wilkinson Divider
- Example – Design of a Wilkinson Power Divider

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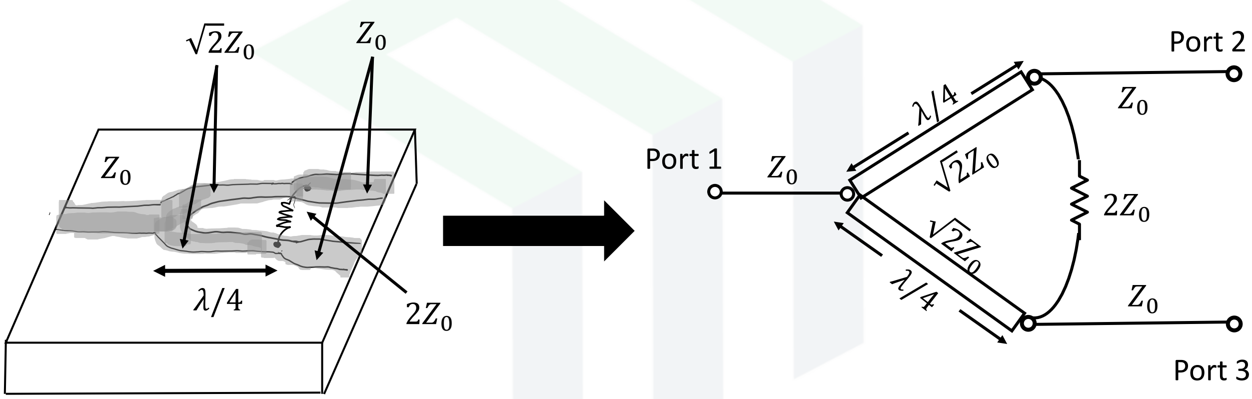
Basic Properties of Wilkinson Power Dividers

- The Wilkinson Power Divider is a lossy three-port network that has all port matched, and the ports are isolated
- The Wilkinson Power Divider was developed by Ernest J. Wilkinson in 1960
- It splits an input signal into two equal output signals in phase
- It can combine two inputs constructively to a single output
- If the output ports are matched, the Wilkinson Power Divider will appear lossless

Basic Properties of Wilkinson Power Dividers

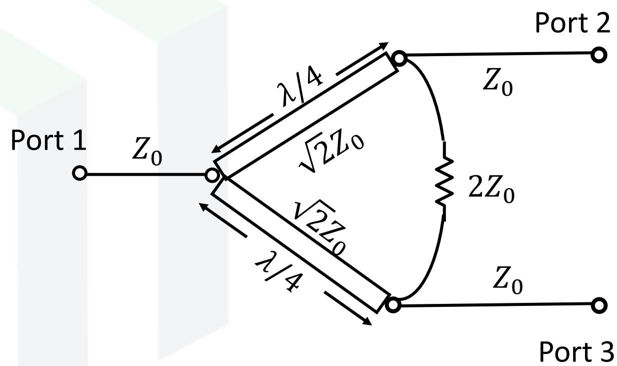
Equal-Split Wilkinson Power Divider



Basic Properties of Wilkinson Power Dividers

How to obtain S-parameters of the Wilkinson Power Divider?

- Taking advantage of the symmetry of the circuit, we can redraw the power divider in normalized and symmetric form.
- This will allow to analyze the circuit by simplifying to two simpler circuits driven by symmetric and antisymmetric sources (Even- and Odd-Mode Analysis).
- All impedances will be normalized to the characteristic impedance Z_0.



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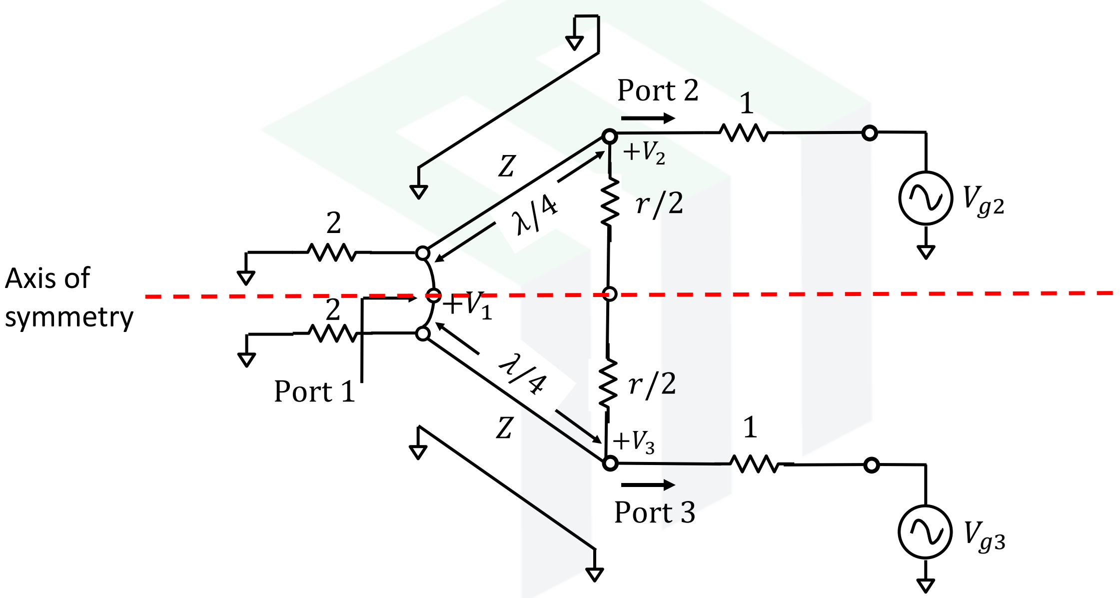
Normalized and Symmetric Wilkinson Power Divider



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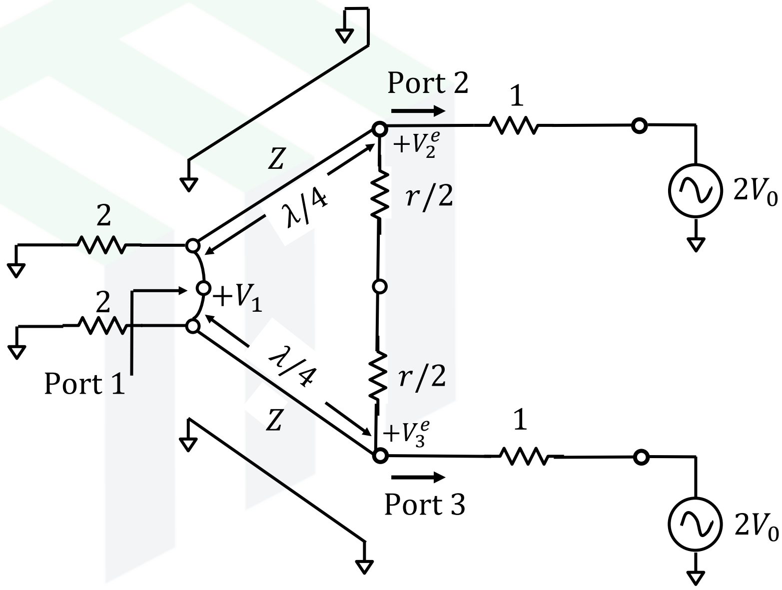
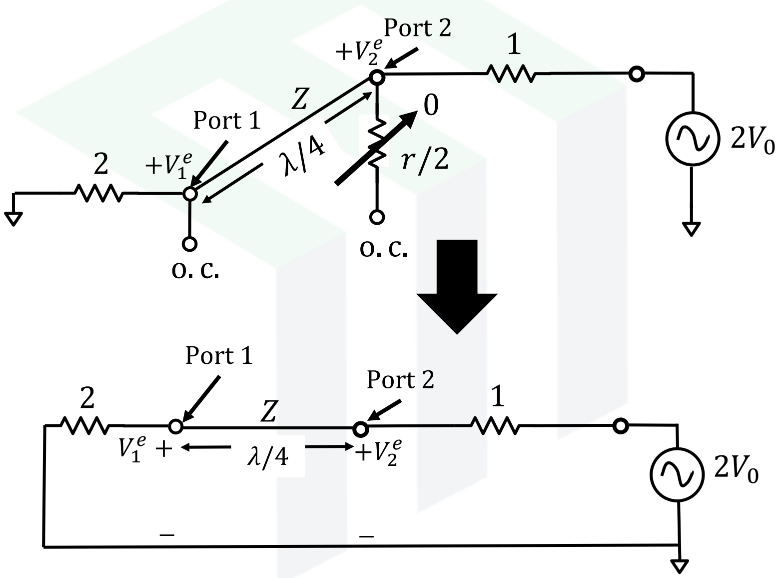
Even-Mode Analysis ($V_{g2} = V_{g3} = 2V_0$)

- $V_2^e = V_3^e$
- No current goes through $r/2$ resistors
- The circuit can be separated at the middle and replace the bisections with open circuits



Even-Mode Analysis ($V_{g2} = V_{g3} = 2V_0$)



Even-Mode Analysis ($V_{g2} = V_{g3} = 2V_0$)

- The input impedance at Port 2 with the $\lambda/4$ T.L. and the 2 resistor

$$Z_{in}\left(-\frac{\lambda}{4}\right) = \frac{Z^2}{Z_L} = \frac{Z^2}{2}$$

- For port 2 to be matched,

$$Z = \sqrt{2}$$

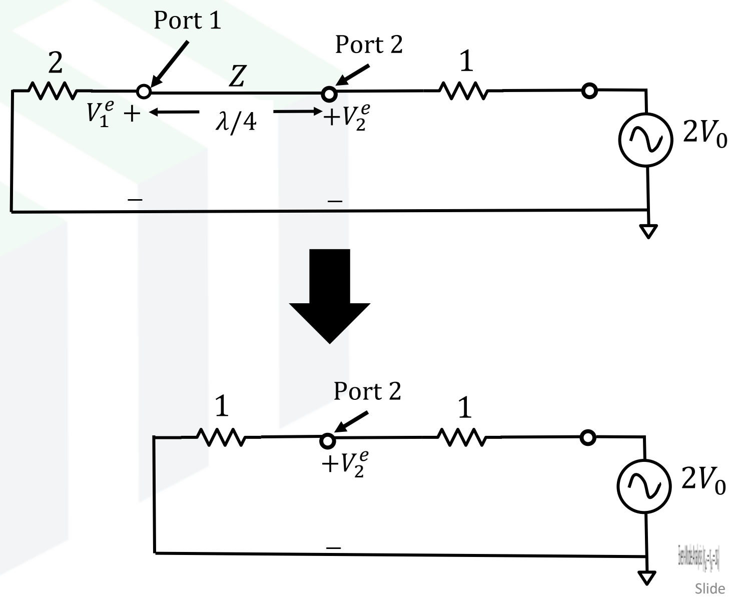
And $Z_{in} = 1$

- Now we obtain

$$\boxed{V_2^e = V_0}$$

And by even symmetry,

$$\boxed{V_3^e = V_0}$$



Even-Mode Analysis ($V_{g2} = V_{g3} = 2V_0$)

- To find the voltage at Port 1,

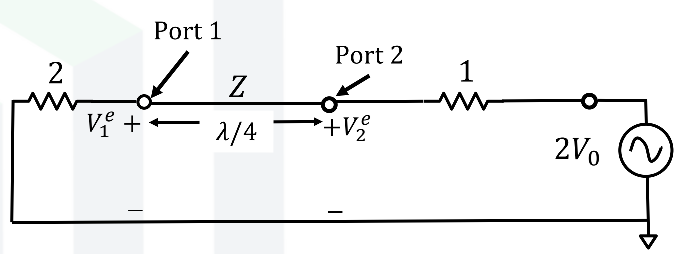
$x = 0$ at Port 1
$x = -\lambda/4$ at Port 2

The expression for voltage is

$$V(x) = V^+\left(e^{-j\beta x} + \Gamma e^{j\beta x}\right)$$
$$V_2^e = V(-\lambda/4) = jV^+(1 - \Gamma) = V_0$$
$$V_1^e = V(0) = V^+(1 + \Gamma) = jV_0\frac{\Gamma + 1}{\Gamma - 1}$$

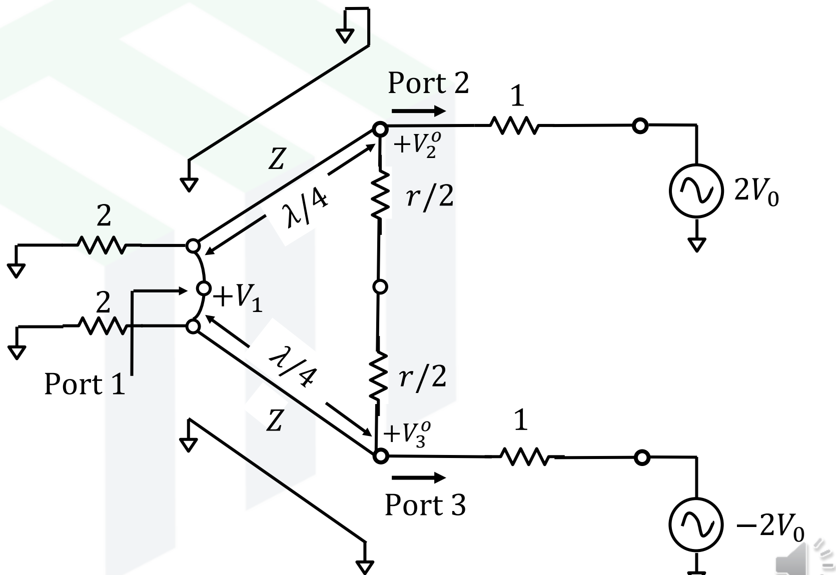
$$\Gamma = \frac{2 - \sqrt{2}}{2 + \sqrt{2}}$$

Then we obtain $\boxed{V_1^e = -jV_0\sqrt{2}}$



Odd-Mode Analysis ($V_{g2} = -V_{g3} = 2V_0$)

- $V_2^o = -V_3^o$
- A voltage null is present along the middle of the circuit
- The circuit can be separated at the middle and replace the bisections with ground



Odd-Mode Analysis ($V_{g2} = -V_{g3} = 2V_0$)

- The voltage at Port 2 is the voltage present at the $r/2$ resistor
- To match Port 2 for odd-mode $r = 2$.
- We obtain

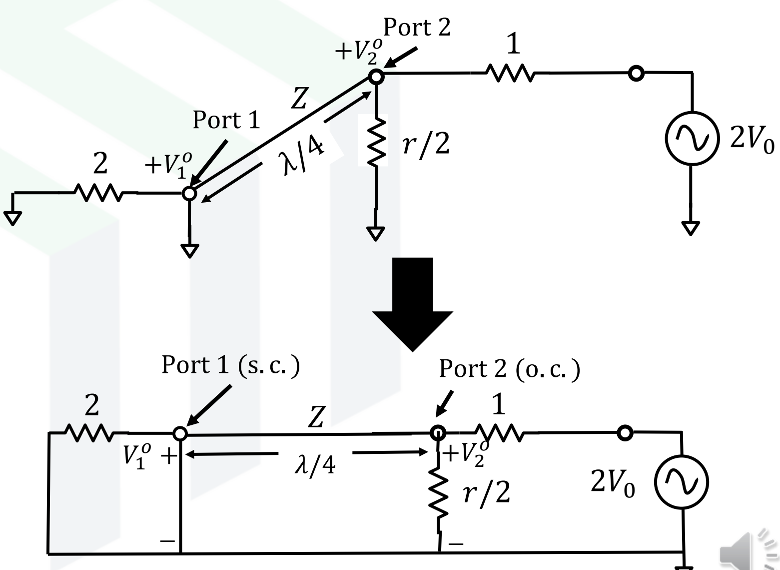
$$\boxed{V_2^o = V_0}$$

and by odd symmetry

$$\boxed{V_3^o = -V_0}$$

- Since Port 1 is short-circuited,

$$\boxed{V_1^o = 0}$$



Scattering Parameters for Wilkinson Power Divider

Now with superposition the total voltage can be obtained

Total Voltage at Port 1:

$$V_1^t = V_1^e + V_1^o = -jV_0\sqrt{2}$$

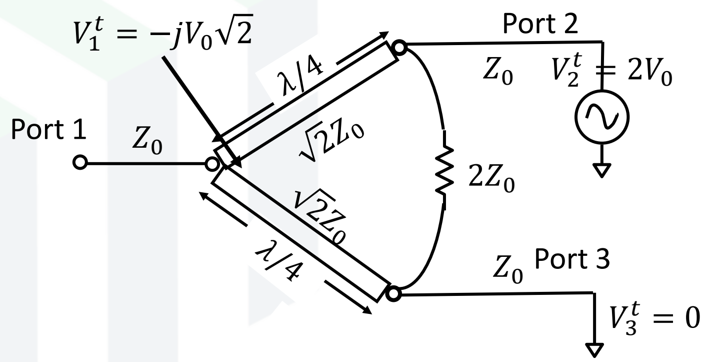
Total Voltage at Port 2:

$$V_2^t = V_2^e + V_2^o = 2V_0$$

Total Voltage at Port 3:

$$V_3^t = V_3^e + V_3^o = 0$$

The total voltages can correspond to a matched source $2V_0$ at Port 2, and Ports 1 and 3 also matched.



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Scattering Parameters for Wilkinson Power Divider

The incident and reflected voltages are then

Ports	V^+	V^-
Port 1	$V_1^+ = 0$	$V_1^- = -jV_0\sqrt{2}$
Port 2	$V_2^+ = 2V_0$	$V_2^- = 0$
Port 3	$V_3^+ = 0$	$V_3^- = 0$

And the scattering parameters can be obtained:

$$S_{12} = \frac{V_1^-}{V_2^+} = \frac{-jV_0\sqrt{2}}{2V_0} = -\frac{j}{\sqrt{2}} = S_{21} \quad \text{(symmetry due to reciprocity)}$$

$$S_{13} = S_{31} = -\frac{j}{\sqrt{2}} \quad \text{(symmetry of ports 2 and 3)}$$

$$S_{23} = \frac{V_2^-}{V_3^+} = 0 = S_{32} \quad \text{(ports are isolated due to short or open circuit at bisection)}$$

$$S_{22} = \frac{V_2^-}{V_2^+} = 0 = S_{33} \quad \text{(symmetry of ports 2 and 3)}$$

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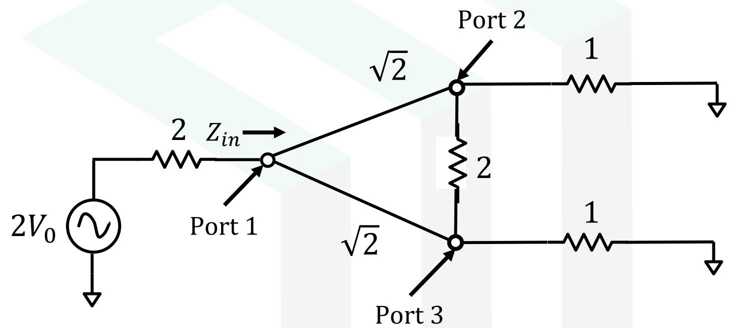
Scattering Parameters for Wilkinson Power Divider

To find S_{11}, we drive Port 1 with voltage source $2V_0$ with matched loads at Ports 2 and 3 and find the normalized input impedance Z_{in}:



The input impedance is then

$$Z_{in} = \frac{(\sqrt{2})^2}{2} = 1$$

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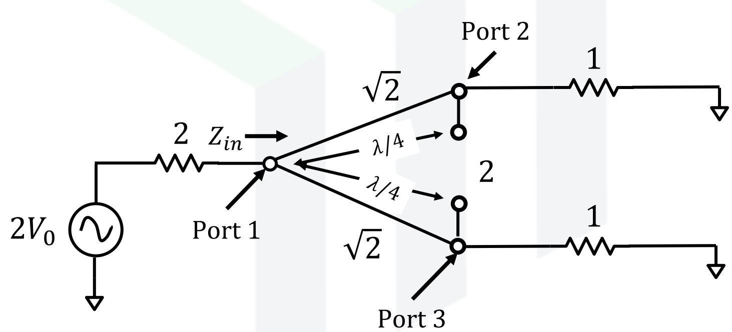
Scattering Parameters for Wilkinson Power Divider

Since $Z_{in} = 1$, the source is matched, and we have:

$$S_{11} = 0$$

Furthermore, the voltage is split equally and with the same phase at Ports 2 and 3, so no power is dissipated in the resistor, making the divider lossless when the outputs are matched.



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Scattering Parameters for Wilkinson Power Divider

Putting everything together, the scattering matrix for the Wilkinson Power Divider is:

$$[S] = \frac{-j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

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Unequal Power Division in Wilkinson Divider

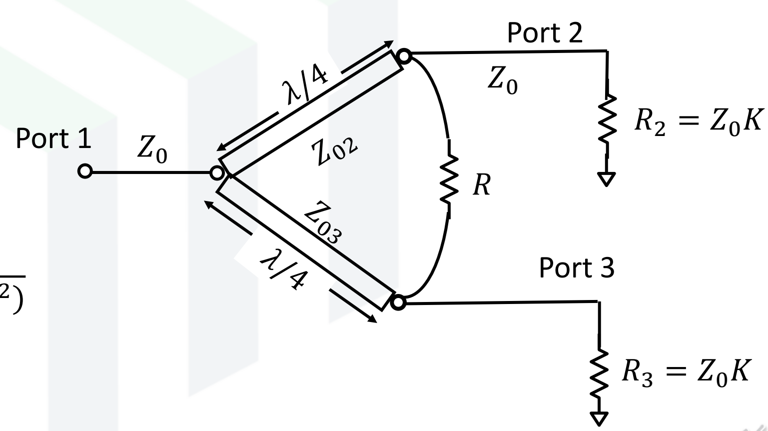
Wilkinson power dividers can also be made with unequal power splits. Considering the power ratio between ports 2 and 3,

$$K^2 = P_3/P_2$$

$$Z_{03} = Z_0\sqrt{\frac{1+K^2}{K^3}}$$

$$Z_{02} = K^2 Z_{03} = Z_0\sqrt{K(1+K^2)}$$

$$R = Z_0\left(K + \frac{1}{K}\right)$$



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Example – Design of a Wilkinson Power Divider

Design an equal-split Wilkinson Power Divider for a $50\ \Omega$ system impedance at a frequency f_0.

Solution:

The quarter-wave transmission lines in the divider have a characteristic impedance of

$$Z = \sqrt{2}Z_0 = 70.7\ \Omega$$

The shunt resistor value is

$$R = 2Z_0 = 100\ \Omega$$

And the transmission lines are $\lambda/4$ long at frequency f_0