Electromagnetics:

Microwave Engineering

The Quadrature (90°) Hybrid Directional Coupler

Slide 1

Lecture Outline

- Properties of the Quadrature (90°) Hybrid
- Normalized and Symmetric Form
- Even- and Odd- Mode Analysis
- Transmission and Reflection Coefficients
- Characteristics of Quadrature Hybrid Coupler
- Example – Design of a Quadrature Hybrid

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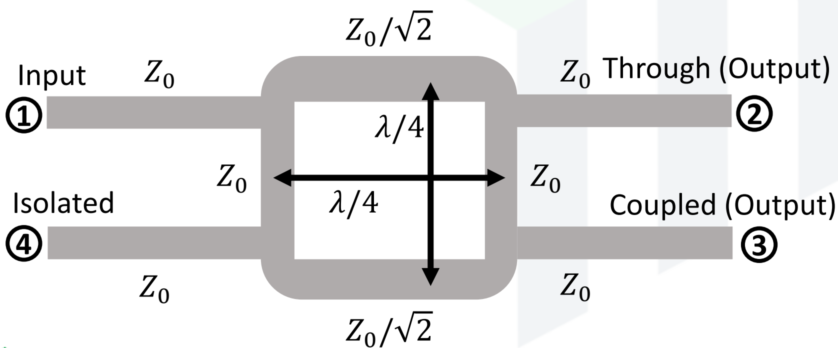
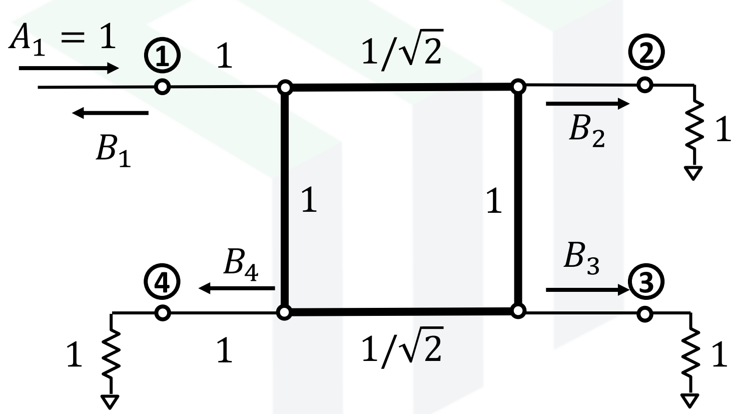
Properties of the Quadrature (90°) Hybrid

- The Quadrature Hybrid is a 3 dB directional coupler with a 90° phase difference in the outputs of the through and coupled arms.
- It is also called branch-line hybrid.
- Due to the high symmetry of the coupler, any port can be used as input port, and even-odd analysis can be performed

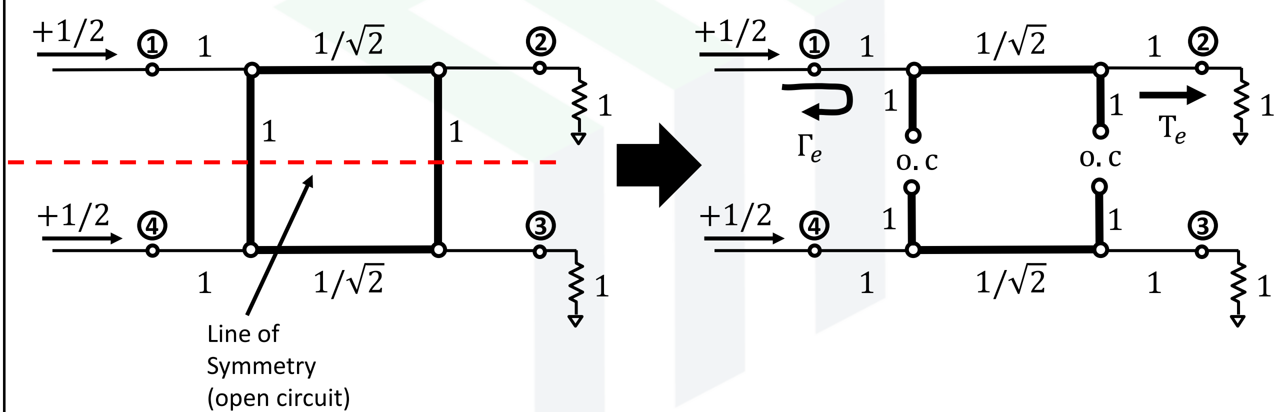


$$[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

Normalized and symmetric form of Quadrature Hybrid



Even-Mode of Quadrature Hybrid



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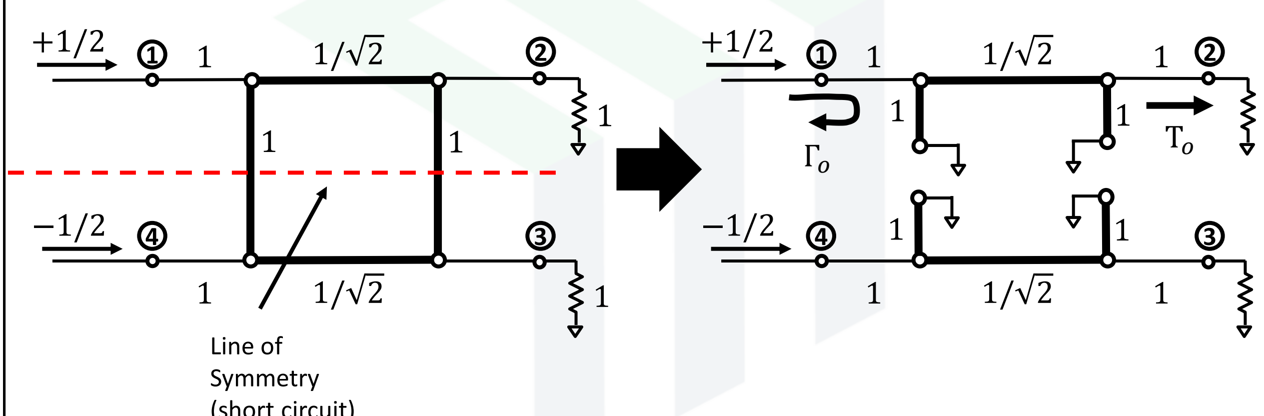
Odd-Mode of Quadrature Hybrid



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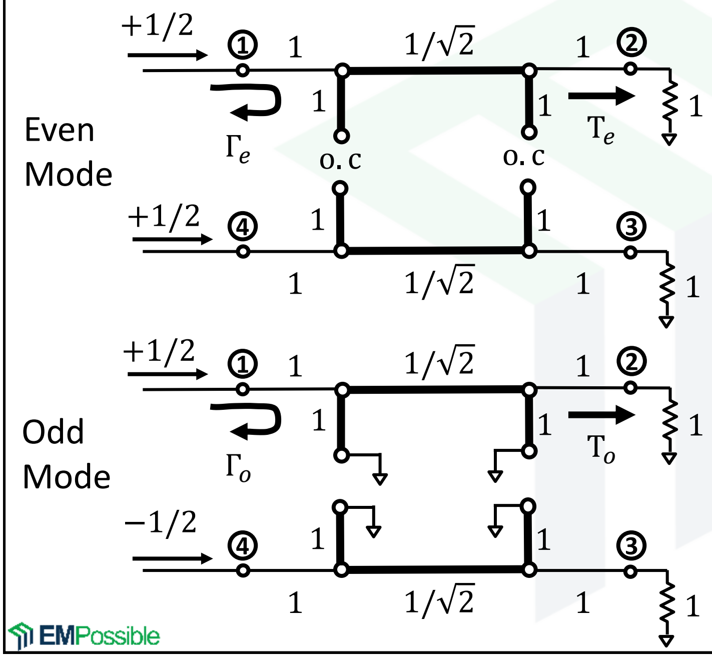
Even-Odd Mode of Quadrature Hybrid



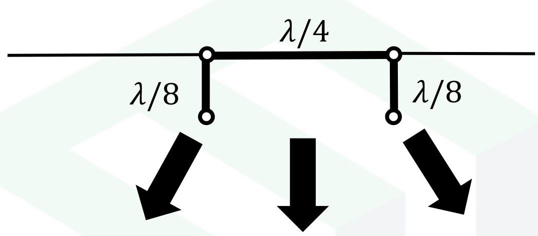
$$B_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o$$

$$B_2 = \frac{1}{2}\mathrm{T}_e + \frac{1}{2}\mathrm{T}_o$$

$$B_3 = \frac{1}{2}\mathrm{T}_e - \frac{1}{2}\mathrm{T}_o$$

$$B_4 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o$$

Transmission and Reflection Coefficients – Even Mode



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{even} = \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix} \begin{bmatrix} 0 & j/\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & j \\ j & -1 \end{bmatrix}$$

Open Circuit Shunt $\lambda/8$ T.L.
$Y = j\tan(\beta l) = j$

$\lambda/4$ T.L.

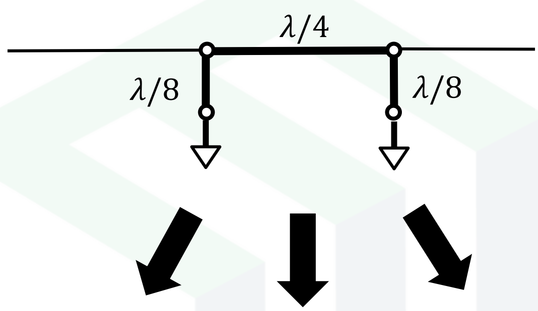
Open Circuit Shunt $\lambda/8$ T.L.
$Y = j\tan(\beta l) = j$

Transmission and Reflection Coefficients – Even Mode

$$\Gamma_e = \frac{A+B-C-D}{A+B+C+D} = \frac{(1+j-j+1)/\sqrt{2}}{(1+j+j-1)/\sqrt{2}} = 0$$

$$T_e = \frac{2}{A+B+C+D} = \frac{2}{(1+j+j-1)/\sqrt{2}} = -\frac{1}{\sqrt{2}}(1+j)$$

Transmission and Reflection Coefficients - Odd Mode



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{odd} = \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix} \begin{bmatrix} 0 & j/\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$$

Short-Circuit Shunt $\lambda/8$ T.L.
$Y = -j\cot(\beta l) = -j$

$\lambda/4$ T.L.

Short-Circuit Shunt $\lambda/8$ T.L.
$Y = -j\cot(\beta l) = -j$

Transmission and Reflection Coefficients – Odd Mode

$$\Gamma_o = \frac{A+B-C-D}{A+B+C+D} = \frac{(1+j-j-1)/\sqrt{2}}{(1+j+j+1)/\sqrt{2}} = 0$$

$$T_o = \frac{2}{A+B+C+D} = \frac{2}{(1+j+j+1)/\sqrt{2}} = \frac{1}{\sqrt{2}}(1-j)$$

Characteristics of Quadrature Hybrid Coupler

$$B_1 = \frac{1}{2}(0) + \frac{1}{2}(0) = 0$$

Port 1 is matched

$$B_2 = \frac{1}{2}\left(-\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) + \frac{1}{2}\left(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) = -\frac{j}{\sqrt{2}}$$

Half of power with $-90°$ phase shift is transmitted from Port 1 to Port 2

$$B_3 = \frac{1}{2}\left(-\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) - \frac{1}{2}\left(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}$$

Half of power with $-180°$ phase shift is transmitted from Port 1 to Port 3

$$B_4 = \frac{1}{2}(0) - \frac{1}{2}(0) = 0$$

No power to Port 4 (isolated)

Example – Design of a Quadrature Hybrid

Design a $50\ \Omega$ branch-line quadrature hybrid junction at a frequency f_0.

Solution:

The transmission lines will be $\lambda/4$ long at the design frequency f_0. The branch-line impedances are

$$\frac{Z_0}{\sqrt{2}} = \frac{50}{\sqrt{2}} = 35.4\ \Omega$$

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