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Electromagnetics:
Microwave Engineering

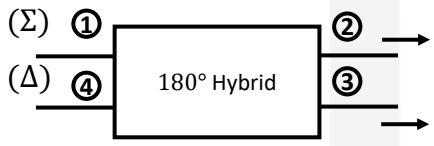
The 180° Hybrid Directional Coupler

Lecture Outline

- Properties of the 180° Hybrid
- Even- and Odd- Modes of the Ring Hybrid – Ports 1 and 3
- Ring Hybrid Coefficients: Even- and Odd- Mode Ports 1 and 3
- Even- and Odd- Modes of the 180° Hybrid – Ports 4 and 2
- Ring Hybrid Coefficients: Even- and Odd- Mode Ports 4 and 2
- Example – Design of a Ring Hybrid

Properties of the 180° Hybrid

- The 180° Hybrid junction is a four-port network with a 180° phase shift between the two outputs.
- A signal applied to Port 1 will be evenly split in two in-phase components at Ports 2 and 3, and Port 4 is isolated
- A signal applied to Port 4 will be evenly split in two out-of-phase components at Ports 2 and 3, and Port 1 is isolated
- If a signal applied to Ports 2 and 3, it operates as a combiner. The sum of the inputs will be present at Port 1 (Σ, sum port), and the difference will be present in Port 4 (Δ, difference port)



$$[S] = \frac{-j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

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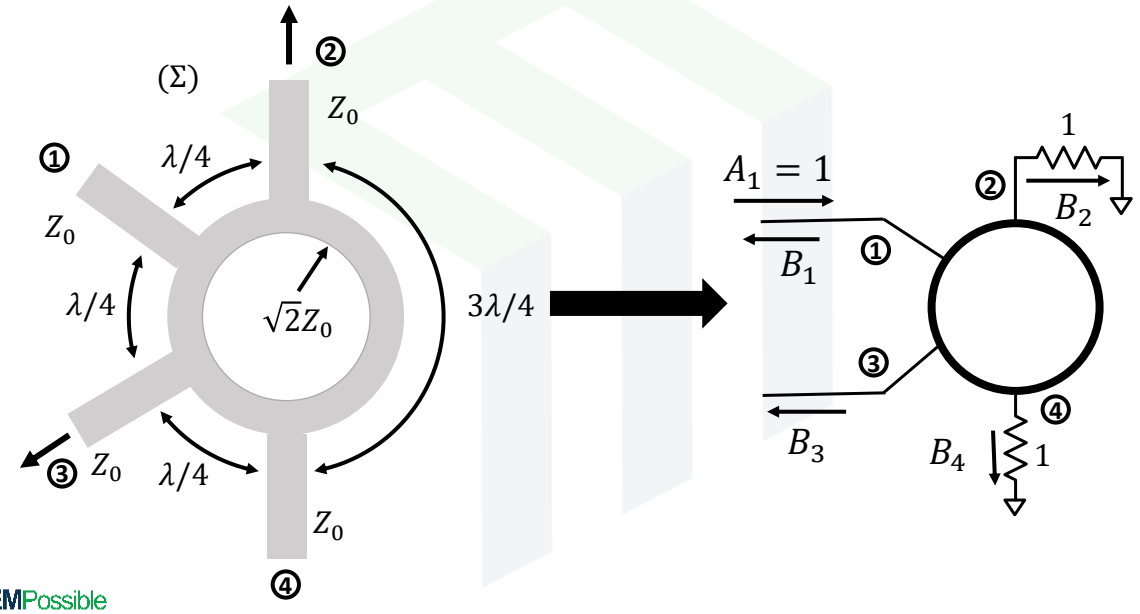
The Ring (Rat-Race) Hybrid



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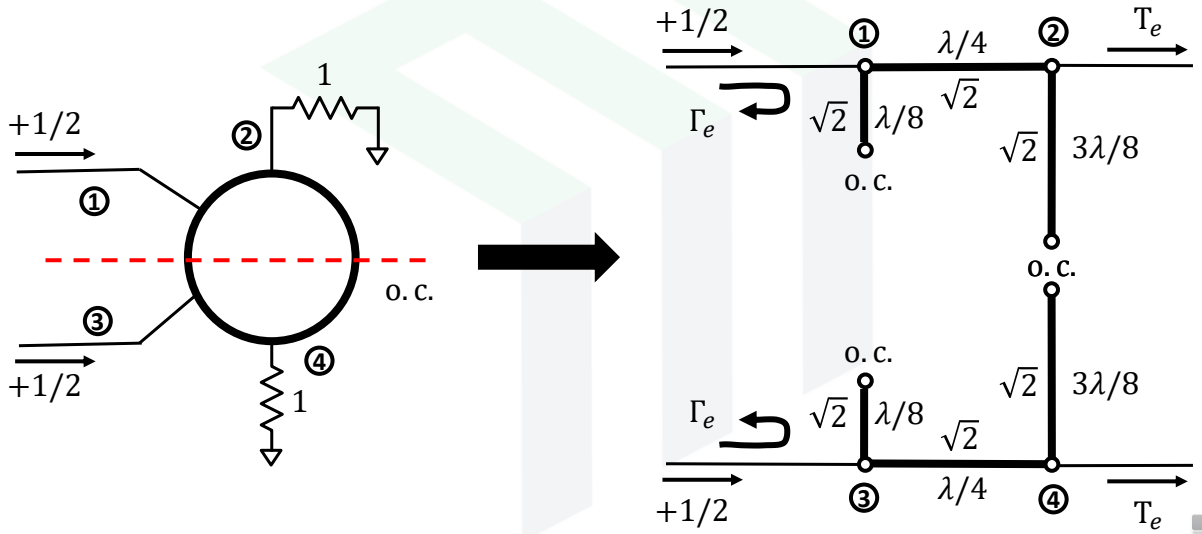
Even-Mode Analysis of the Ring Hybrid – Ports 1 and 3



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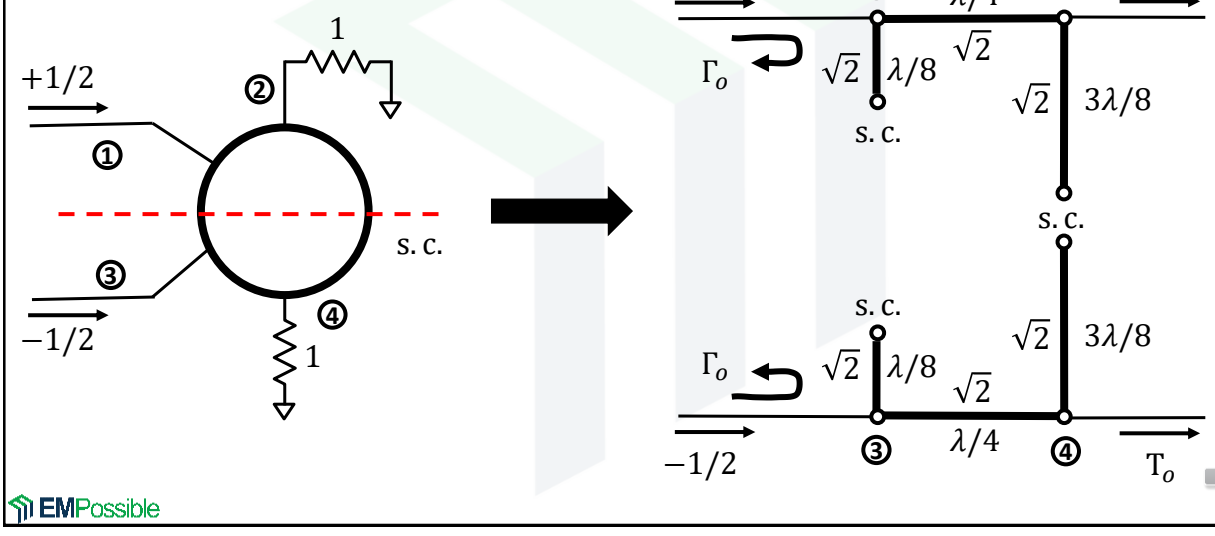
Odd-Mode Analysis of the Ring Hybrid – Ports 1 and 3



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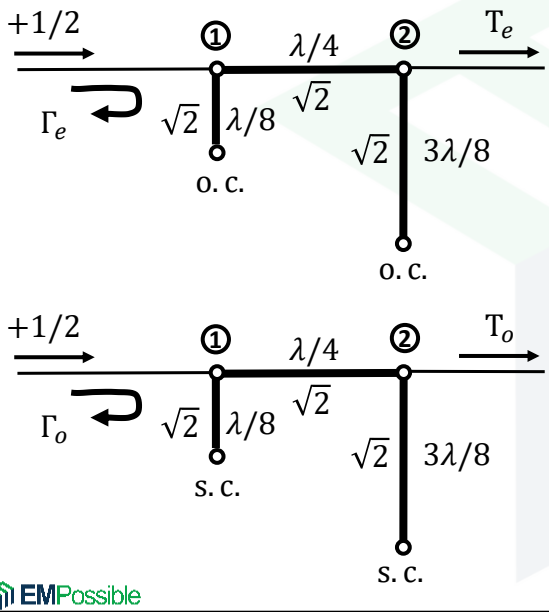
Even- Odd-Mode Analysis of Ring Hybrid – Ports 1 and 3



$$B_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o$$

$$B_2 = \frac{1}{2}\mathrm{T}_e + \frac{1}{2}\mathrm{T}_o$$

$$B_3 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o$$

$$B_4 = \frac{1}{2}\mathrm{T}_e - \frac{1}{2}\mathrm{T}_o$$

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Even-Mode Analysis of the Ring Hybrid – Ports 1 and 3

The corresponding $ABCD$ parameters of the even- and odd-mode two-port circuits are

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{even} = \begin{bmatrix} 1 & j\sqrt{2} \\ j\sqrt{2} & -1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{odd} = \begin{bmatrix} -1 & j\sqrt{2} \\ j\sqrt{2} & 1 \end{bmatrix}$$

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Ring Hybrid Coefficients: Even- and Odd- Mode – Ports 1 and 3

$$\Gamma_e = \frac{A_e + B_e - C_e - D_e}{A_e + B_e + C_e + D_e} = \frac{1 + j\sqrt{2} - j\sqrt{2} + 1}{1 + j\sqrt{2} + j\sqrt{2} - 1} = -\frac{j}{\sqrt{2}}$$

$$T_e = \frac{2}{A_e + B_e + C_e + D_e} = \frac{2}{1 + j\sqrt{2} + j\sqrt{2} - 1} = -\frac{j}{\sqrt{2}}$$

$$\Gamma_o = \frac{A_o + B_o - C_o - D_o}{A_o + B_o + C_o + D_o} = \frac{-1 + j\sqrt{2} - j\sqrt{2} - 1}{-1 + j\sqrt{2} + j\sqrt{2} + 1} = \frac{j}{\sqrt{2}}$$

$$T_o = \frac{2}{A_o + B_o + C_o + D_o} = \frac{2}{-1 + j\sqrt{2} + j\sqrt{2} + 1} = -\frac{j}{\sqrt{2}}$$

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Ring Hybrid Coefficients: Even- and Odd- Mode – Ports 1 and 3

$$B_1 = \frac{1}{2}\left(-\frac{j}{\sqrt{2}}\right) + \frac{1}{2}\left(\frac{j}{\sqrt{2}}\right) = 0$$

Port 1 is matched

$$B_2 = \frac{1}{2}\left(-\frac{j}{\sqrt{2}}\right) + \frac{1}{2}\left(-\frac{j}{\sqrt{2}}\right) = -\frac{j}{\sqrt{2}}$$

$$B_3 = \frac{1}{2}\left(-\frac{j}{\sqrt{2}}\right) - \frac{1}{2}\left(\frac{j}{\sqrt{2}}\right) = -\frac{j}{\sqrt{2}}$$

Power is evenly distributed and in phase between ports 2 and 3

$$B_4 = \frac{1}{2}\left(-\frac{j}{\sqrt{2}}\right) - \frac{1}{2}\left(-\frac{j}{\sqrt{2}}\right) = 0$$

No power to Port 4 (isolated)

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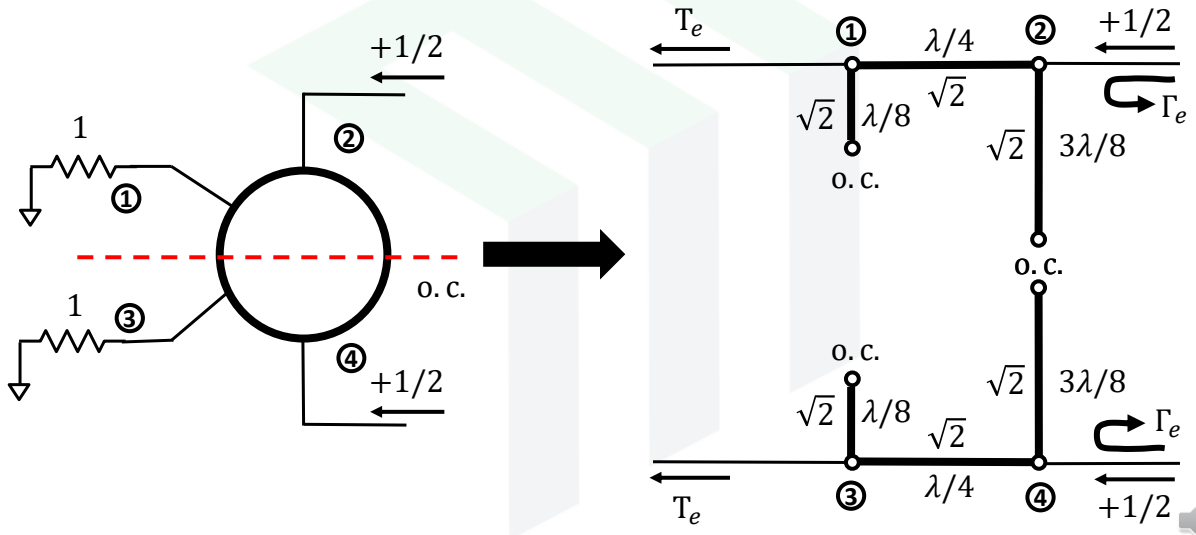
Even-Mode Analysis of the Ring Hybrid – Ports 4 and 2



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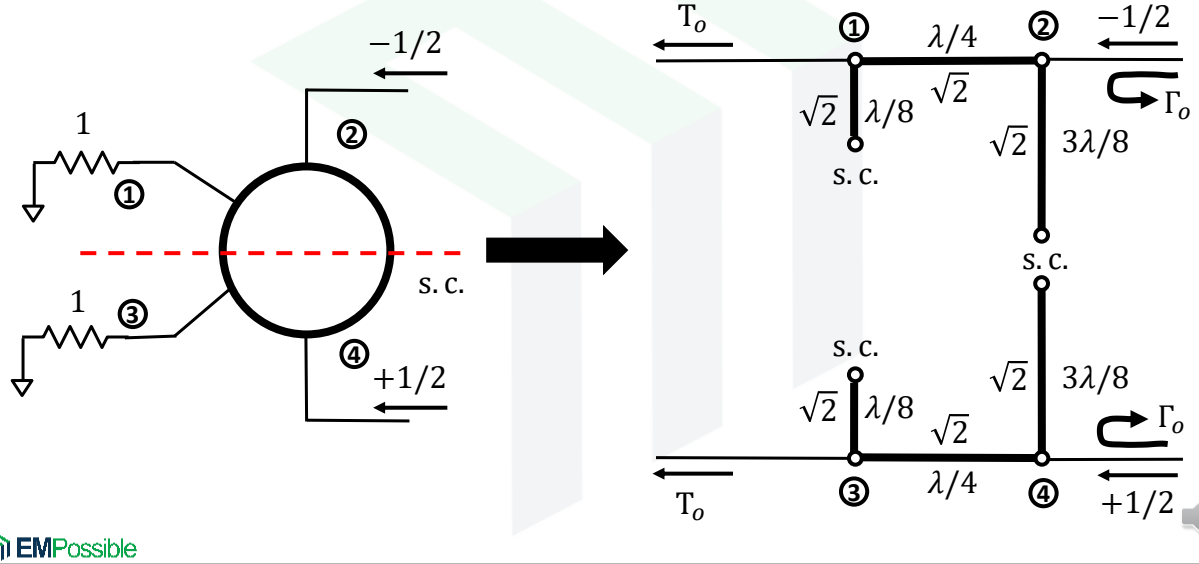
Odd-Mode Analysis of the Ring Hybrid – Ports 4 and 2



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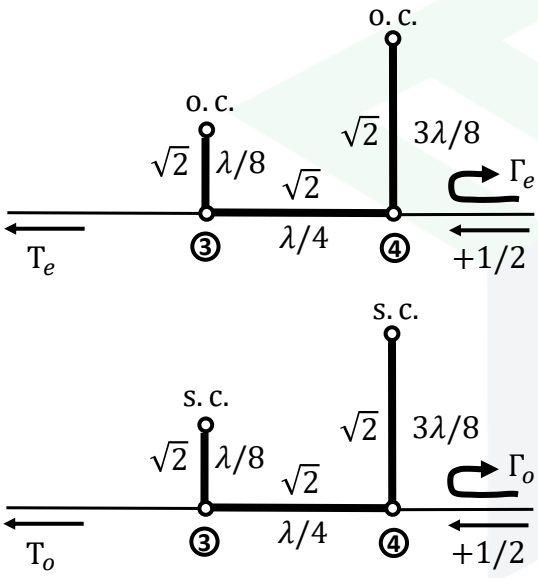
Odd-Mode Analysis of the Ring Hybrid – Ports 4 and 2



$$B_1 = \frac{1}{2}\mathrm{T}_e - \frac{1}{2}\mathrm{T}_o$$

$$B_2 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o$$

$$B_3 = \frac{1}{2}\mathrm{T}_e + \frac{1}{2}\mathrm{T}_o$$

$$B_4 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o$$

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Even-Mode Analysis of the Ring Hybrid – Ports 4 and 2

The corresponding $ABCD$ parameters of the even- and odd-mode two-port circuits are

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{even} = \begin{bmatrix} -1 & j\sqrt{2} \\ j\sqrt{2} & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{odd} = \begin{bmatrix} 1 & j\sqrt{2} \\ j\sqrt{2} & -1 \end{bmatrix}$$

And the corresponding reflection and transmission coefficients are:

$$\Gamma_e = \frac{j}{\sqrt{2}} \qquad \Gamma_o = -\frac{j}{\sqrt{2}}$$

$$\mathrm{T}_e = -\frac{j}{\sqrt{2}} \qquad \mathrm{T}_o = -\frac{j}{\sqrt{2}}$$

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Even-Mode Analysis of the Ring Hybrid – Ports 4 and 2

Using these results to calculate the backward components we obtain:

$$B_1 = 0$$ Port 1 is matched

$$B_2 = \frac{j}{\sqrt{2}}$$ Power is evenly distributed with $180°$ phase between ports 2 and 3

$$B_3 = -\frac{j}{\sqrt{2}}$$

$$B_4 = 0$$ No power to Port 4 (isolated)

Example – Design of a Ring Hybrid

Design a $180°$ hybrid for a $50\ \Omega$ system impedance at a design frequency f_0

Solution:

The characteristic impedance of the ring transmission line is

$$\sqrt{2}Z_0 = 70.7\ \Omega$$