Advanced Electromagnetics:
21st Century Electromagnetics

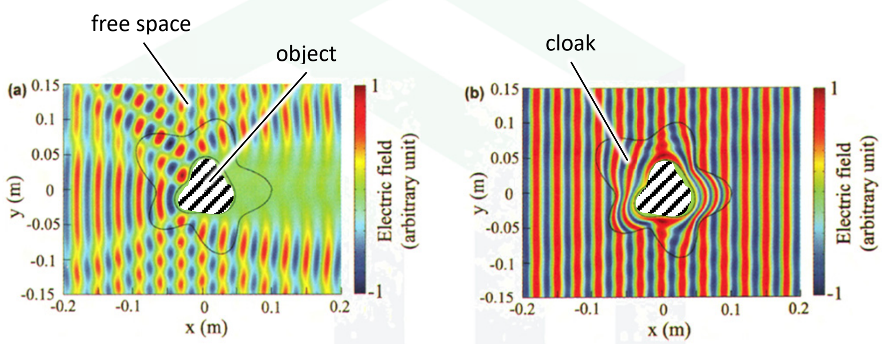
Cloaking with Transformation Optics

Lecture Outline

- Traditional Cloaking
- Carpet Cloaking

Cloaking

What is Cloaking?



A ***perfect*** cloak must have the following properties:

1. Cannot reflect or scatter waves.
2. Must perfectly reconstruct the wave front on the other side of the object.
3. Must work for waves applied from any direction.

D.-H. Kwon, D. H. Werner, “Transformation Electromagnetics: An Overview of the Theory and Applications,” IEEE Ant. Prop. Mag., Vol. 52, No. 1, 24-46 (2020).

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Famous Cylinder Cloak

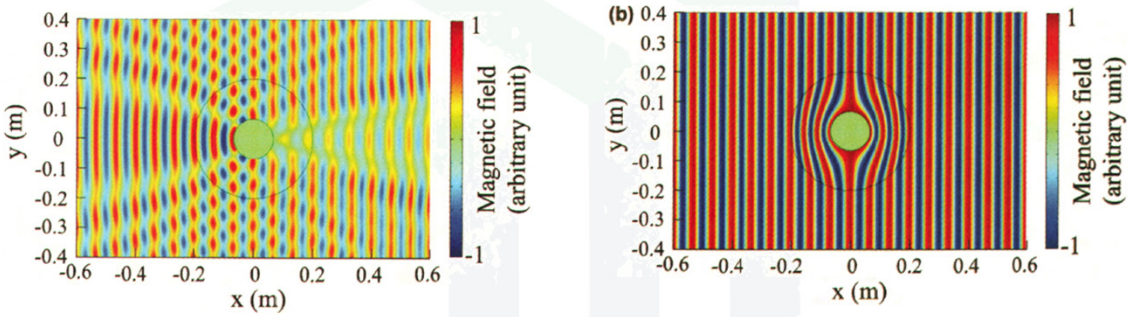
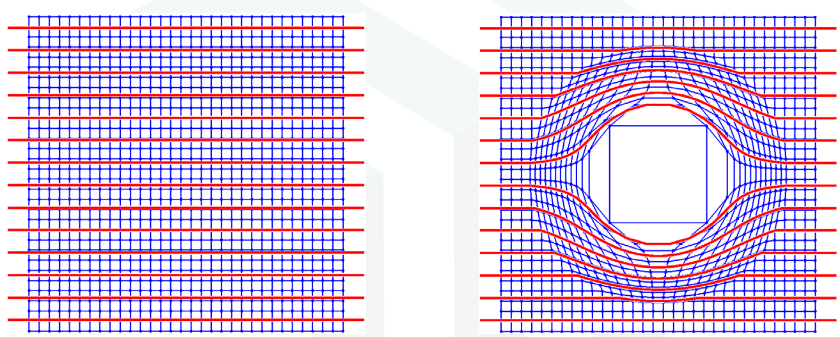


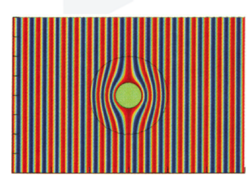
Figure 3. A two-dimensional circular annular cloak: (a) A conducting cylinder subject to a TM-mode plane-wave illumination, (b) the same conducting cylinder enclosed within a cloak.

D.-H. Kwon, D. H. Werner, "Transformation Electromagnetics: An Overview of the Theory and Applications," IEEE Ant. Prop. Mag., Vol. 52, No. 1, 24-46 (2020).

Path of Rays for Invisibility



To render a region in the center of the grid invisible, the wave must be made to flow around it.



Coordinate Transformation Geometry

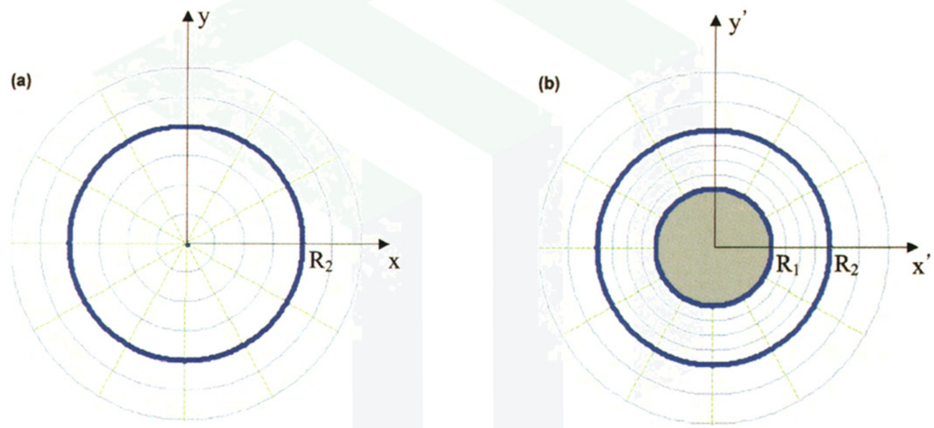


Figure 2. A coordinate transformation for a two-dimensional circular annular cloak: (a) the original coordinate system, (b) the transformed coordinate system.

D.-H. Kwon, D. H. Werner, "Transformation Electromagnetics: An Overview of the Theory and Applications," IEEE Ant. Prop. Mag., Vol. 52, No. 1, 24-46 (2020).

Deriving the Coordinate Transformation

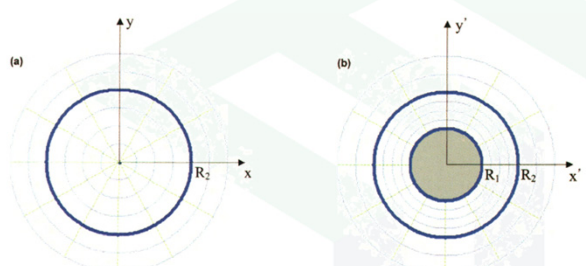


Figure 2. A coordinate transformation for a two-dimensional circular annular cloak: (a) the original coordinate system, (b) the transformed coordinate system.

Be observation, we wish to map our coordinates as follows.

$$\rho'(\rho = 0) = R_1$$
$$\rho'(\rho = R_2) = R_2$$

This transform can be done with a simple straight line.

$$y = mx + b \quad \rightarrow \quad \rho' = m\rho + b$$

y-intercept: $b = R_1$

Slope: $m = \frac{R_2 - R_1}{R_2}$ $\Rightarrow$ $\boxed{\rho' = \frac{R_2 - R_1}{R_2}\rho + R_1}$

D.-H. Kwon, D. H. Werner, "Transformation Electromagnetics: An Overview of the Theory and Applications," IEEE Ant. Prop. Mag., Vol. 52, No. 1, 24-46 (2020).

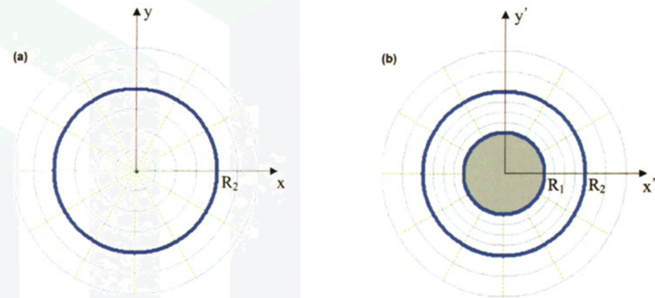
The Jacobian Matrix

Due to the cylindrical geometry, the coordinate transformation will be in cylindrical coordinates.

$$\rho' = R_1 + \frac{R_2 - R_1}{R_2}\rho$$

$$\phi' = \phi$$

$$z' = z$$



It follows that the Jacobian matrix is

$$[J] = \begin{bmatrix} \frac{1}{1}\frac{\partial \rho'}{\partial \rho} & \frac{1}{\rho}\frac{\partial \rho'}{\partial \phi} & \frac{1}{1}\frac{\partial \rho'}{\partial z} \\ \frac{\rho'}{1}\frac{\partial \phi'}{\partial \rho} & \frac{\rho'}{\rho}\frac{\partial \phi'}{\partial \phi} & \frac{\rho'}{1}\frac{\partial \phi'}{\partial z} \\ \frac{1}{1}\frac{\partial z'}{\partial \rho} & \frac{1}{\rho}\frac{\partial z'}{\partial \phi} & \frac{1}{1}\frac{\partial z'}{\partial z} \end{bmatrix} = \begin{bmatrix} (R_2 - R_1)/R_2 & 0 & 0 \\ 0 & \rho'/\rho & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The Material Tensors

The material tensors are

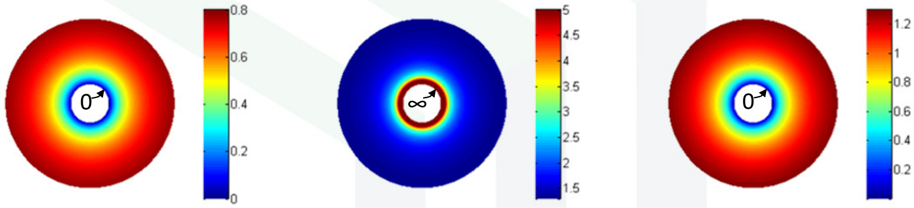
$$\left[\mu'(\vec{r}\,')\right] = \frac{[J]\left[\mu(\vec{r})\right][J]^T}{\det[J]} \qquad \left[\varepsilon'(\vec{r}\,')\right] = \frac{[J]\left[\varepsilon(\vec{r})\right][J]^T}{\det[J]}$$

Assuming we start with free space, we will have

$$\left[\mu'(\vec{r}\,')\right] = \left[\varepsilon'(\vec{r}\,')\right] = \begin{bmatrix} \frac{r' - R_1}{r'} & 0 & 0 \\ 0 & \frac{r'}{r' - R_1} & 0 \\ 0 & 0 & \left(\frac{R_2}{R_2 - R_1}\right)^2 \frac{r' - R_1}{r'} \end{bmatrix}$$

Cylindrical Tensors

$\mu'_{rr}(\vec{r}\,') = \varepsilon'_{rr}(\vec{r}\,')$ $\mu'_{\theta\theta}(\vec{r}\,') = \varepsilon'_{\theta\theta}(\vec{r}\,')$ $\mu'_{zz}(\vec{r}\,') = \varepsilon'_{zz}(\vec{r}\,')$



$$\frac{r'-R_1}{r'} \qquad \frac{r'}{r'-R_1} \qquad \left(\frac{R_2}{R_2-R_1}\right)^2 \frac{r'-R_1}{r'}$$

$$R_1 = 0.10 \qquad R_2 = 0.45$$

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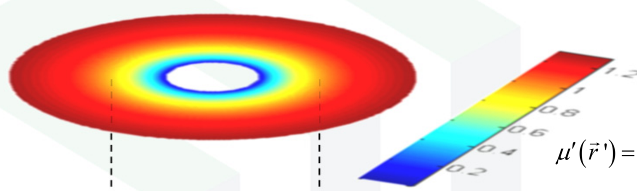
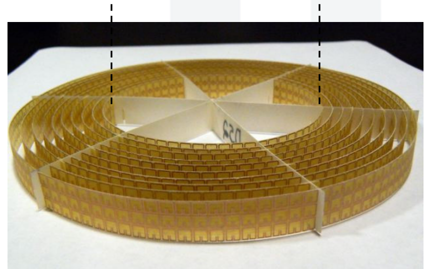
Physical Device



$\mu'(\vec{r}\,') = \varepsilon'(\vec{r}\,')$



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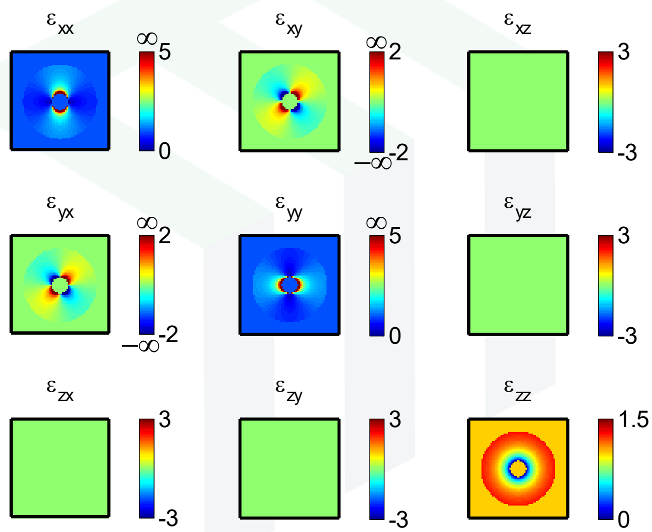
Cartesian Tensors



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Carpet Cloaking

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Problems with Cloaking by Transformation Electromagnetics

- The resulting materials:
  - Are anisotropic
  - Require dielectric and magnetic properties
  - Require extreme and singular values
- Particularly problematic at optical frequencies

Three Cases for Cloaking

Squish an object to a point.

Squish an object to a line.

Object becomes infinitely conducting.

This is not a problem because the objects have zero size.

These can be rendered invisible, but require extreme and singular values as well as being anisotropic.

Squish an object to a sheet.

A sheet is highly visible.

It can only be made invisible if it sits on another conducting sheet so they cannot be distinguished.

While more limited, invisibility can be realized without extreme values and with isotropic materials.

Carpet Cloak Concept

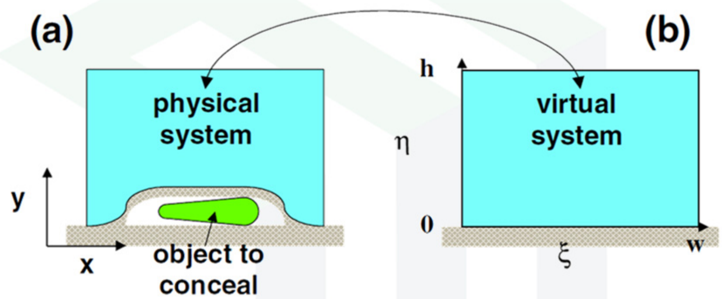


FIG. 1 (color online). The virtual and the physical systems. The regions in cyan are transformed into each other. Shaded regions represent the ground planes. The observer perceives the physical system as the virtual one with a flat ground plane.

J. Li, J. B. Pendry, "Hiding under the Carpet: A New Strategy for Cloaking," Phys. Rev. Lett. **101**, 203901 (2008).

The Jacobian Matrix $[J]$ and Covariant Matrix $[g]$

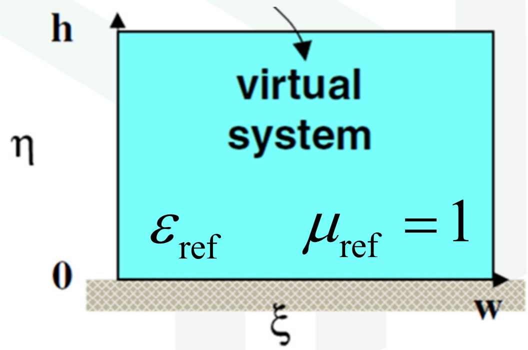
Define the Jacobian matrix $[J]$ just like before.

$$A_{ij} = \frac{\partial x_i}{\partial x_i'}$$

The Covariant matrix $[g]$ is then

$$[g] = [J]^T [J]$$

The Original System



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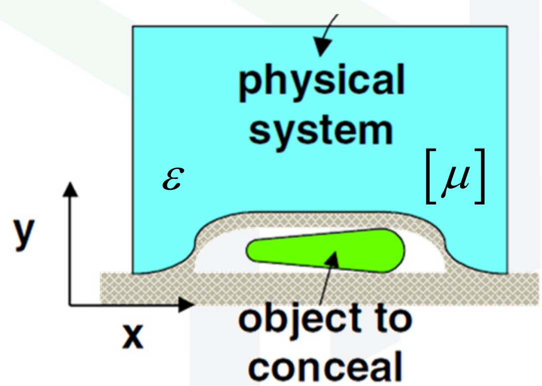
The Transformed System



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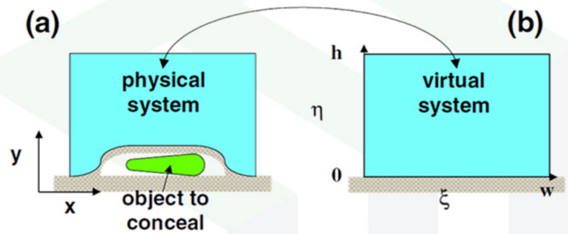
The Transformation



$$\varepsilon = \frac{\varepsilon_{\text{ref}}}{\sqrt{\det[g]}}$$

$$[\mu] = \frac{[J][J]^T}{\sqrt{\det[g]}}$$

ε_{ref} anything

$\mu_{\text{ref}} = 1$

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A Preliminary Tensor Description

The permeability is still a tensor quantity. For a given wave, it will have two principle values.

$$[\mu] \rightarrow \mu_{\text{T}} \text{ and } \mu_{\text{L}} \qquad \mu_{\text{T}}\mu_{\text{L}} = 1$$

This gives two refractive indices to characterize the medium.

$$n_{\text{T}} = \sqrt{\mu_{\text{T}}\varepsilon} \qquad n_{\text{L}} = \sqrt{\mu_{\text{L}}\varepsilon}$$

The anisotropy can be characterized using the anisotropy factor α.

$$\alpha = \max\left[\frac{n_{\text{T}}}{n_{\text{L}}}, \frac{n_{\text{L}}}{n_{\text{T}}}\right]$$

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Isotropic Medium

By picking a suitable transform, the permeability can be neglected.

$$\varepsilon = \frac{\varepsilon_{\text{ref}}}{\sqrt{\det[g]}} \qquad [\mu] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The optimal transformation is generated by minimizing the Modified-Liao functional

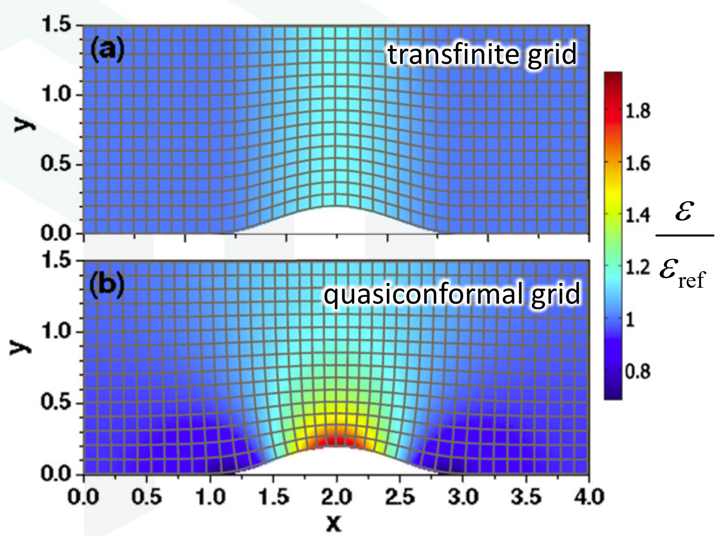
$$\Phi = \frac{1}{hw}\int_0^w dx' \int_0^h \frac{\left(\text{Tr}[g]\right)^2}{\det[g]} dy'$$

This leads to high ε and low anisotropy.

Impact of Grid on Isotropic Approximation

Low ε so this medium will be poorly approximated as isotropic.

High ε so this medium is well approximated as isotropic.



Example Carpet Cloak

Scattering from cloaked object.

Scattering from just the object.