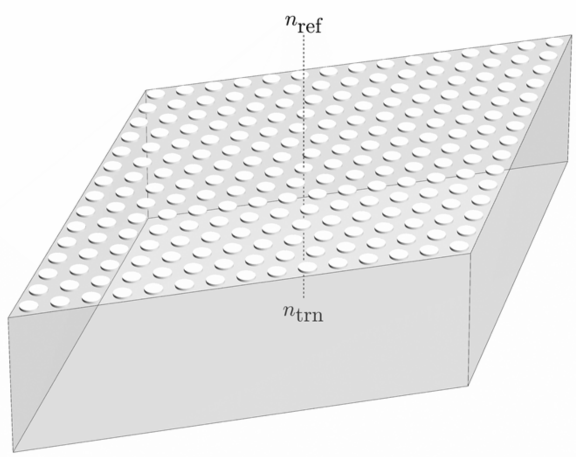
Advanced Electromagnetics:
21st Century Electromagnetics

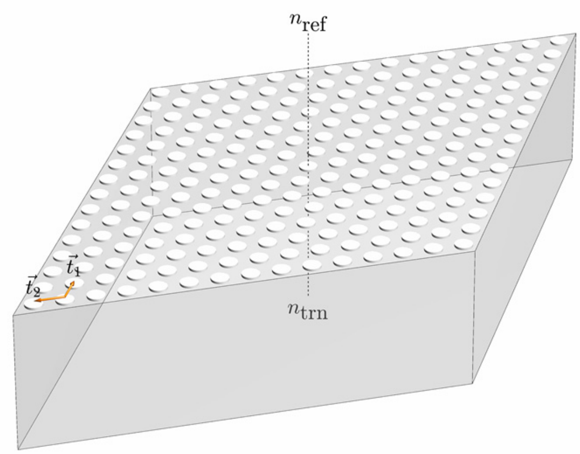
Diffraction from Oblique Gratings

Oblique Grating



Slide 2

Direct Lattice Vectors $\vec{t}_1$ and $\vec{t}_2$



For a hexagonal grating, the direct lattice vectors are

$$\vec{t}_1 = \frac{a}{2}\hat{a}_x - \frac{a\sqrt{3}}{2}\hat{a}_y$$

$$\vec{t}_2 = \frac{a}{2}\hat{a}_x + \frac{a\sqrt{3}}{2}\hat{a}_y$$

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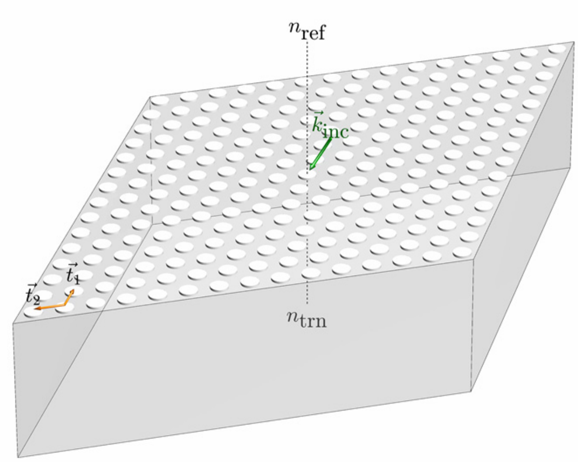
Incident Wave



Let there be an incident wave with wave vector $\vec{k}_{\text{inc}}$.

$$\vec{k}_{\text{inc}} = k_0 n_{\text{ref}} \begin{pmatrix} \sin\theta\cos\phi\hat{a}_x \\ +\sin\theta\sin\phi\hat{a}_y \\ +\cos\theta\hat{a}_z \end{pmatrix}$$

Wave polarization does not impact the directions of the diffraction orders.

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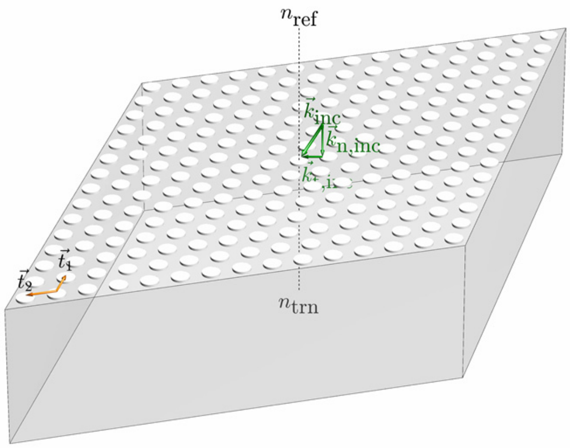
Tangential & Normal Components of $\vec{k}_{\mathrm{inc}}$



The incident wave vector can be decomposed into tangential $\vec{k}_{\mathrm{t,inc}}$ and normal $\vec{k}_{\mathrm{n,inc}}$ components.

$$\vec{k}_{\mathrm{inc}} = \vec{k}_{\mathrm{t,inc}} + \vec{k}_{\mathrm{n,inc}}$$

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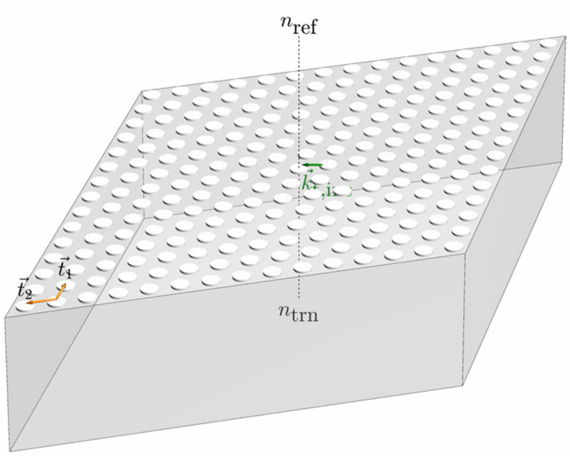
Tangential Component $\vec{k}_{\mathrm{t,inc}}$



It is only the tangential component of the incident wave vector that determines the directions of the diffraction orders.

$$\vec{k}_{\mathrm{inc}} = \boxed{\vec{k}_{\mathrm{t,inc}}} + \vec{k}_{\mathrm{n,inc}}$$

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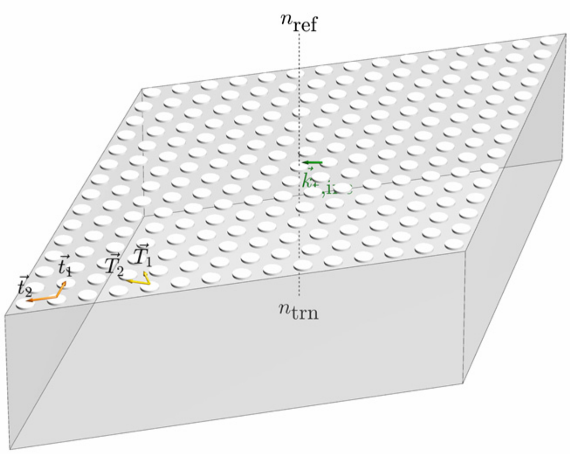
Reciprocal Lattice Vectors $\vec{T}_1$ and $\vec{T}_2$



The reciprocal lattice vectors $\vec{T}_1$ and $\vec{T}_2$ are calculated from the direct lattice vectors $\vec{t}_1$ and $\vec{t}_2$.

$$\vec{T}_1 = \frac{2\pi}{t_{1,x}t_{2,y} - t_{2,x}t_{1,y}} \begin{bmatrix} t_{2,y} \\ -t_{2,x} \end{bmatrix}$$

$$\vec{T}_2 = \frac{2\pi}{t_{1,x}t_{2,y} - t_{2,x}t_{1,y}} \begin{bmatrix} -t_{1,y} \\ t_{1,x} \end{bmatrix}$$

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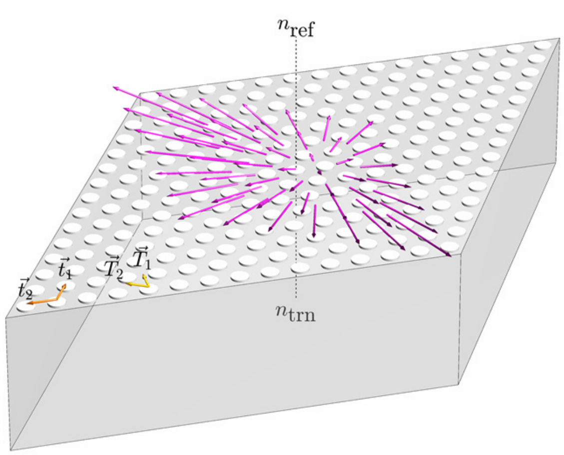
Tangential Components of Diffraction Orders



The tangential components are the same for both the reflected and transmitted diffraction orders.

They are expanded about $\vec{k}_{\text{t,inc}}$ in integer multiples of $\vec{T}_1$ and $\vec{T}_2$.

$$\vec{k}_{\text{t}}(p,q) = \vec{k}_{\text{t,inc}} - p\vec{T}_1 - q\vec{T}_2$$

Diffraction Order (p,q)

$p = -\infty, \ldots, -3, -2, -1, 0, +1, +2, +3, \ldots + \infty$

$q = -\infty, \ldots, -3, -2, -1, 0, +1, +2, +3, \ldots + \infty$

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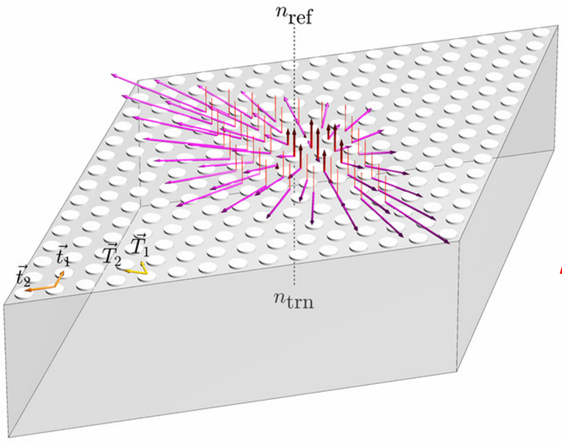
Normal Components of Reflected Waves



The normal components of the diffraction orders are calculated from the dispersion relation. For the reflected waves, this is

$$\vec{k}_{\text{n,ref}}(p,q) = -\hat{a}_n\sqrt{(k_0 n_{\text{ref}})^2 - \left|\vec{k}_{\text{t}}(p,q)\right|^2}$$

The square-root can become imaginary

Imaginary $\vec{k}_{\text{n,ref}}$ indicates diffraction orders that are cutoff and evanescent.

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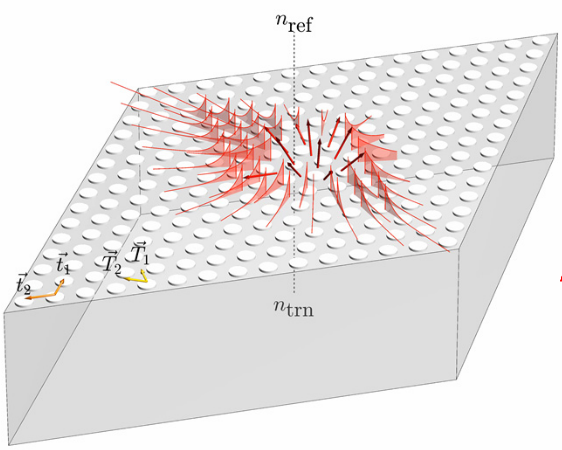
Overall Reflected Diffraction Orders



The normal components of the diffraction orders are calculated from the dispersion relation. For the reflected waves, this is

$$\vec{k}_{\text{n,ref}}(p,q) = -\hat{a}_n\sqrt{(k_0 n_{\text{ref}})^2 - \left|\vec{k}_{\text{t}}(p,q)\right|^2}$$

The square-root can become imaginary

Imaginary $\vec{k}_{\text{n,ref}}$ indicates diffraction orders that are cutoff and evanescent.

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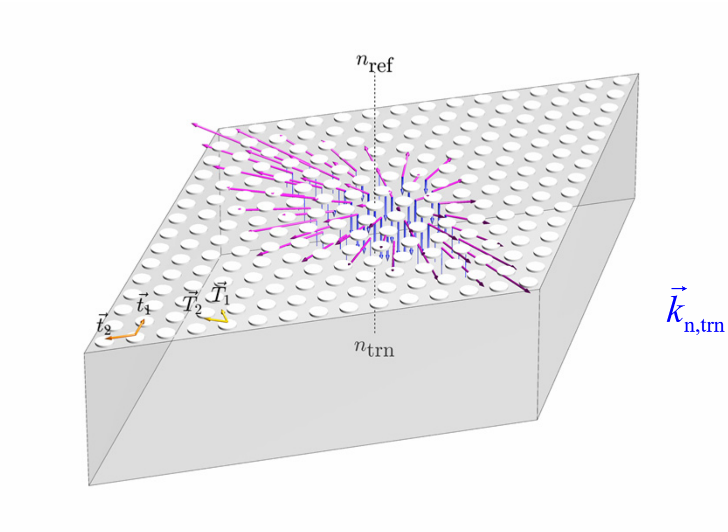
Normal Components of Transmitted Waves



The normal components of the diffraction orders are calculated from the dispersion relation. For the transmitted waves, this is

$$\vec{k}_{\text{n,trn}}(p,q) = \hat{a}_n \sqrt{\left(k_0 n_{\text{trn}}\right)^2 - \left|\vec{k}_{\text{t}}(p,q)\right|^2}$$

The square-root can become imaginary

Imaginary $\vec{k}_{\text{n,trn}}$ indicates diffraction orders that are cutoff and evanescent.

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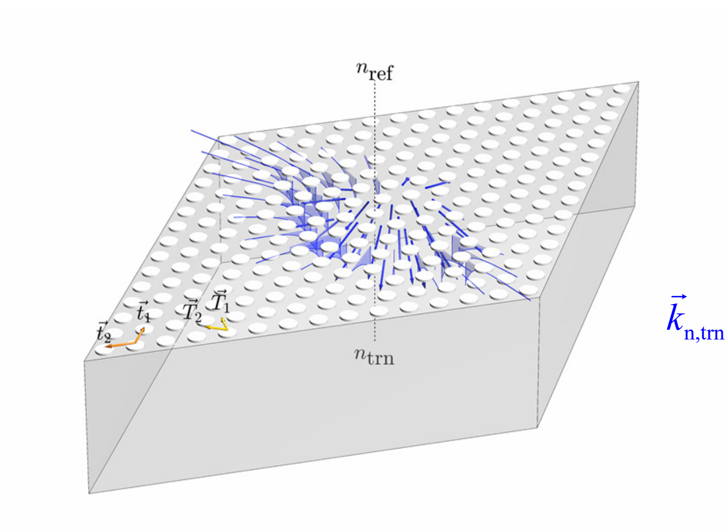
Overall Transmitted Diffraction Orders



The normal components of the diffraction orders are calculated from the dispersion relation. For the transmitted waves, this is

$$\vec{k}_{\text{n,trn}}(p,q) = \hat{a}_n \sqrt{\left(k_0 n_{\text{trn}}\right)^2 - \left|\vec{k}_{\text{t}}(p,q)\right|^2}$$

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Imaginary $\vec{k}_{\text{n,trn}}$ indicates diffraction orders that are cutoff and evanescent.

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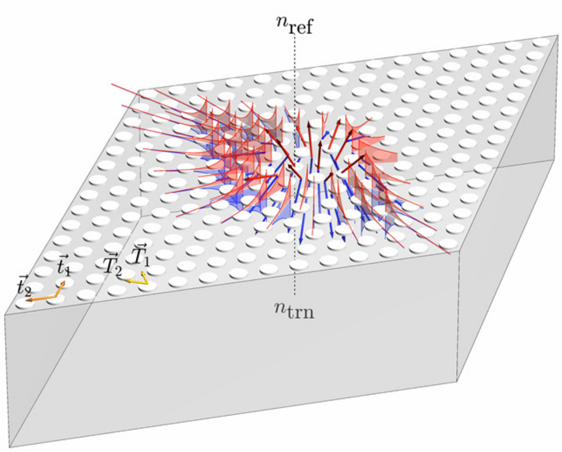
Comparison of Reflected & Transmitted



Assuming $n_{\text{trn}} > n_{\text{ref}}$, there will be more transmitted waves and they will have smaller angle.

Summary of Calculating Direction of Diffraction Orders

Step 1 – Calculate lattice vectors

$$\vec{t}_1 = ? \qquad \vec{t}_2 = ?$$

$$\vec{T}_1 = \frac{2\pi}{t_{1,x}t_{2,y} - t_{2,x}t_{1,y}} \begin{bmatrix} t_{2,y} \\ -t_{2,x} \end{bmatrix} \qquad \vec{T}_2 = \frac{2\pi}{t_{1,x}t_{2,y} - t_{2,x}t_{1,y}} \begin{bmatrix} -t_{1,y} \\ t_{1,x} \end{bmatrix}$$

Step 2 – Calculate incident wave vector

$$\vec{k}_{\text{inc}} = k_0 n_{\text{ref}} \left(\sin\theta\cos\phi\,\hat{a}_x + \sin\theta\sin\phi\,\hat{a}_y + \cos\theta\,\hat{a}_z \right)$$

Step 3 – Extract tangential component

$$\vec{k}_{\text{t,inc}} = k_0 n_{\text{ref}} \left(\sin\theta\cos\phi\,\hat{a}_x + \sin\theta\sin\phi\,\hat{a}_y \right)$$

Step 4 – Calculate tangential components of diffraction orders

$$\vec{k}_{\text{t}}(p,q) = \vec{k}_{\text{t,inc}} - p\vec{T}_1 - q\vec{T}_2$$

$$p = -\infty, \ldots, -3, -2, -1, 0, +1, +2, +3, \ldots + \infty$$
$$q = -\infty, \ldots, -3, -2, -1, 0, +1, +2, +3, \ldots + \infty$$

Step 5 – Calculate normal components of diffraction orders

$$\vec{k}_{\text{n,ref}}(p,q) = -\hat{a}_{\text{n}} \sqrt{\left(k_0 n_{\text{ref}}\right)^2 - \left|\vec{k}_{\text{t}}(p,q)\right|^2}$$

$$\vec{k}_{\text{n,trn}}(p,q) = \hat{a}_{\text{n}} \sqrt{\left(k_0 n_{\text{trn}}\right)^2 - \left|\vec{k}_{\text{t}}(p,q)\right|^2}$$