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Electromagnetics:
Electromagnetic Field Theory

# Differential Length, Area & Volume

# Outline

- Cartesian Coordinates
- Cylindrical Coordinates
- Spherical Coordinates

## Cartesian Coordinates

| Differential Element | Formula | Notes |
|---|---|---|
| **Length** | $d\vec{\ell} = dx \cdot \hat{x} + dy \cdot \hat{y} + dz \cdot \hat{z}$ | Vector is tangential to line |
| **Normal Area** | $d\vec{s} = dydz \cdot \hat{x} + dxdz \cdot \hat{y} + dxdy \cdot \hat{z}$ | Vector is normal to surface |
| **Volume** | $dv = dxdydz$ | Scalar |

## Cylindrical Coordinates

| Differential Element | Formula | Notes |
|---|---|---|
| **Length** | $d\vec{\ell} = d\rho \cdot \hat{\rho} + \rho d\phi \cdot \hat{\phi} + dz \cdot \hat{z}$ | Vector is tangential to line |
| **Normal Area** | $d\vec{s} = \rho d\phi dz \cdot \hat{\rho} + d\rho dz \cdot \hat{\phi} + \rho d\rho d\phi \cdot \hat{z}$ | Vector is normal to surface |
| **Volume** | $dv = \rho d\rho d\phi dz$ | Scalar |

## Spherical Coordinates

| Differential Element | Formula | Notes |
|---|---|---|
| **Length** | $d\vec{\ell} = dr \cdot \hat{r} + rd\theta \cdot \hat{\theta} + r\sin\theta d\phi \cdot \hat{\phi}$ | Vector is tangential to line |
| **Normal Area** | $d\vec{s} = r^2 \sin\theta d\theta d\phi \cdot \hat{r} + r\sin\theta drd\phi \cdot \hat{\theta} + rdrd\theta \cdot \hat{\phi}$ | Vector is normal to surface |
| **Volume** | $dv = r^2 \sin\theta drd\theta d\phi$ | Scalar |

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# Cartesian Coordinates

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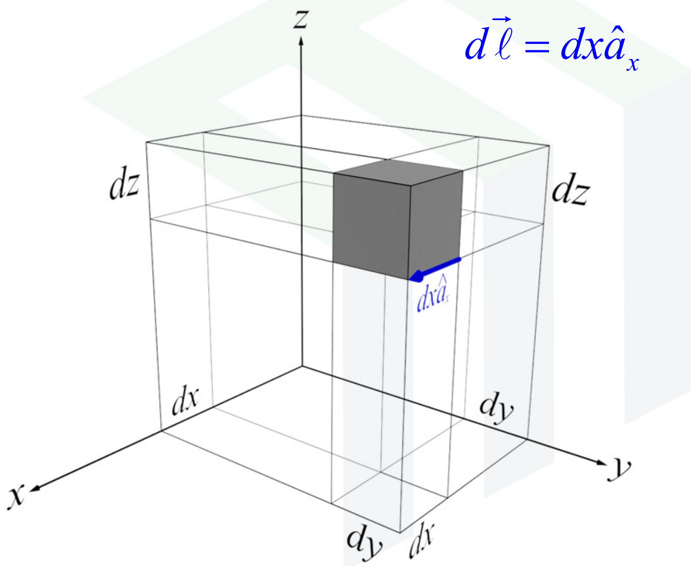
# Cartesian: *Differential Lengths*

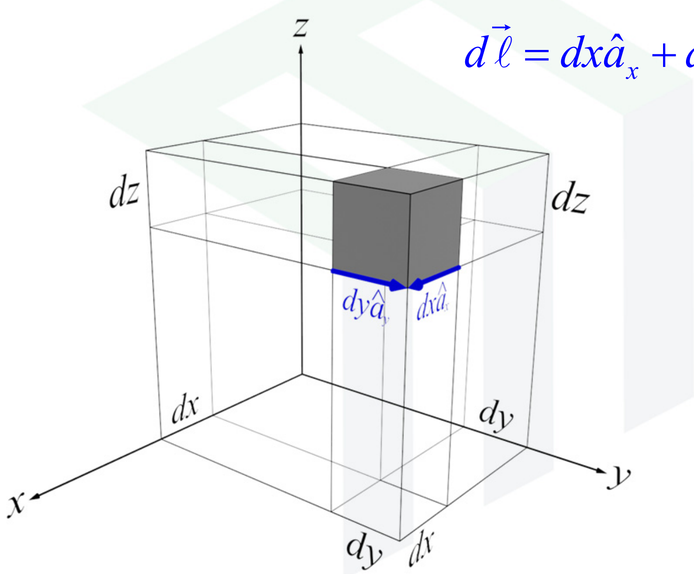
$$d\vec{\ell} = dx\hat{a}_x$$





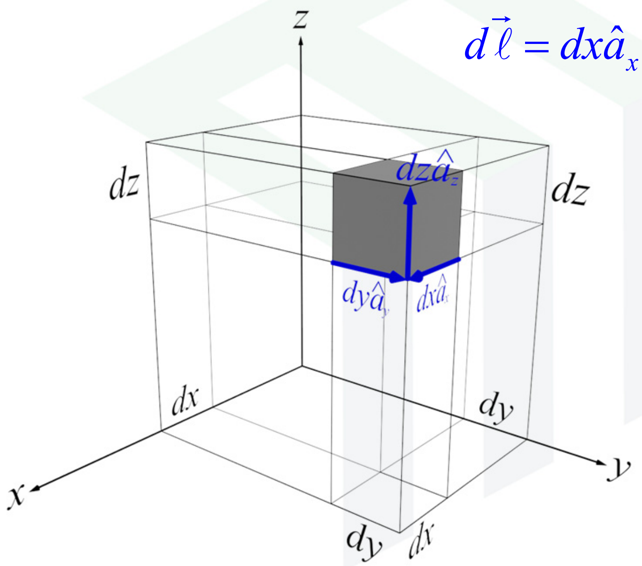

# Cartesian: *Differential Lengths*

$$d\vec{\ell} = dx\hat{a}_x + dy\hat{a}_y$$



# Cartesian: *Differential Lengths*

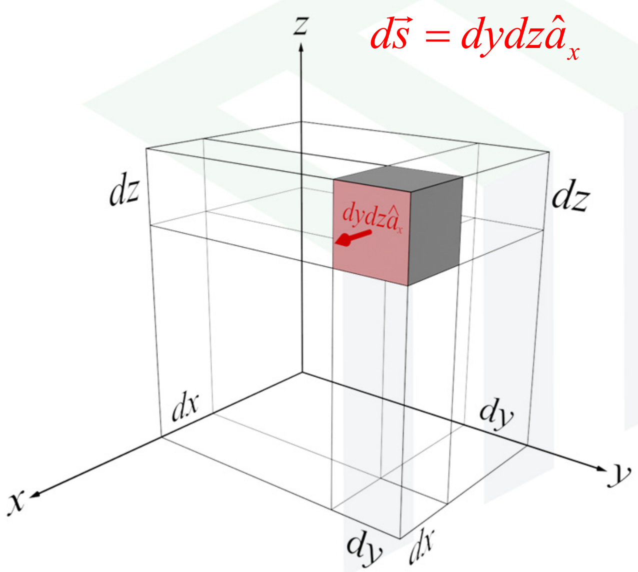
$$d\vec{\ell} = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z$$





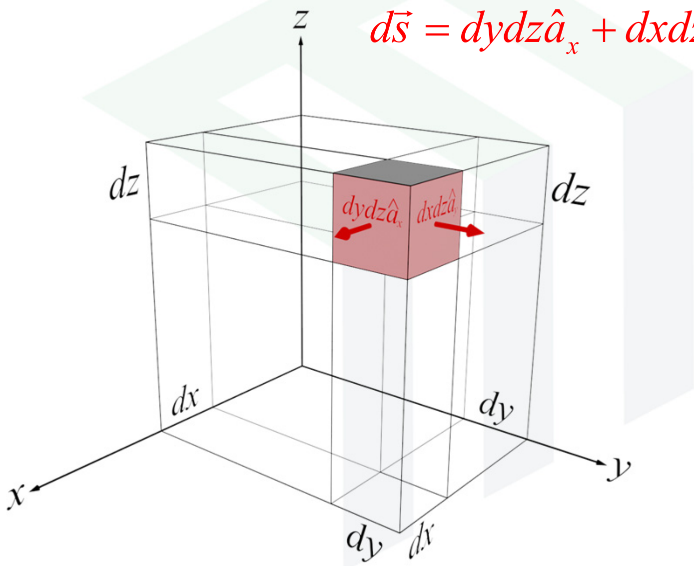

# Cartesian: *Differential Areas*

$$d\vec{s} = dydz\hat{a}_x$$



# Cartesian: *Differential Areas*

$$d\vec{s} = dydz\hat{a}_x + dxdz\hat{a}_y$$

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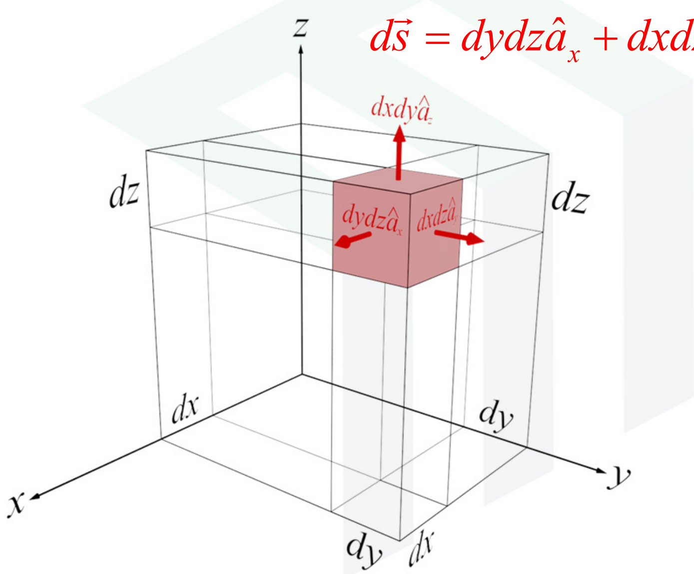
# Cartesian: *Differential Areas*

$$d\vec{s} = dydz\hat{a}_x + dxdz\hat{a}_y + dxdy\hat{a}_z$$

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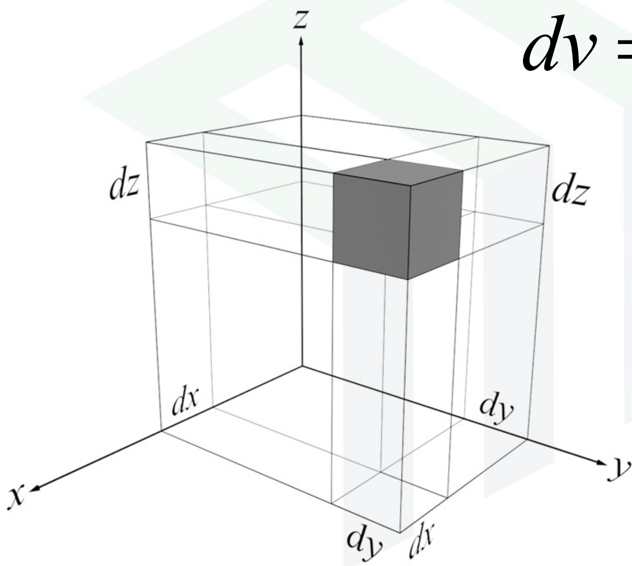
## Cartesian: *Differential Volume*

$$dv = dx\,dy\,dz$$

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# Cylindrical Coordinates

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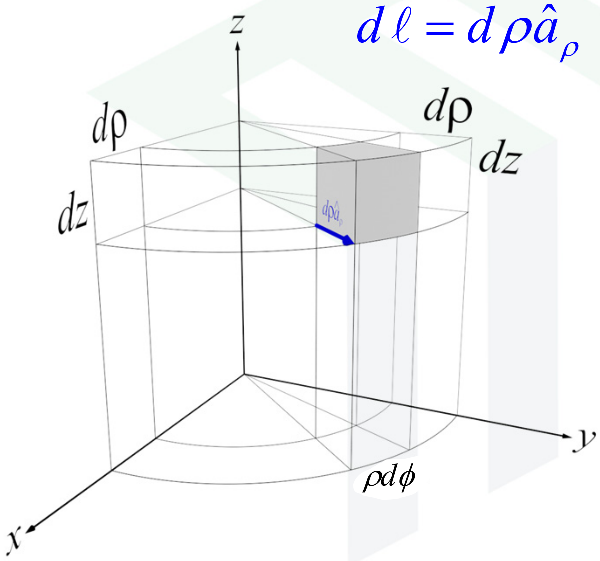
# Cylindrical: *Differential Lengths*

$$d\vec{\ell} = d\rho\hat{a}_\rho$$

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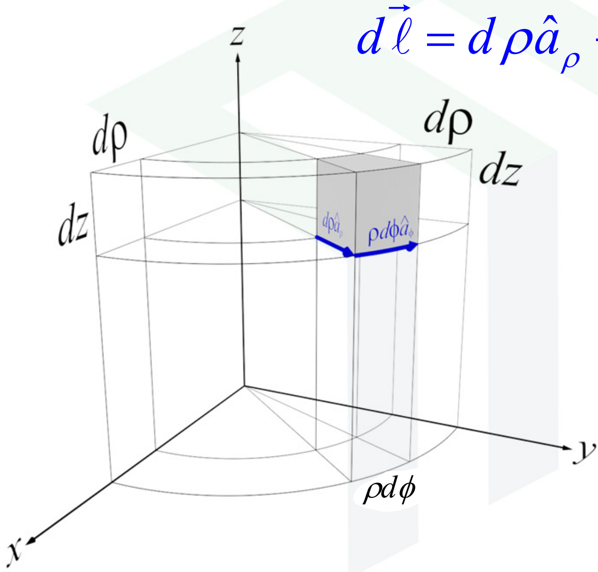
# Cylindrical: *Differential Lengths*

$$d\vec{\ell} = d\rho\hat{a}_\rho + \rho d\phi\hat{a}_\phi$$

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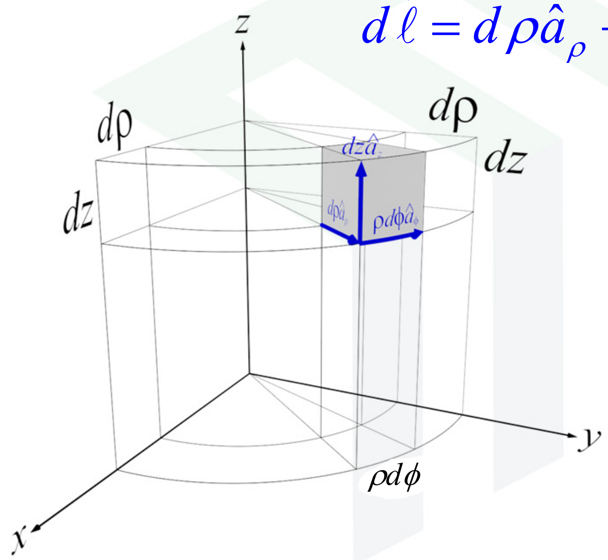
# Cylindrical: *Differential Lengths*

$$d\vec{\ell} = d\rho\hat{a}_{\rho} + \rho d\phi\hat{a}_{\phi} + dz\hat{a}_{z}$$

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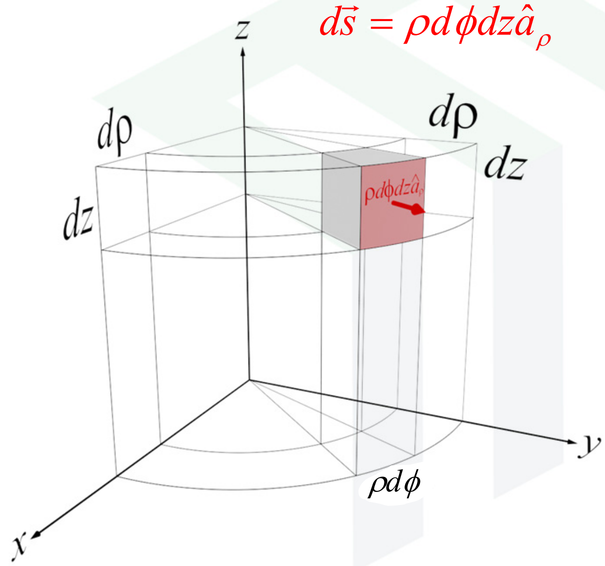
# Cylindrical: *Differential Areas*

$$d\vec{s} = \rho d\phi dz\hat{a}_{\rho}$$

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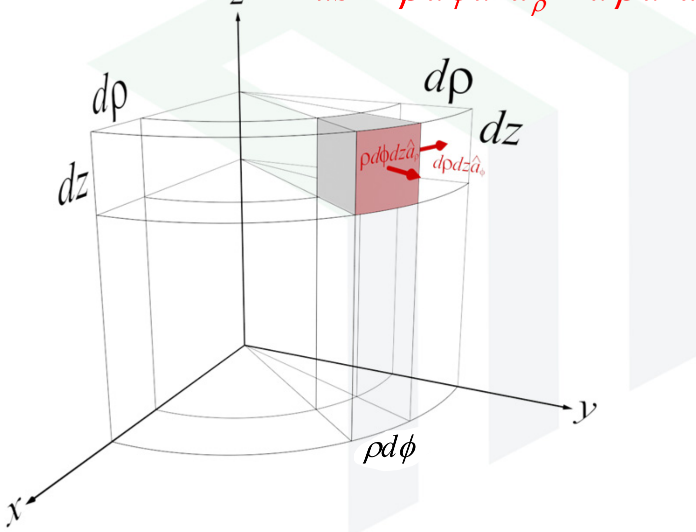
# Cylindrical: *Differential Areas*

$$d\vec{s} = \rho d\phi dz\hat{a}_\rho + d\rho dz\hat{a}_\phi$$

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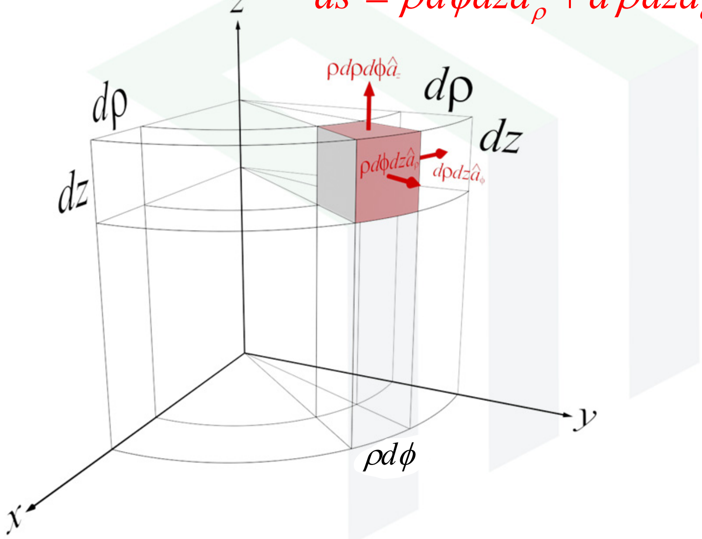
# Cylindrical: *Differential Areas*

$$d\vec{s} = \rho d\phi dz\hat{a}_\rho + d\rho dz\hat{a}_\phi + \rho d\rho d\phi\hat{a}_z$$

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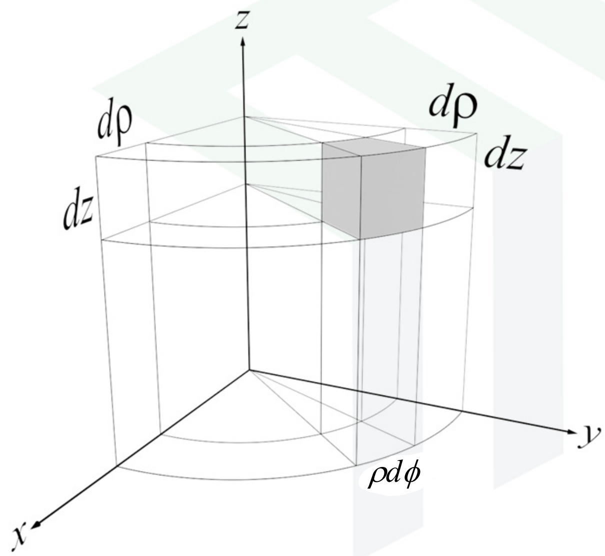
# Cylindrical: *Differential Volume*

$$dv = \rho d\rho d\phi dz$$

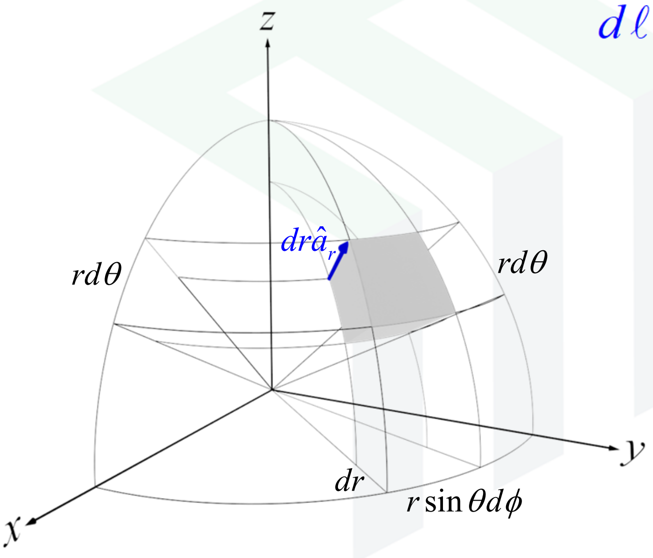





# Spherical Coordinates

# Spherical: *Differential Lengths*

$$d\vec{\ell} = dr\hat{a}_r$$

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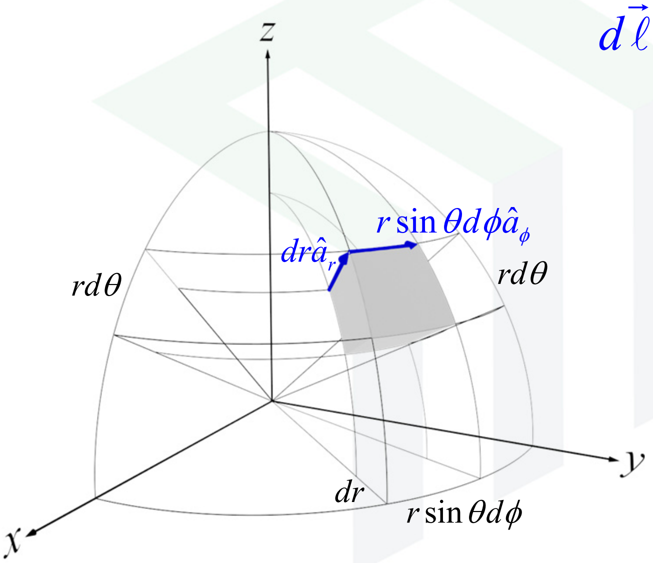
# Spherical: *Differential Lengths*

$$d\vec{\ell} = dr\hat{a}_r + r\sin\theta d\phi\hat{a}_\phi$$

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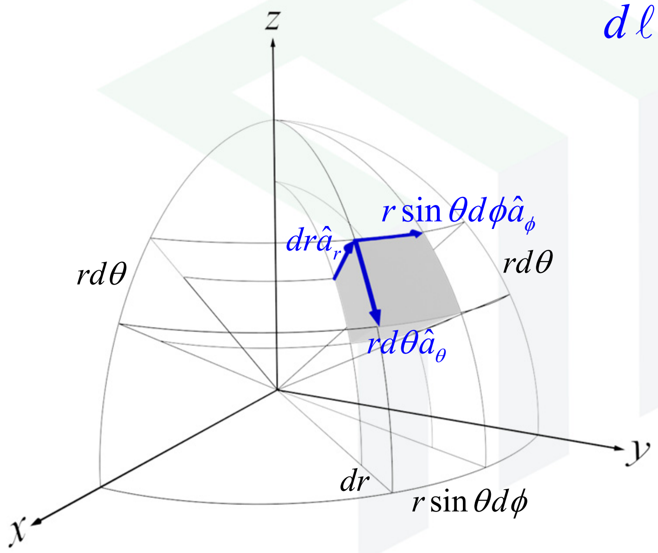
# Spherical: *Differential Lengths*

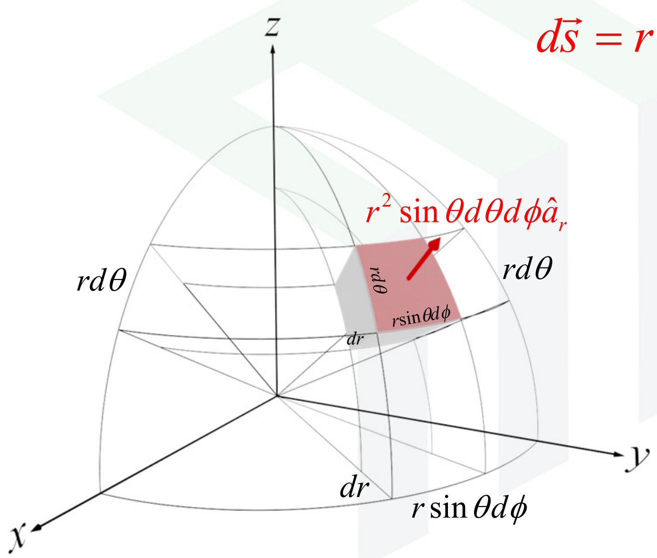
$$d\vec{\ell} = dr\hat{a}_r + rd\theta\hat{a}_\theta + r\sin\theta d\phi\hat{a}_\phi$$





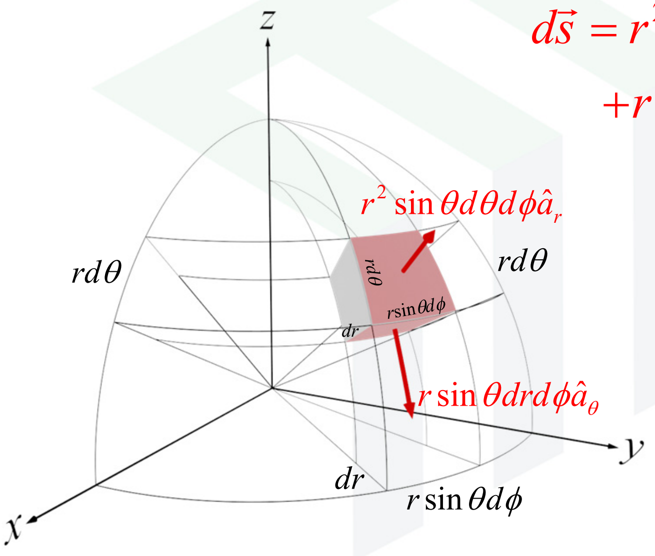

# Spherical: *Differential Areas*

$$d\vec{s} = r^2\sin\theta d\theta d\phi\hat{a}_r$$



# Spherical: *Differential Areas*

$$d\vec{s} = r^2 \sin\theta d\theta d\phi \hat{a}_r + r\sin\theta dr d\phi \hat{a}_\theta$$

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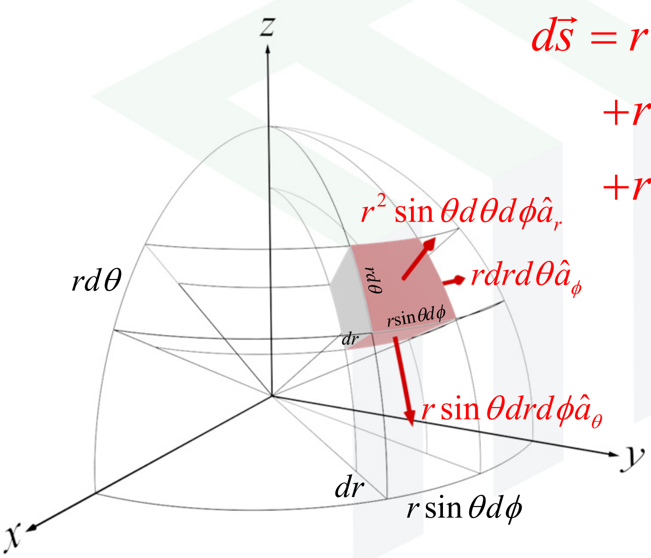
# Spherical: *Differential Areas*


$$d\vec{s} = r^2 \sin\theta d\theta d\phi \hat{a}_r + r\sin\theta dr d\phi \hat{a}_\theta + r dr d\theta \hat{a}_\phi$$

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# Spherical: *Differential Volume*

$$dv = r^2 \sin\theta dr d\theta d\phi$$