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Electromagnetics:
Electromagnetic Field Theory

# Integrations & Constant Coordinate Lines & Surfaces

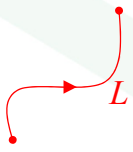
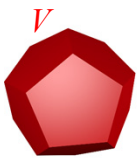
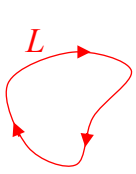
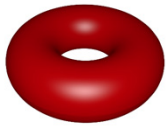
## Outline

- Integrations
- Constant Coordinate Lines & Areas

# Integrations



## Types of Integrations

| Ordinary Line Integral | Ordinary Surface Integral | Ordinary Volume Integral |
|---|---|---|
| $\int_L d\ell$   | $\iint_S ds$   | $\iiint_V dv$   |
| **Closed-Contour Line Integral** | **Closed-Contour Surface Integral** | **Closed-Contour Volume Integral** |
| $\oint_L d\ell$   | $\oiint_S ds$   | $\oiiint_V dV$ ? |

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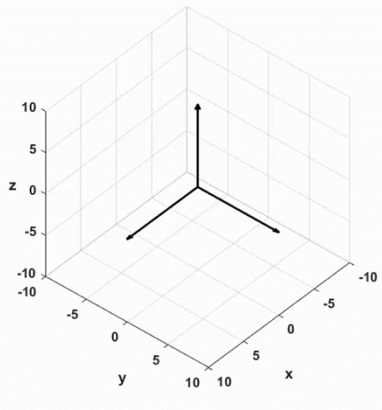
# Cube Integration




$$f(x, y_0, z_0) = \int_{x=0}^{0.0} dx \hat{a}_x$$

$$y_0 = z_0 = 0$$

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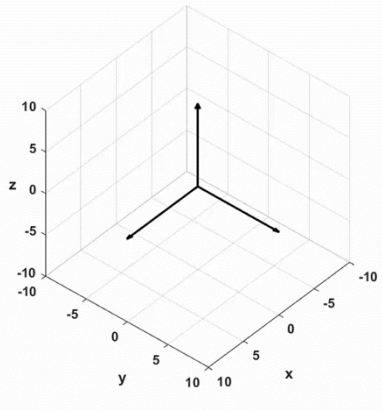
# Cylinder Integration




$$f(\rho, \phi_0, z_0) = \int_{\rho=0}^{0.0} d\rho \hat{a}_\rho$$

$$\phi_0 = z_0 = 0$$

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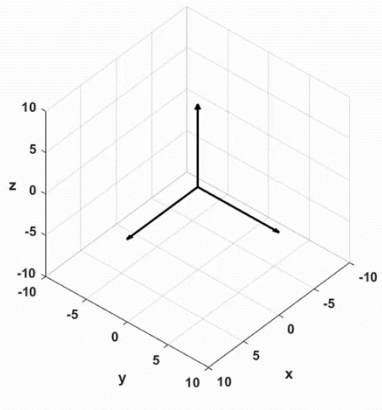
# Sphere Integration

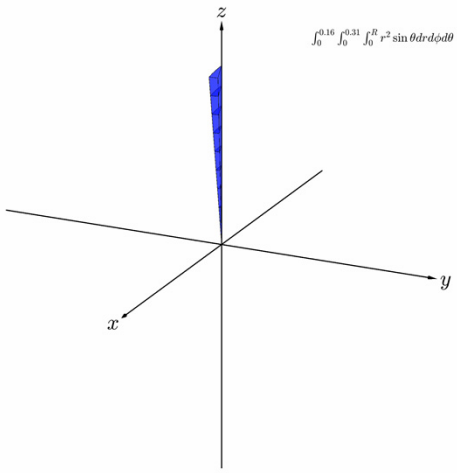



$$f(r,\theta_0,\phi_0)=\int_{r=0}^{0.0} dr\hat{a}_r$$

$$\theta_0=\pi/2,\phi_0=0$$

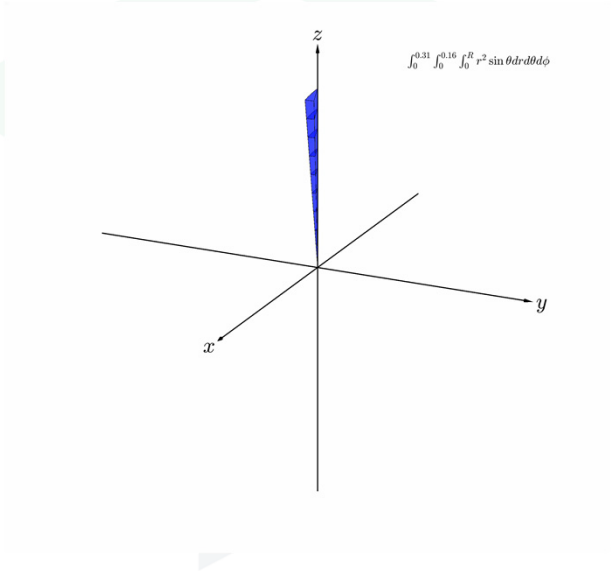


# Sphere Integration




$$\int_0^{0.16}\int_0^{0.31}\int_0^{R} r^2\sin\theta dr d\phi d\theta$$

# Sphere Integration

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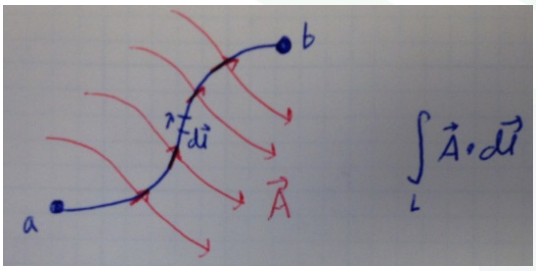
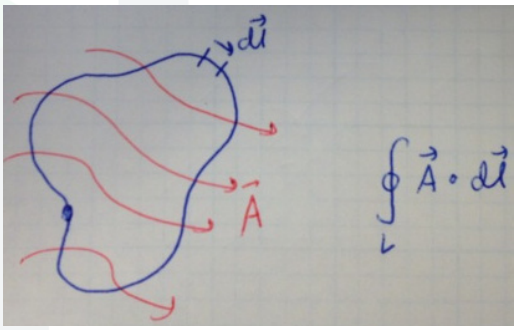
# Constant Coordinate Lines and Areas

# Line Integrals

Ordinary Line Integral




Closed-Contour Line Integral

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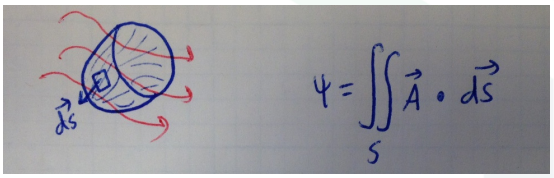
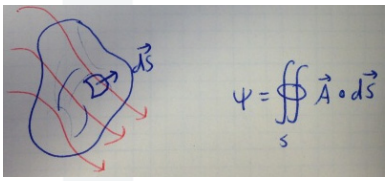
Slide 11

# Surface Integrals

Ordinary Surface Integral




Closed-Surface Integral

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Slide 12

# Volume Integrals

Ordinary Volume Integral

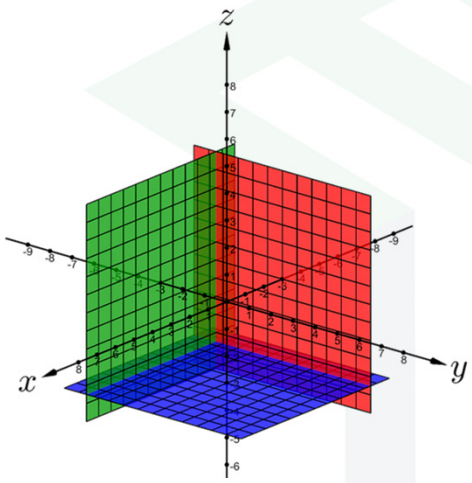
$$\iiint_V dV$$

Closed-Volume Integral

?



# Constant Coordinate Surfaces: *Cartesian* $(x,y,z)$




Constant $x$

$$\int_{z_1}^{z_2}\int_{y_1}^{y_2} f(x_0,y,z)\,dydz$$

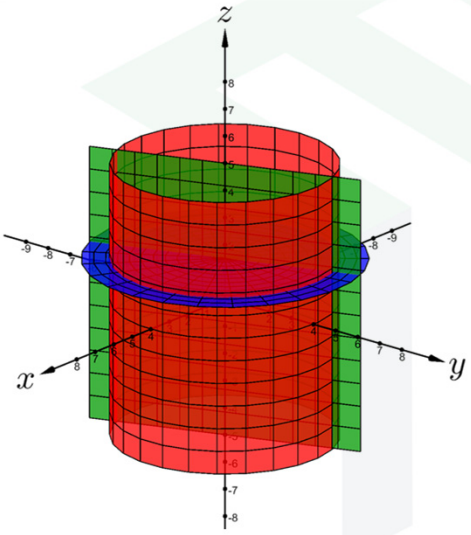
Constant $y$

$$\int_{z_1}^{z_2}\int_{x_1}^{x_2} f(x,y_0,z)\,dxdz$$

Constant $z$

$$\int_{y_1}^{y_2}\int_{x_1}^{x_2} f(x,y,z_0)\,dxdy$$

# Constant Coordinate Surfaces: *Cylindrical* $(\rho,\phi,z)$




Constant $\rho$

$$\int_{z_1}^{z_2}\int_{0}^{2\pi} f(\rho_0,\phi,z)\,\rho_0 d\phi dz$$

Constant $\phi$

$$\int_{z_1}^{z_2}\int_{\rho_1}^{\rho_2} f(\rho,\phi_0,z)\,d\rho dz$$

Constant $z$

$$\int_{\phi=0}^{2\pi}\int_{\rho=0}^{\rho_0} f(\rho,\phi,z_0)\,\rho d\rho d\phi$$

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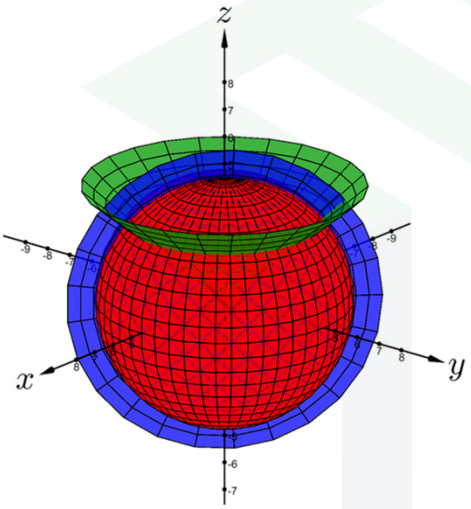
# Constant Coordinate Surfaces: *Spherical* $(r,\theta,\phi)$




Constant $r$

$$\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} f(r_0,\theta,\phi)\,r_0^2\sin\theta d\theta d\phi$$

Constant $\theta$

$$\int_{\phi=0}^{2\pi}\int_{r=0}^{r_0} f(r,\theta_0,\phi)\,r\sin\theta dr d\phi$$

Constant $\phi$

$$\int_{\theta=0}^{\pi}\int_{r=0}^{r_0} f(r,\theta,\phi_0)\,r dr d\theta$$

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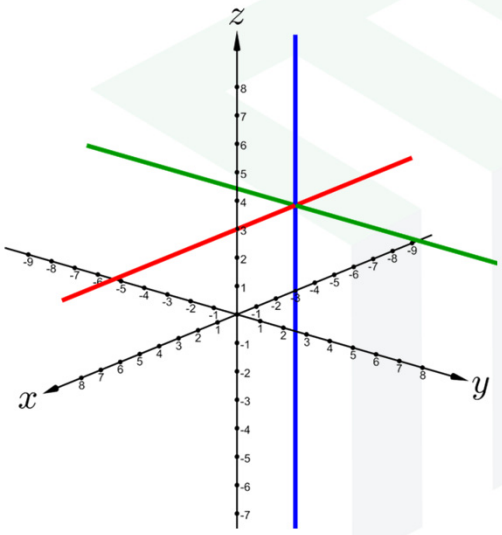
# Constant Coordinate Lines: *Cartesian* $(x,y,z)$




Constant $y$ & $z$

$$\int_{x_1}^{x_2} f(x, y_0, z_0)\, dx$$

Constant $x$ & $z$

$$\int_{y_1}^{y_2} f(x_0, y, z_0)\, dy$$

Constant $x$ and $y$

$$\int_{z_1}^{z_2} f(x_0, y_0, z)\, dz$$

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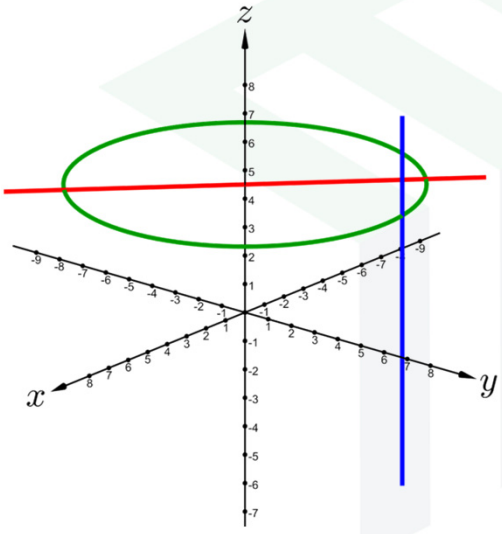
# Constant Coordinate Lines: *Cylindrical* $(\rho,\phi,z)$




Constant $\phi$ & $z$

$$\int_{\rho_1}^{\rho_2} f(\rho, \phi_0, z_0)\, d\rho$$

Constant $\rho$ & $z$

$$\int_{\phi=0}^{2\pi} f(\rho_0, \phi, z_0)\, \rho_0 d\phi$$

Constant $\rho$ and $\phi$

$$\int_{z_1}^{z_2} f(\rho_0, \phi_0, z)\, dz$$

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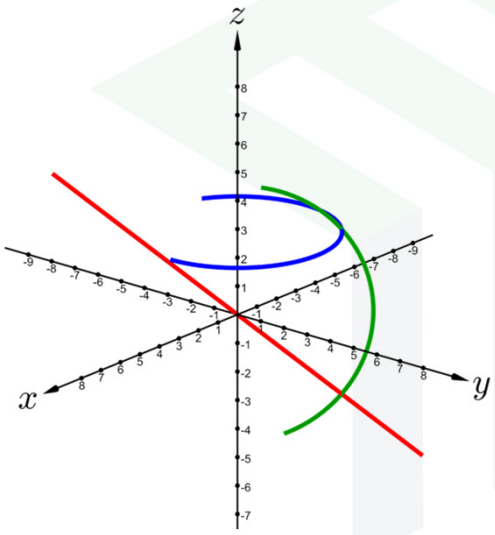
# Constant Coordinate Lines: *Spherical* $(r,\theta,\phi)$




Constant $\theta$ & $\phi$

$$\int_{r_1}^{r_2} f(r,\theta_0,\phi_0)\,dr$$

Constant $r$ & $\phi$

$$\int_{\theta_1}^{\theta_2} f(r_0,\theta,\phi_0)\,r_0 d\theta$$

Constant $r$ and $\theta$

$$\int_{\phi_1}^{\phi_2} f(r_0,\theta_0,\phi)\,r_0 \sin\theta_0 d\phi$$

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Slide 19