



Electromagnetics:
Electromagnetic Field Theory

Charge Distributions



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Outline

- Charge Distributions
- Fields Around Charge Distributions
- Recipe to Calculate Field Around Charge Distributions

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Charge Distributions

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Point Charge

 Q

Size: infinitely small

Total Charge: Q

$$\text{Total Charge: } Q = \sum_{i=1}^N Q_i$$

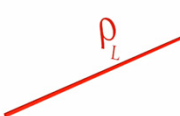
$$\text{Total Field: } \vec{D}_{\text{Total}} = \sum_{i=1}^N \frac{Q_i}{4\pi R_i^2} \hat{a}_{R_i}$$

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Line Charge



Size: Length L

Charge Density: Coulombs/Meter (C/m)

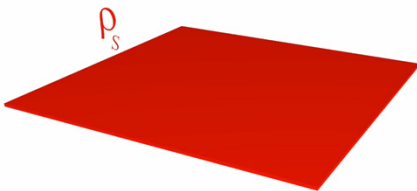
Total Charge: $Q = \begin{cases} \int_L \rho_\ell d\ell & \text{general case} \\ \rho_\ell L & \text{uniform } \rho_\ell \end{cases}$

Total Field: $\vec{D}_{\text{Total}} = \int_\ell \frac{\rho_\ell d\ell}{4\pi R^2} \hat{a}_R$

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Surface Charge



Size: Length L
Width W

Charge Density: Coulombs/Meter² (C/m²)

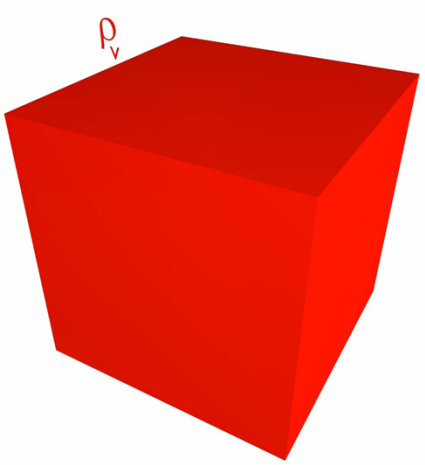
Total Charge: $Q = \begin{cases} \iint_S \rho_s ds & \text{general case} \\ \rho_s S & \text{uniform } \rho_s \end{cases}$

Total Field: $\vec{D}_{\text{Total}} = \iint_S \frac{\rho_s ds}{4\pi R^2} \hat{a}_R$

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Volume Charge



Size: Length L
Width W
Height H

Charge Density: Coulombs/Meter³ (C/m³)

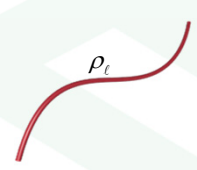

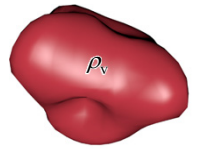
Total Charge: $Q = \begin{cases} \iiint_v \rho_v dv & \text{general case} \\ \rho_v v & \text{uniform } \rho_v \end{cases}$

Total Field: $\vec{D}_{\text{Total}} = \iiint_v \frac{\rho_v dv}{4\pi R^2} \hat{a}_R$

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Charge Distribution Summary

Point Charge	Line Charge	Sheet Charge	Volume Charge
Q			
Charge Q (C)	Line Charge Density ρ_l (C/m)	Surface Charge Density ρ_s (C/m ²)	Volume Charge Density ρ_v (C/m ³)
Total Charge $Q_{\text{Total}} = \sum_{i=1}^N Q_i$	Total Charge $Q_{\text{Total}} = \int_l \rho_l dl \cong \rho_l L$	Total Charge $Q_{\text{Total}} = \iint_S \rho_s ds \cong \rho_s S$	Total Charge $Q_{\text{Total}} = \iiint_v \rho_v dv \cong \rho_v V$
Total Field $\vec{D}_{\text{Total}} = \sum_{i=1}^N \frac{Q_i}{4\pi R_i^2} \hat{a}_{R_i}$	Total Field $\vec{D}_{\text{Total}} = \int_l \frac{\rho_l dl}{4\pi R^2} \hat{a}_R$	Total Field $\vec{D}_{\text{Total}} = \iint_s \frac{\rho_s ds}{4\pi R^2} \hat{a}_R$	Total Field $\vec{D}_{\text{Total}} = \iiint_v \frac{\rho_v dv}{4\pi R^2} \hat{a}_R$

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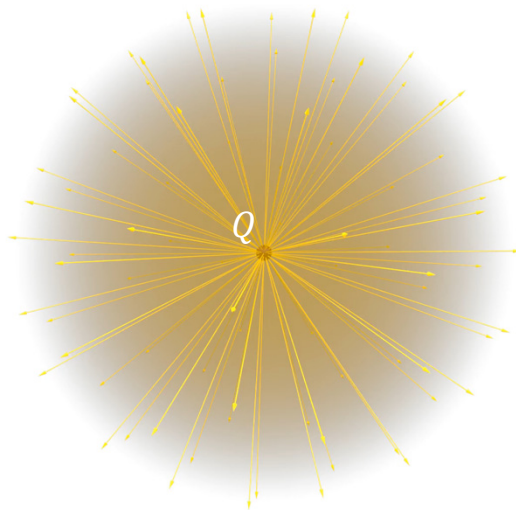
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Fields Around Charge Distributions

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Point Charge



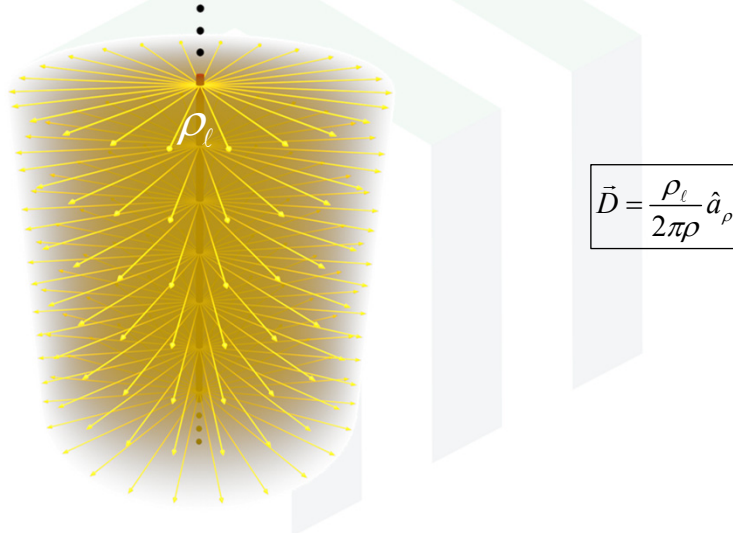
$$\vec{D} = \frac{Q}{4\pi r^2} \hat{a}_r$$

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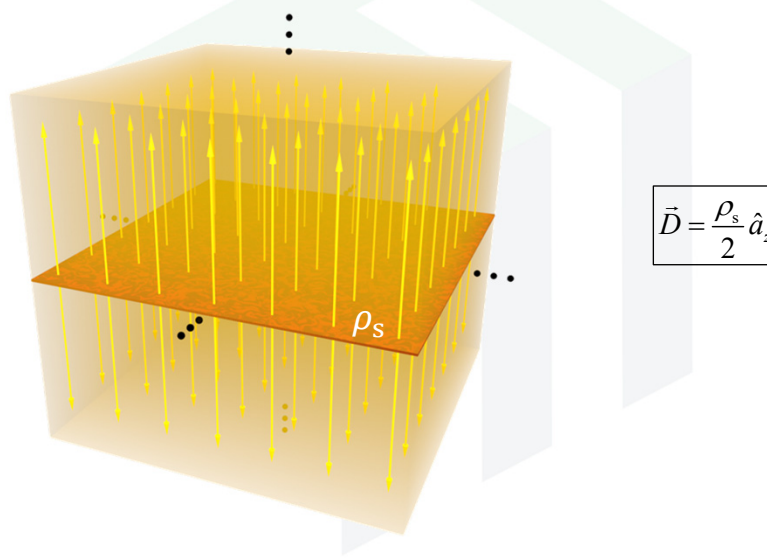
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Infinite Line Charge

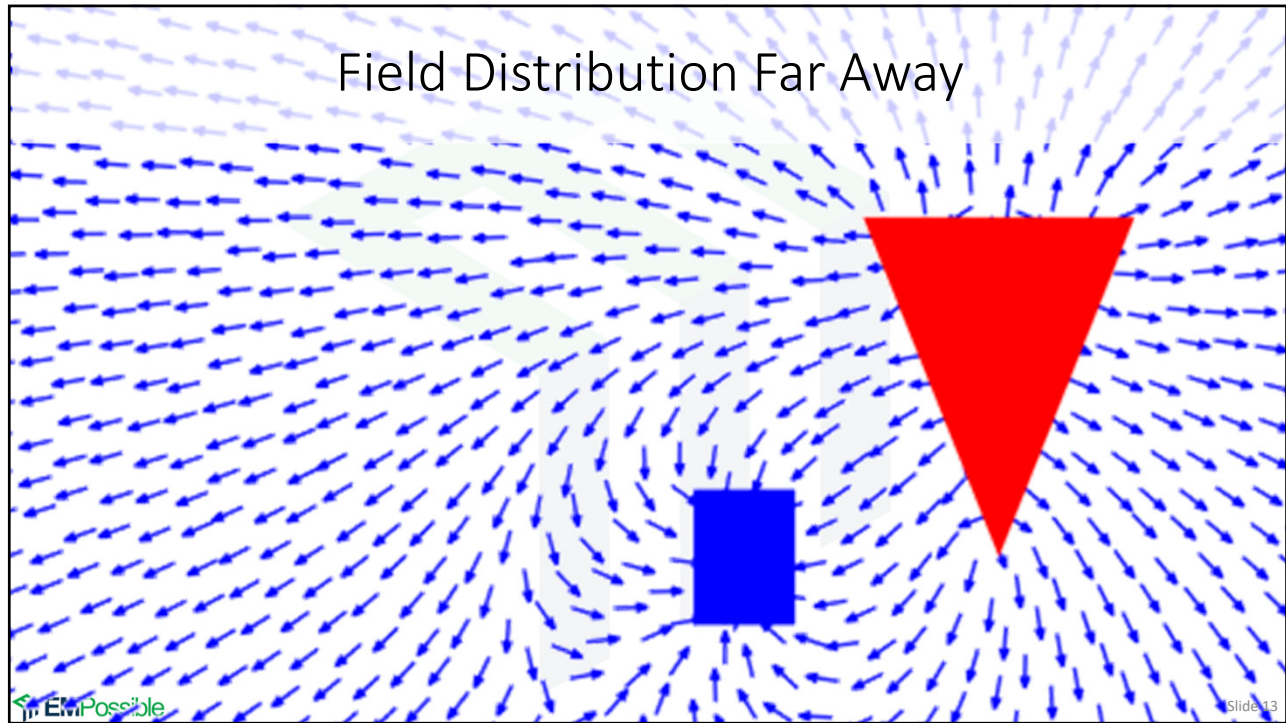


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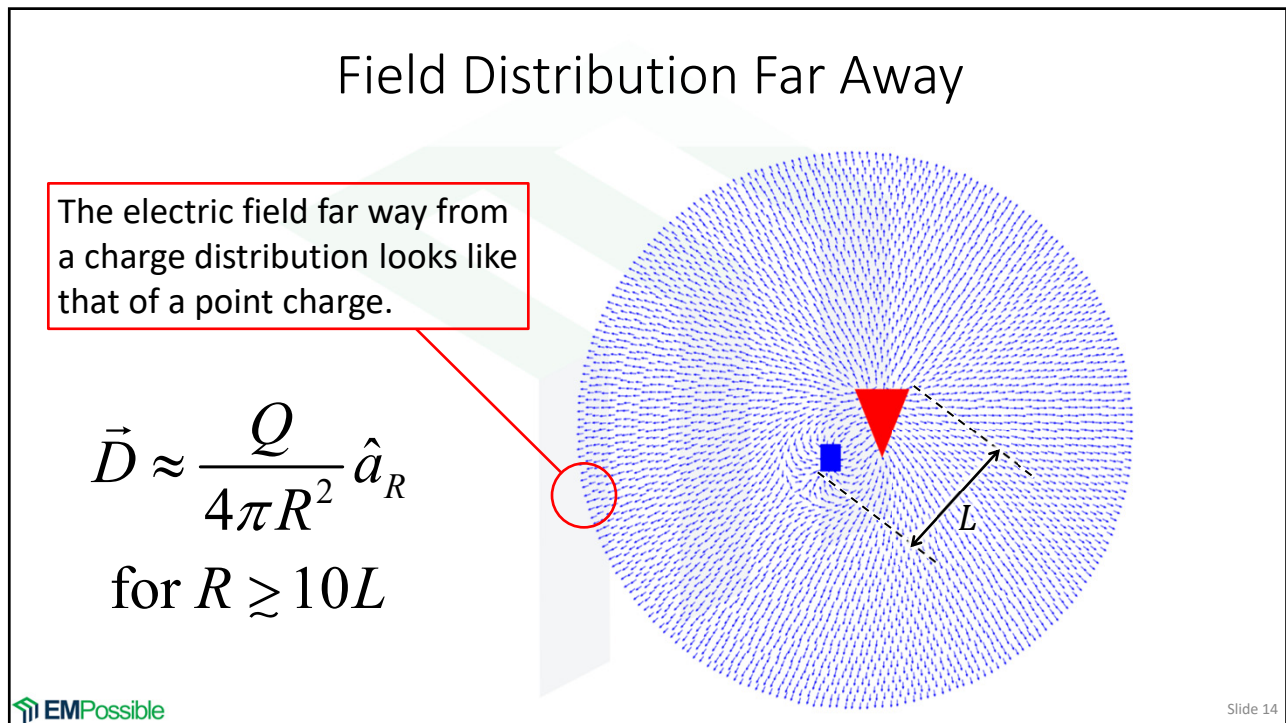
Infinite Surface Charge



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Recipe to Calculate Field Around Charge Distributions

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Recipe for Solving Problems

1. Draw the problem and label with all dimensions and parameters.
2. Choose a coordinate system that makes the math easiest.
3. Write the general equation.

	Point	Line	Surface	Volume
Q_{total}	$\sum_{i=1}^N q_i$	$\int \rho \, dl$	$\iint \rho \, ds$	$\iiint \rho \, dv$
\vec{D}_{total}	$\sum_{i=1}^N \frac{q_i}{4\pi R_i^2} \hat{a}_n$	$\int \frac{\rho \, dl}{4\pi R^2} \hat{a}_n$	$\iint \frac{\rho \, ds}{4\pi R^2} \hat{a}_n$	$\iiint \frac{\rho \, dv}{4\pi R^2} \hat{a}_n$

4. Write expressions for each term in the integral.
5. Choose limits of integration.
6. Solve the integral.
7. Interpret the result.

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