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Electromagnetics:
Electromagnetic Field Theory

Forces on Point Charges

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Force on a Point Charge

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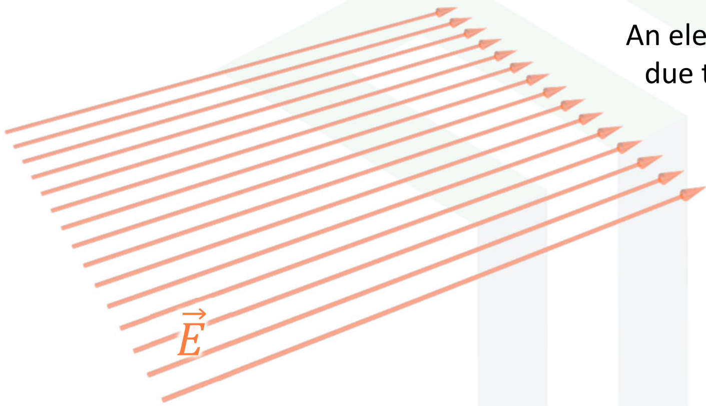
Force $\vec{F}$ on a Point Charge Q



An electric field $\vec{E}$ may be present due to other charges, an applied voltage, a wave, etc.

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3

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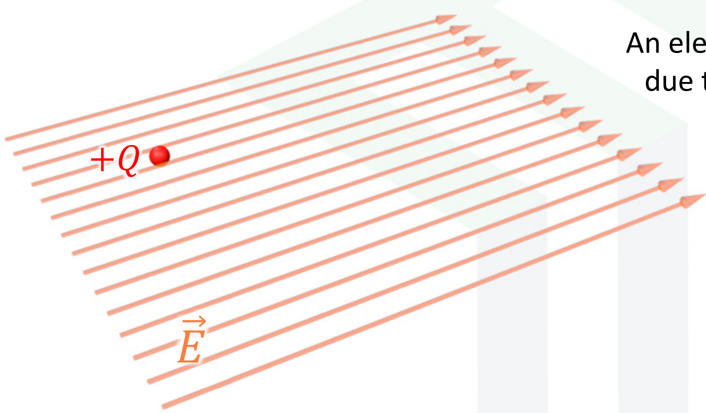

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Let there be a charge Q.

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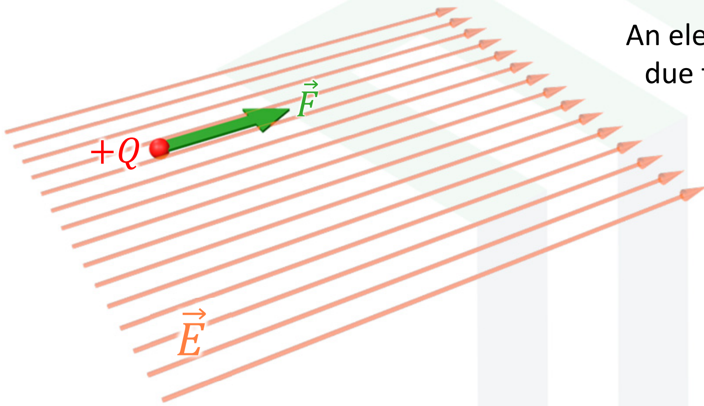

An electric field $\vec{E}$ may be present due to other charges, an applied voltage, a wave, etc.

Let there be a charge Q.

$$\vec{F} = Q\vec{E}$$

$\vec{F} \equiv$ force vector (N)

Force is most closely associated with electric field intensity $\vec{E}$.

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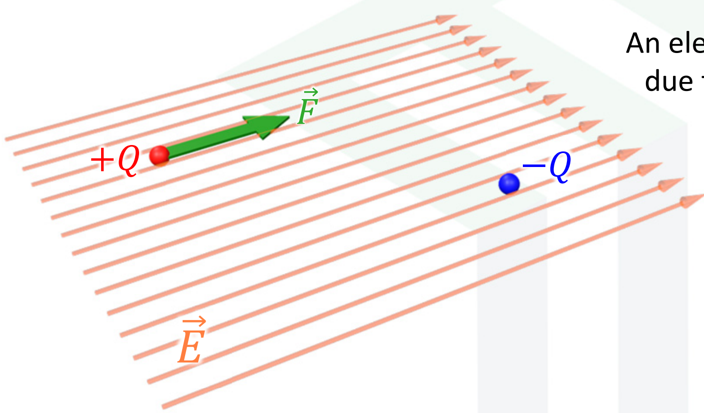
Force $\vec{F}$ on a Point Charge Q



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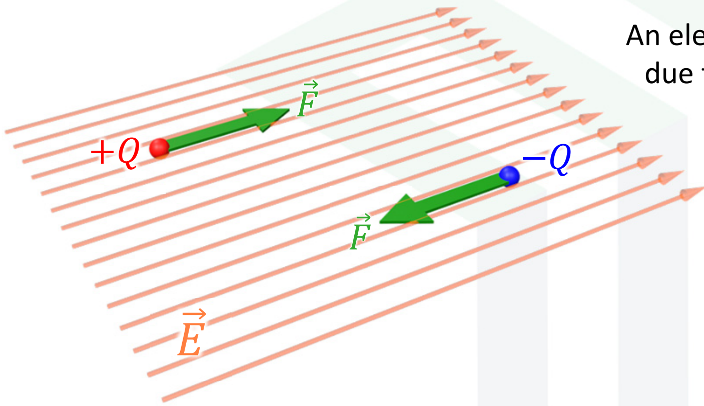

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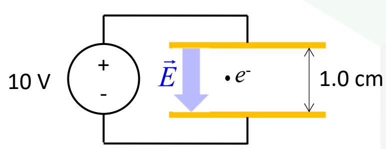
Example

What is the force $\vec{F}$ on an electron located between two metal plates separated by 1 cm and 10 volts is applied?

<u>Solution</u>

Sketch the problem.



Calculate the electric field intensity $\vec{E}$.

$$\vec{E} = -\frac{V}{d}\hat{a}_z = -\frac{10\text{ V}}{0.01\text{ m}}\hat{a}_z = -1000\hat{a}_z\ \text{V/m}$$

Calculate the force on the electron.

$$\begin{aligned}\vec{F} &= Q\vec{E}\\ &= \left(-1.60\times10^{-19}\text{ C}\right)\left(-1000\hat{a}_z\ \text{V/m}\right)\\ &= 1.60\times10^{-16}\hat{a}_z\ \text{N}\\ &= 160\hat{a}_z\ \text{aN}\end{aligned}$$

Attonewtons (wow, that is small!)

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Coulomb's Law

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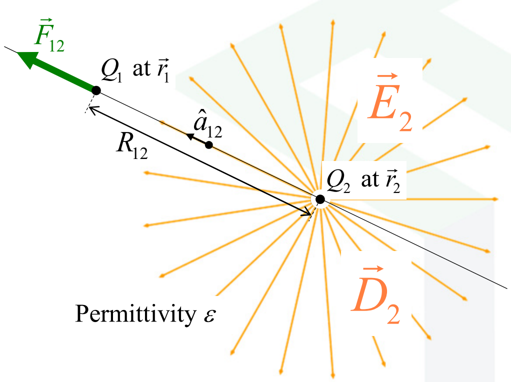
Coulomb's Law



Force on Q_1 due to Q_2 is

$$\vec{F}_{21} = \frac{Q_1 Q_2}{4\pi\varepsilon R_{12}^2}\hat{a}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon}\frac{\vec{r}_2 - \vec{r}_1}{\left|\vec{r}_2 - \vec{r}_1\right|^3}$$

Given two charges, Q_1 and Q_2.

Charge Q_2 creates an electric flux $\vec{D}_2$.

$$\vec{D}_2 = \frac{Q_2\left(\vec{r} - \vec{r}_{Q_2}\right)}{4\pi\left|\vec{r} - \vec{r}_{Q_2}\right|^3}$$

The electric field intensity $\vec{E}_2$ is calculated from the electric flux density $\vec{D}_2$.

$$\vec{E}_2 = \frac{1}{\varepsilon}\vec{D}_2$$

Electric field $\vec{E}_2$ puts a force on charge Q_1.

$$\vec{F}_{21} = Q_1\vec{E}_2$$

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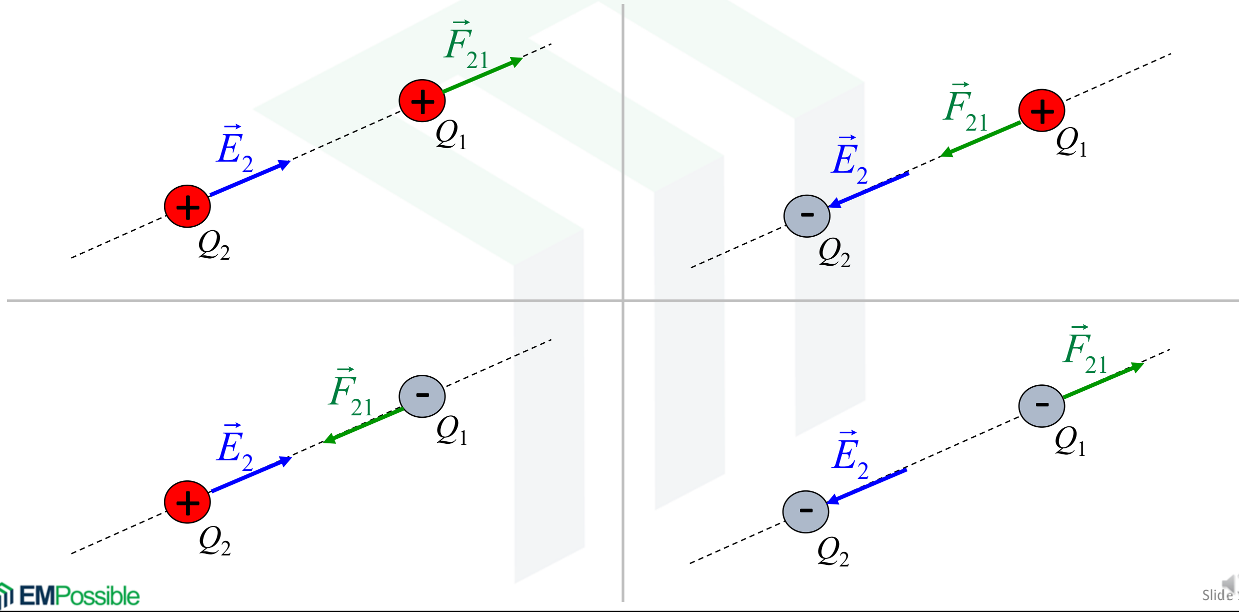
Summary of Direction of Force on Q_1 Due to Q_2



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Notes About Coulomb's Law

- Direction of force is along the line joining the two charges.
- Force is directly proportional to $Q_1 Q_2$.
- Force is inversely proportional to R^2.
- Charges must be stationary for these equations to be valid (i.e. electrostatics).

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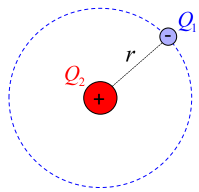
Example

Calculate the attractive force between the electron and proton in a hydrogen atom.

Solution

Sketch the problem.



Charge:

$Q_1 = -1.60\times10^{-19}\ \text{C}$

$Q_2 = +1.60\times10^{-19}\ \text{C}$

Distance:

$r \approx 53\ \text{pm}$

Calculate the force

$$\vec{F}_{21} = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r R_{12}^2}\hat{a}_{12}$$

$$= \frac{\left(-1.60\times10^{-19}\ \text{C}\right)\left(+1.60\times10^{-19}\ \text{C}\right)}{4\pi\left(8.854\times10^{-12}\ \text{F/m}\right)(1.0)\left(53\times10^{-12}\ \text{m}\right)^2}\hat{a}_r$$

$$= -8.19\times10^{-8}\hat{a}_r\ \text{N}$$

$$\boxed{= 82\ \text{nN}}$$

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What About Time t?

At $t = 0$, a charge Q with mass m is located at the origin and moving with at an initial velocity of v_0 in the $+x$ direction. At this instant in time, a uniform electric field $\vec{E} = E_0\hat{a}_y$ is applied. Derive an expression for the position of the charge as a function of time for $t \geq 0$.

Force due to electric field. $\vec{F}(t) = Q(t)\vec{E}(t)$

Acceleration $\vec{F}(t) = m(t)a(t)$ $\quad a(t) = \dfrac{\vec{F}(t)}{m(t)} = \dfrac{Q\vec{E}}{m} = \dfrac{QE_0\hat{a}_y}{m}$

Velocity $\vec{v}(t) = \vec{v}_0 + \displaystyle\int_{-\infty}^{t} a(\tau)d\tau = v_0\hat{a}_x + \int_0^t \frac{QE_0\hat{a}_y}{m}d\tau = v_0\hat{a}_x + \frac{QE_0t\hat{a}_y}{m}$

Position $\vec{r}(t) = \vec{r}_0 + \displaystyle\int_{-\infty}^{t} v(\tau)d\tau = \int_0^t \left(v_0\hat{a}_x + \frac{QE_0\tau\hat{a}_y}{m}\right)d\tau = v_0t\hat{a}_x + \frac{QE_0t^2}{2m}\hat{a}_y$

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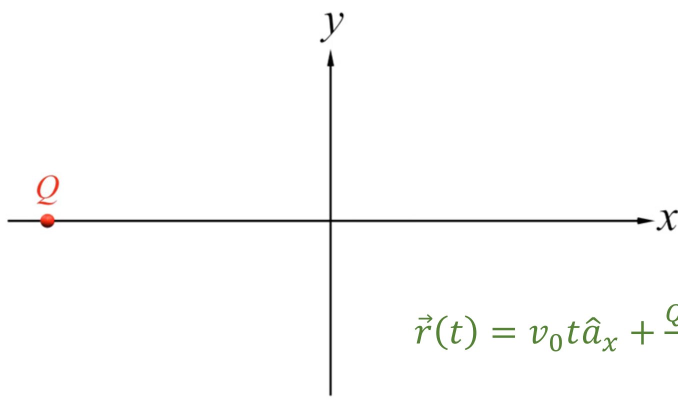

$$\vec{r}(t) = v_0 t \hat{a}_x + \frac{Q E_0 t^2}{2m} \hat{a}_y \qquad t \geq 0$$

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Coulomb’s Law for Multiple Point Charges

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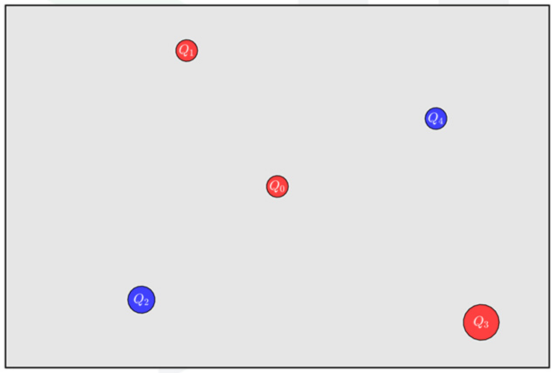
Coulomb's Law for Multiple Charges

The force on charge Q_0 due to charges Q_1 to Q_N is

$$\vec{F}_0 = \sum_{i=1}^{N} \vec{F}_{0i} = \sum_{i=1}^{N} \frac{Q_0 Q_i}{4\pi\varepsilon R_{0i}^2} \hat{a}_{0i} = \sum_{i=1}^{N} \frac{Q_0 Q_i}{4\pi\varepsilon} \frac{\vec{r}_0 - \vec{r}_i}{\left|\vec{r}_0 - \vec{r}_i\right|^3}$$



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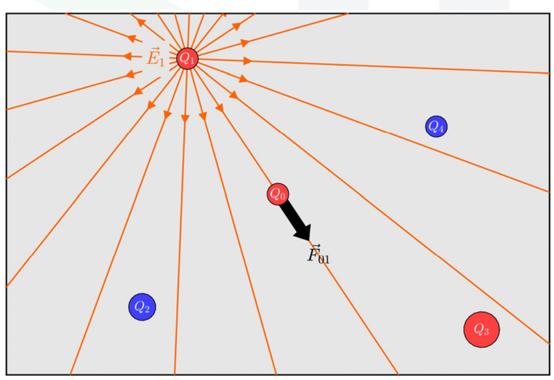

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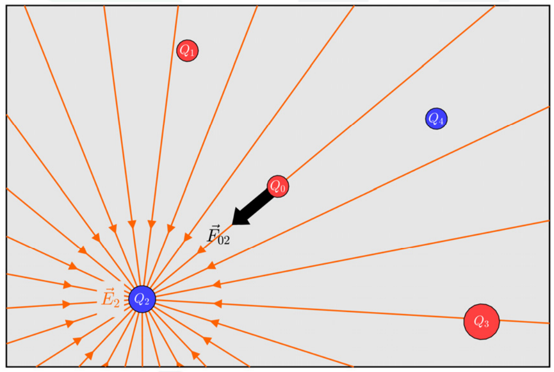

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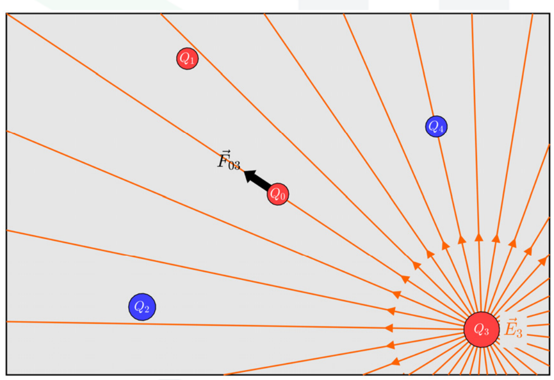

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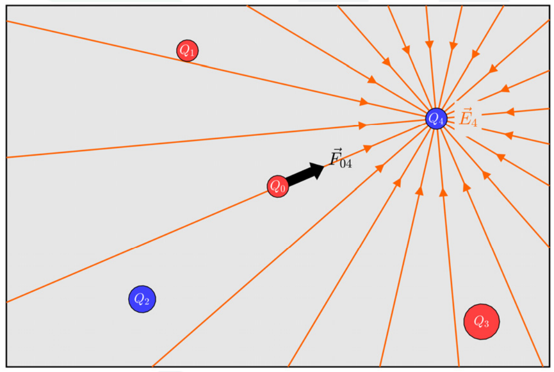

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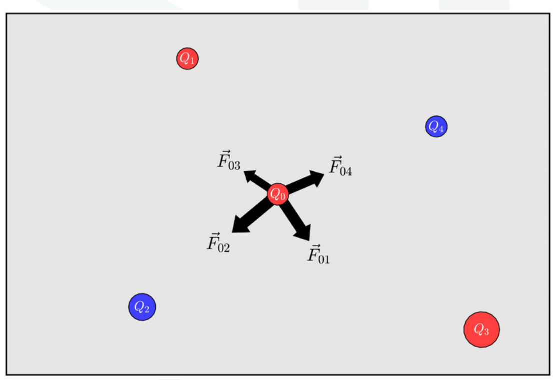

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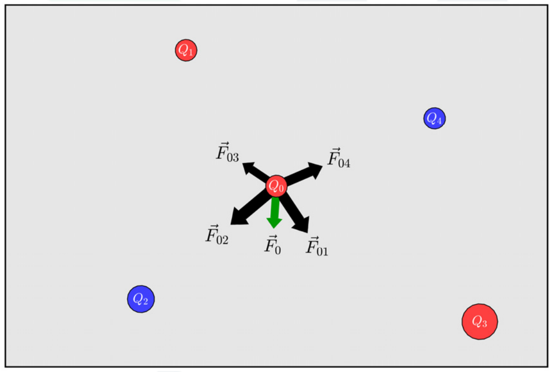

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