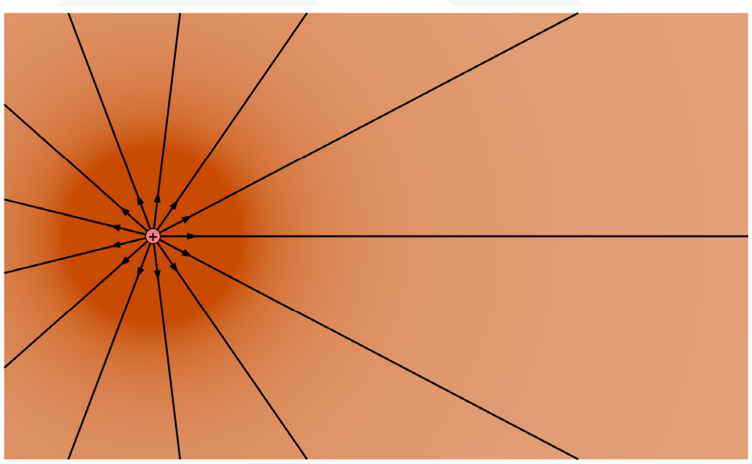
Electromagnetics:
Electromagnetic Field Theory

Multiple Point Charges

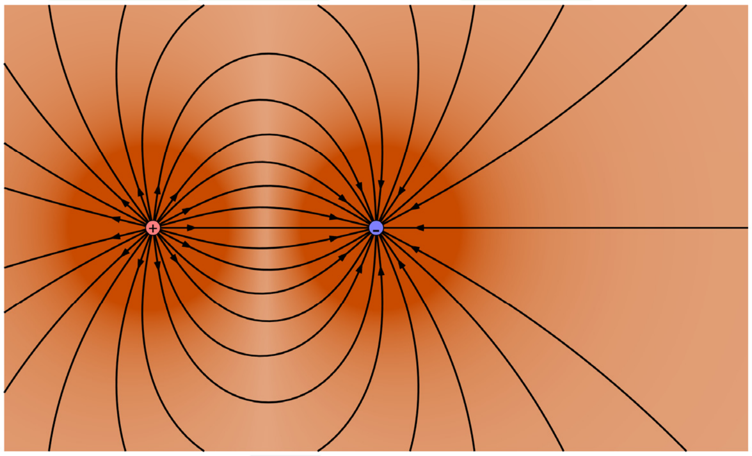
1

Single Charge



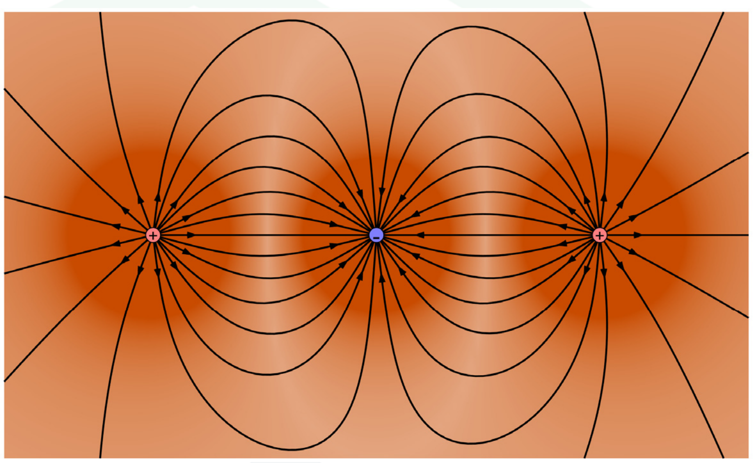
2

Two Charges



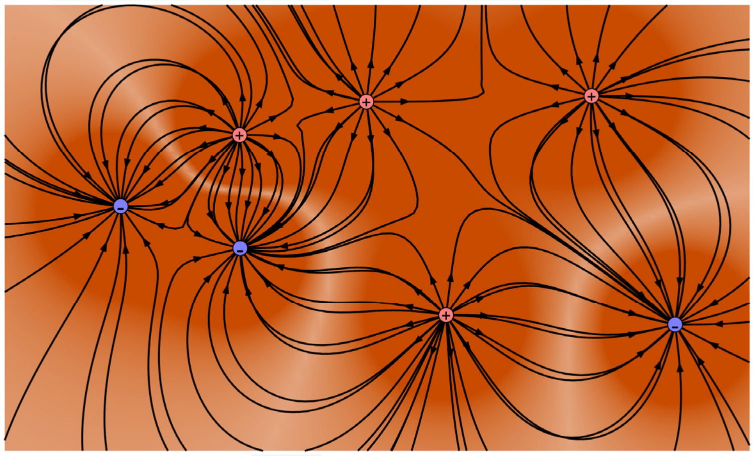
3

Three Charges



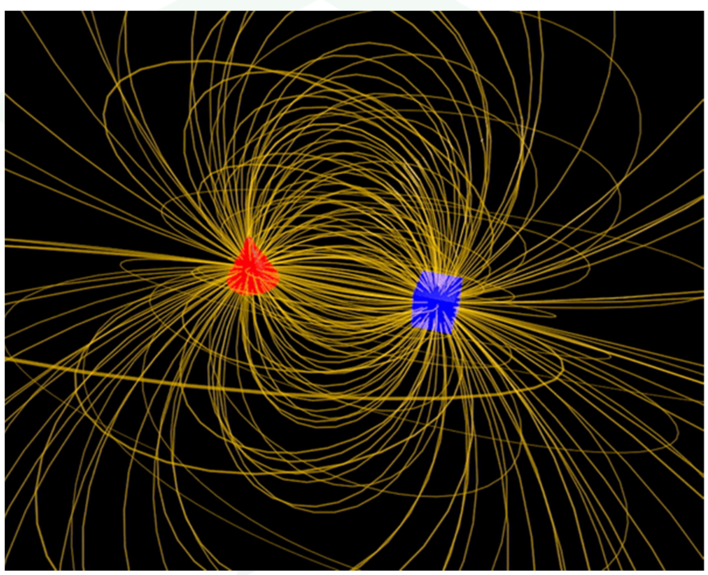
4

Craziness



5

Multiple Charges in 3D



6

Total Electric Flux Density $\vec{D}$

Superposition – the total electric flux arising from multiple point charges is the sum of the electric flux produced by the charges independently.

$$\vec{D}_{\text{Total}} = \sum_{i=1}^{N} \vec{D}_i = \sum_{i=1}^{N} \frac{Q_i}{4\pi R_i^2} \hat{a}_{R_i} = \sum_{i=1}^{N} \frac{Q_i \left(\vec{r} - \vec{r}_{Q_i} \right)}{4\pi \left| \vec{r} - \vec{r}_{Q_i} \right|^3}$$

$N \equiv$ total number of point charges

$$\vec{E}_{\text{Total}} = \frac{\vec{D}_{\text{Total}}}{\varepsilon}$$

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Slide 7

7

Example

8

Problem Statement

Given the following three point charges, calculate the electric flux density at the origin.

$$Q_1 = 30\text{ C} \quad \text{at} \quad \vec{r}_{Q_1} = -1.0\hat{a}_x - 1.5\hat{a}_y + 1.0\hat{a}_z \text{ m}$$
$$Q_2 = -100\text{ C} \quad \text{at} \quad \vec{r}_{Q_2} = -2.0\hat{a}_x + 1.0\hat{a}_y - 1.5\hat{a}_z \text{ m}$$
$$Q_3 = 75\text{ C} \quad \text{at} \quad \vec{r}_{Q_3} = 1.0\hat{a}_x - 1.5\hat{a}_y - 1.0\hat{a}_z \text{ m}$$

$$\vec{D}(0,0,0) = ?$$

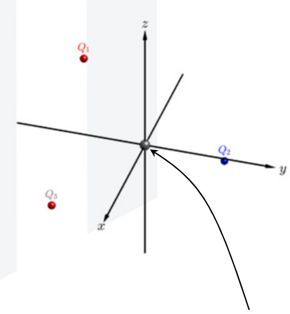
Solution

Step 1 – Sketch the problem

Step 2 – Organize the problem

The total electric flux density is the sum of the electric flux density due to each charge separately.

$$\vec{D} = \vec{D}_1 + \vec{D}_2 + \vec{D}_3$$



Observation point (0,0,0)

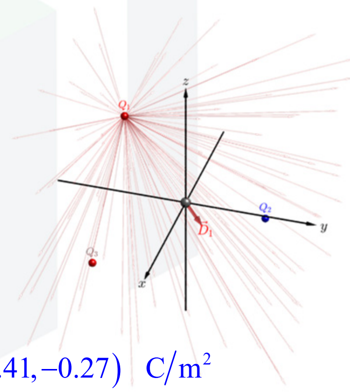
Solution

Step 3 – Calculate $\vec{D}_1$

$$\vec{D}_1 = \frac{Q_1\left(\vec{r}-\vec{r}_{Q_1}\right)}{4\pi\,|\vec{r}-\vec{r}_{Q_1}|^3}$$

$$\begin{aligned}\vec{r}-\vec{r}_{Q_1} &= (0,0,0)-(-1,-1.5,1)\\ &= (1,1.5,-1)\end{aligned}$$

$$\frac{\vec{r}-\vec{r}_{Q_1}}{|\vec{r}-\vec{r}_{Q_1}|^3} = \frac{(1,1.5,-1)}{\left[(1)^2+(1.5)^2+(-1)^2\right]^{3/2}} = (0.11,0.17,-0.11)$$



$$\vec{D}_1 = \frac{Q_1}{4\pi}\frac{\vec{r}-\vec{r}_{Q_1}}{|\vec{r}-\vec{r}_{Q_1}|^3} = \frac{30}{4\pi}(0.11,0.17,-0.11) = (0.27,0.41,-0.27)\quad \text{C/m}^2$$

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Slide 11

11

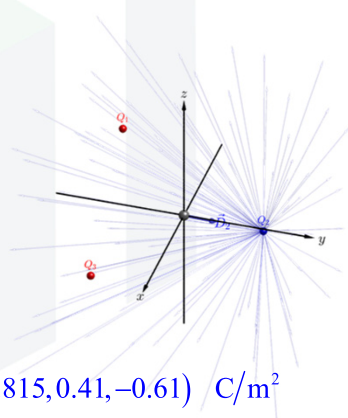
Solution

Step 4 – Calculate $\vec{D}_2$

$$\vec{D}_2 = \frac{Q_2\left(\vec{r}-\vec{r}_{Q_2}\right)}{4\pi\,|\vec{r}-\vec{r}_{Q_2}|^3}$$

$$\begin{aligned}\vec{r}-\vec{r}_{Q_2} &= (0,0,0)-(-2,1,-1.5)\\ &= (2,-1,1.5)\end{aligned}$$

$$\frac{\vec{r}-\vec{r}_{Q_2}}{|\vec{r}-\vec{r}_{Q_2}|^3} = \frac{(2,-1,1.5)}{\left[(2)^2+(-1)^2+(1.5)^2\right]^{3/2}} = (0.10,-0.051,0.077)$$



$$\vec{D}_2 = \frac{Q_2}{4\pi}\frac{\vec{r}-\vec{r}_{Q_2}}{|\vec{r}-\vec{r}_{Q_2}|^3} = \frac{-100}{4\pi}(0.10,-0.051,0.077) = (-0.815,0.41,-0.61)\quad \text{C/m}^2$$

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Slide 12

12

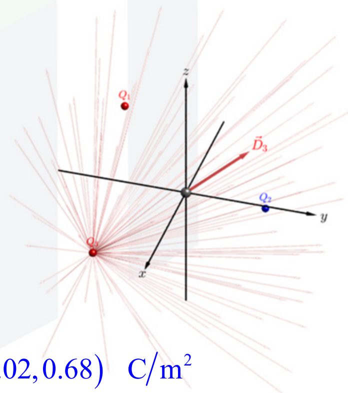
Solution

Step 5 – Calculate $\vec{D}_3$

$$\vec{D}_3 = \frac{Q_3\left(\vec{r}-\vec{r}_{Q_3}\right)}{4\pi\,|\vec{r}-\vec{r}_{Q_3}|^3}$$

$$\begin{aligned}\vec{r}-\vec{r}_{Q_3} &= (0,0,0)-(1,-1.5,-1)\\ &= (-1,1.5,1)\end{aligned}$$

$$\frac{\vec{r}-\vec{r}_{Q_3}}{|\vec{r}-\vec{r}_{Q_3}|^3} = \frac{(-1,1.5,1)}{\left[(-1)^2+(1.5)^2+(1)^2\right]^{3/2}} = (-0.11,0.17,0.11)$$



$$\vec{D}_3 = \frac{Q_3}{4\pi}\frac{\vec{r}-\vec{r}_{Q_3}}{|\vec{r}-\vec{r}_{Q_3}|^3} = \frac{75}{4\pi}(-0.11,0.17,0.11) = (-0.68,1.02,0.68)\quad \text{C/m}^2$$

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Slide 13

13

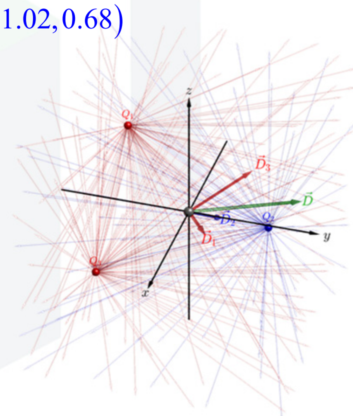
Solution

Step 6 – Calculate the total electric flux density

$$\begin{aligned}\vec{D} &= \vec{D}_1+\vec{D}_2+\vec{D}_3\\ &= (0.27,0.41,-0.27)+(-0.82,0.41,-0.61)+(-0.68,1.02,0.68)\\ &= (-1.22,1.84,-0.20)\quad \text{C/m}^2\end{aligned}$$

The final answer is

$$\boxed{\vec{D} = -1.22\hat{a}_x + 1.84\hat{a}_y - 0.20\hat{a}_z\quad \text{C/m}^2}$$



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Slide 14

14