Electromagnetics:

Electromagnetic Field Theory

Quantifying Electric Flux Around Point Charges

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Quantifying the Electric Flux Around Point Charges

The electric flux density $\vec{D}$ is the quantity most closely associated with electric charge.

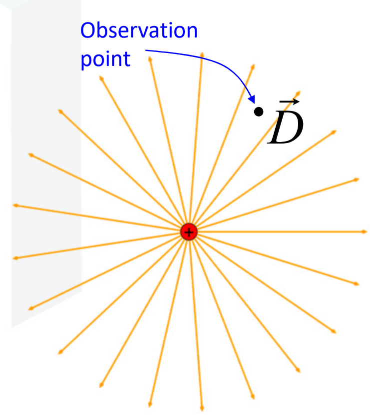
$$\vec{D} = \frac{Q}{4\pi R^2}\hat{a}_R$$

$\vec{D} \equiv$ electric flux density $\left(\text{C/m}^2\right)$

$Q \equiv$ total charge of point (C)

$R \equiv$ distance from charge (m)

$\hat{a}_R \equiv$ unit vector in radial direction (no units)



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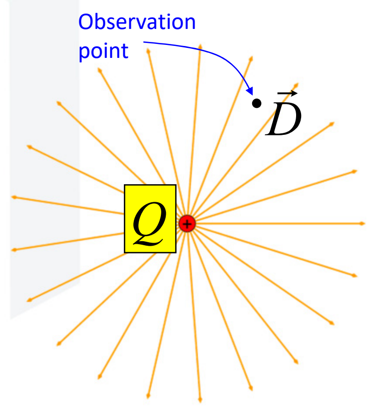

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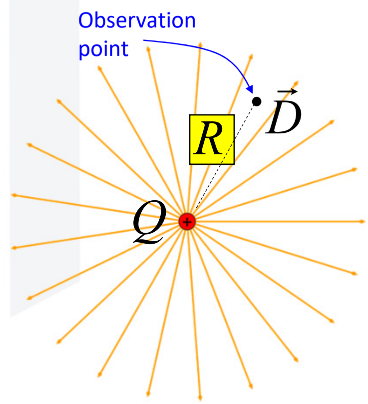

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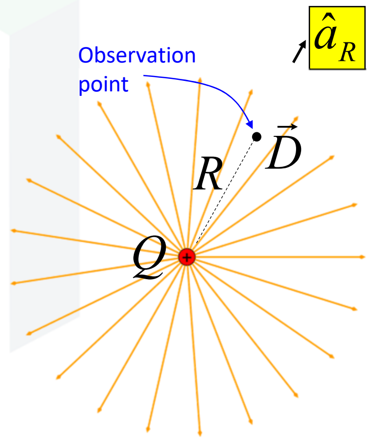

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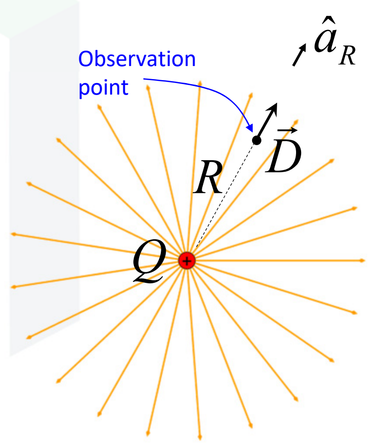

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Derivation of Flux $\vec{D}$ Around a Point Charge

Apply Gauss' law to a point charge located at the origin.

$$Q = \oiint_S \vec{D} \bullet d\vec{s}$$

Choose spherical coordinates and pick the observation point to lie at spherical coordinate $(R, 0, 0)$. Due to symmetry, the electric flux $\vec{D}$ will only have a radial component D_r.

$$Q = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} (D_r \hat{a}_r) \bullet (R^2 \sin\theta d\theta d\phi \hat{a}_r) = D_r R^2 \int_{\theta=0}^{\pi} \left(\int_{\phi=0}^{2\pi} d\phi \right) \sin\theta d\theta$$

$$= 2\pi D_r R^2 \int_{\theta=0}^{\pi} \sin\theta d\theta = -2\pi D_r R^2 (\cos\pi - \cos 0) = 4\pi D_r R^2$$

Solve for D_r and make it a vector equation.

$$Q = 4\pi D_r R^2 \implies D_r = \frac{Q}{4\pi R^2} \implies \boxed{\vec{D} = \frac{Q}{4\pi R^2} \hat{a}_r}$$

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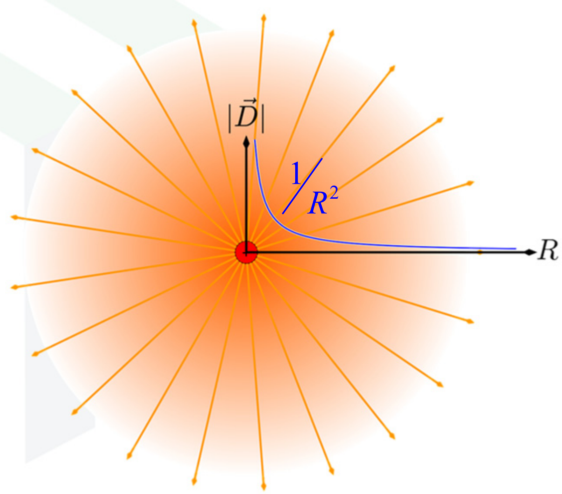
$1/R^2$ Dependence

The amplitude of the electric flux decreases with distance from the charge.

$$\vec{D} = \frac{Q}{4\pi R^2} \hat{a}_R$$

Amplitude of $\vec{D}$ decays as $1/R^2$.



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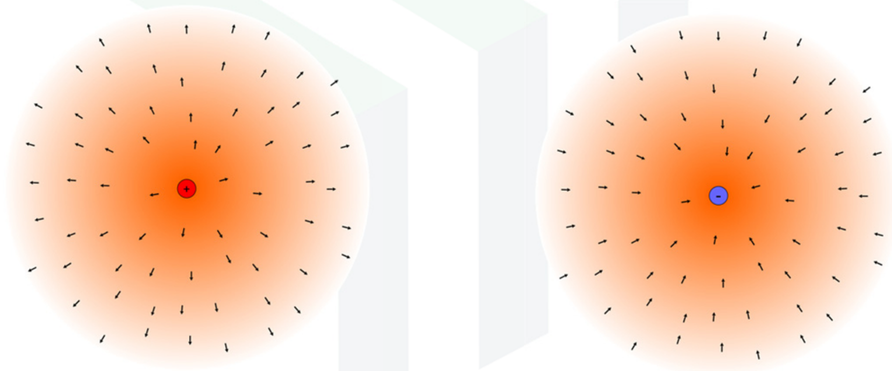
Direction of Electric Flux $\vec{D}$

The electric flux $\vec{D}$ diverges away from positive charge and converges toward negative charge.



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A More Useful Equation for $\vec{D}$

$$\vec{D} = \frac{Q}{4\pi R^2}\hat{a}_R = \frac{Q\left(\vec{r}-\vec{r}_Q\right)}{4\pi\left|\vec{r}-\vec{r}_Q\right|^3}$$

CAUTION: It is easy to forget $1/R^2$ dependence when using this form of the equation by mistakenly thinking it is $1/R^3$.

$\vec{D} \equiv$ electric flux density $\left(\mathrm{C/m^2}\right)$

$Q \equiv$ total charge of point $\left(\mathrm{C}\right)$

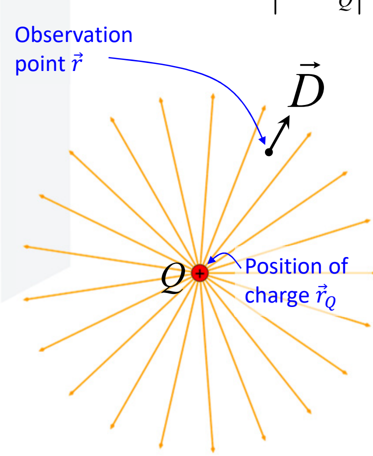
$R \equiv$ distance from charge $\left(\mathrm{m}\right)$

$\vec{r} \equiv$ observation point $\left(\mathrm{m}\right)$

$\vec{r}_Q \equiv$ position of charge $\left(\mathrm{m}\right)$

$\hat{a}_R \equiv$ unit vector in radial direction (no units)

$$\hat{a}_R = \frac{\vec{r}-\vec{r}_Q}{\left|\vec{r}-\vec{r}_Q\right|}$$



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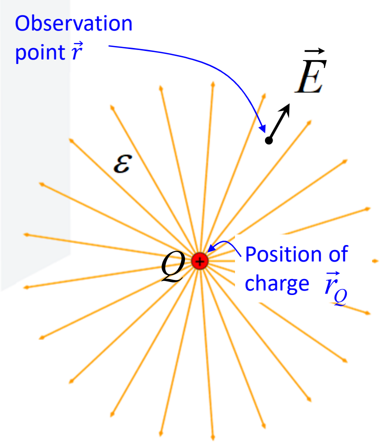
Electric Field Intensity $\vec{E}$

The electric field intensity $\vec{E}$ and electric flux density $\vec{D}$ differ by only a constant, the permittivity ε.

$$\boxed{\vec{D} = \varepsilon \vec{E}}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon R^2}\hat{a}_R = \frac{Q\left(\vec{r} - \vec{r}_Q\right)}{4\pi\varepsilon\left|\vec{r} - \vec{r}_Q\right|^3}$$



CAUTION!
Many textbooks calculate electric fields around point charges as

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

This is confusing and has a hidden assumption built-in that the electric field is being observed in free space ($\varepsilon_r \approx 1$).

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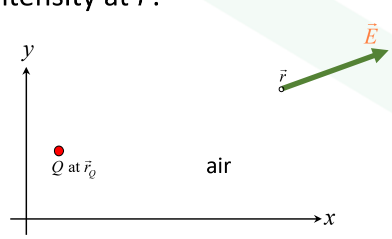
Example

Calculate the electric field intensity at $\vec{r}$.



$$Q = 2.0\ \mu\text{C}$$
$$\vec{r}_Q = 1\hat{a}_x + 2\hat{a}_y\ \text{m}$$
$$\vec{r} = 6\hat{a}_x + 4\hat{a}_y\ \text{m}$$

Solution

First, write the equation for the electric flux density at $\vec{r}$ due to the charge Q.

$$\vec{D} = \frac{Q\left(\vec{r} - \vec{r}_Q\right)}{4\pi\left|\vec{r} - \vec{r}_Q\right|^3}$$

Second, it is usually most convenient to solve the vector part of this equation first.

$$\vec{r} - \vec{r}_Q = \left(6\hat{a}_x + 4\hat{a}_y\right) - \left(1\hat{a}_x + 2\hat{a}_y\right) = 5\hat{a}_x + 2\hat{a}_y\ \text{m}$$

$$\frac{\vec{r} - \vec{r}_Q}{\left|\vec{r} - \vec{r}_Q\right|^3} = \frac{5\hat{a}_x + 2\hat{a}_y}{\left(5^2 + 2^2\right)^{3/2}} = 32.02\times10^{-3}\hat{a}_x + 12.81\times10^{-3}\hat{a}_y\ \frac{1}{\text{m}^2}$$

Third, the electric flux density is

$$\vec{D} = \frac{\left(2.0\times10^{-6}\ \text{C}\right)}{4\pi}\left(32.02\times10^{-3}\hat{a}_x + 12.81\times10^{-3}\hat{a}_y\ \tfrac{1}{\text{m}^2}\right)$$
$$= 5.10\hat{a}_x + 2.04\hat{a}_y\ \tfrac{\text{nC}}{\text{m}^2}$$

Last, the electric field intensity is

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0\varepsilon_r} = \frac{5.10\hat{a}_x + 2.04\hat{a}_y\ \frac{\text{nC}}{\text{m}^2}}{\left(8.854\times10^{-12}\ \frac{\text{F}}{\text{m}}\right)(1.0)} = \boxed{575\hat{a}_x + 230\hat{a}_y\ \tfrac{\text{V}}{\text{m}}}$$

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