Electromagnetics:

Electromagnetic Field Theory

Romeo & Juliet Model of Oscillation

1

Meet Romeo and Juliet

Romeo is a nervous dude. If Juliet expresses too much love, he pulls away from her. If Juliet pulls away from him, he expresses love.

$R \stackrel{\text{def}}{\equiv}$ Love expressed by Romeo

$R < 0$ hate
$R > 0$ love

$$\frac{dR}{dt} = -J$$

Juliet is a great woman and expresses love in proportion to what she receives.

$J \stackrel{\text{def}}{\equiv}$ Love expressed by Juliet

$J < 0$ hate
$J > 0$ love

$$\frac{dJ}{dt} = R$$

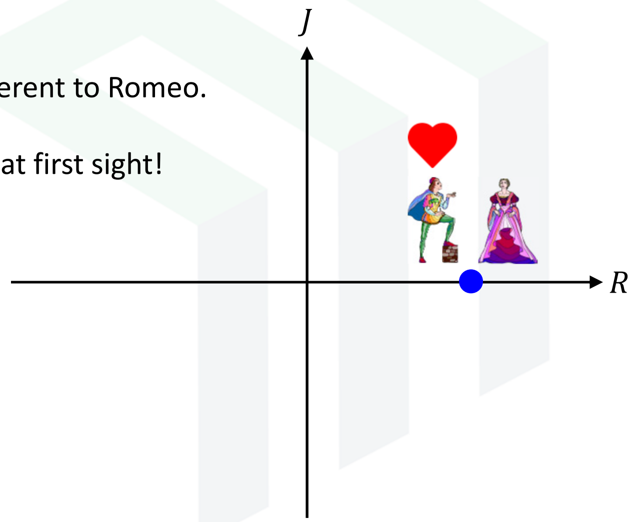
Slide 2

2

A Love Story

Juliet starts off indifferent to Romeo.

For Romeo, it is love at first sight!

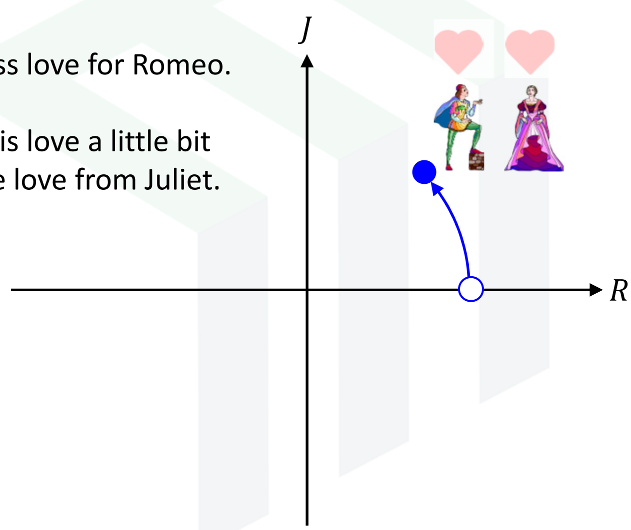


3

A Love Story

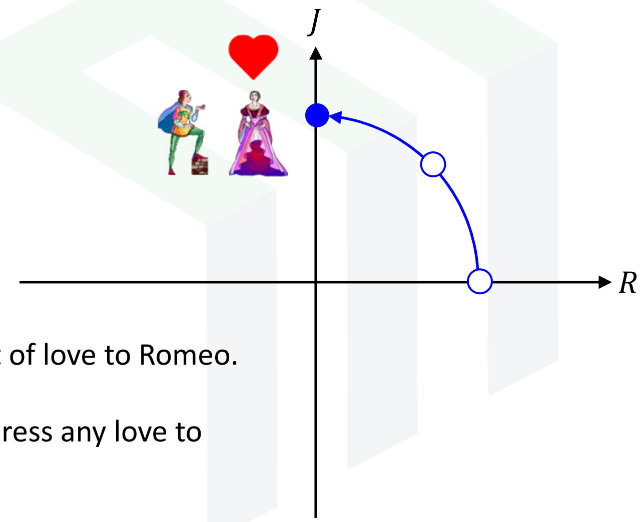
Juliet begins to express love for Romeo.

Romeo backs off on his love a little bit due to receiving some love from Juliet.



4

A Love Story

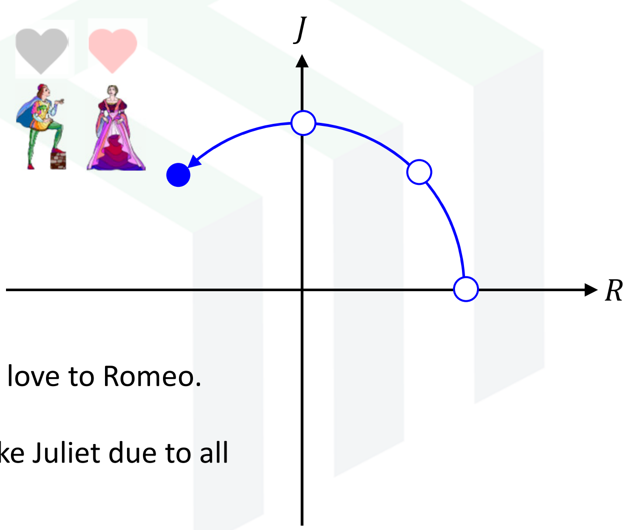


Juliet expresses a lot of love to Romeo.

Romeo does not express any love to Juliet.

5

A Love Story

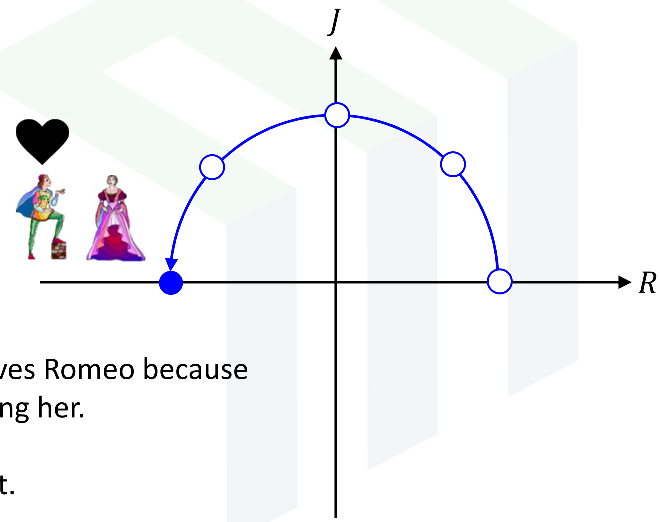


Juliet expresses some love to Romeo.

Romeo begins to dislike Juliet due to all her love.

6

A Love Story



Juliet no longer loves Romeo because of how he is treating her.

Romeo hates Juliet.

EMPossible

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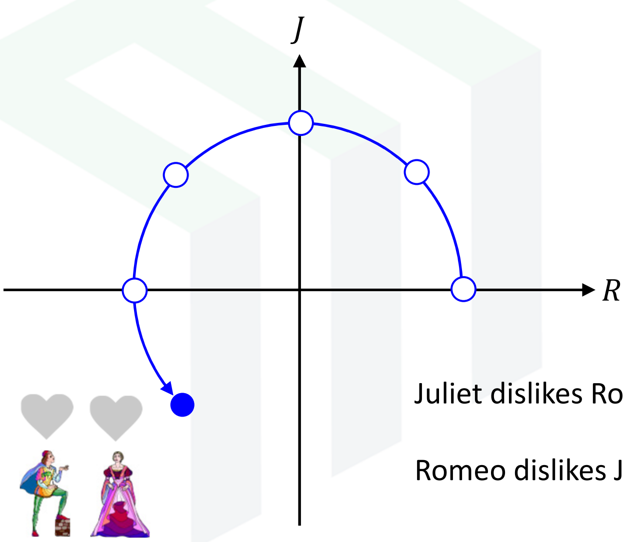
A Love Story



Juliet dislikes Romeo.

Romeo dislikes Juliet.

EMPossible

Slide 8

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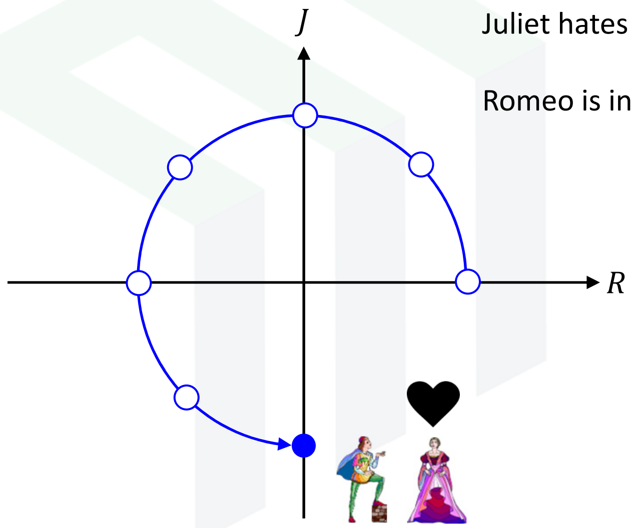
A Love Story



Juliet hates Romeo.

Romeo is indifferent to Juliet.

EMPossible

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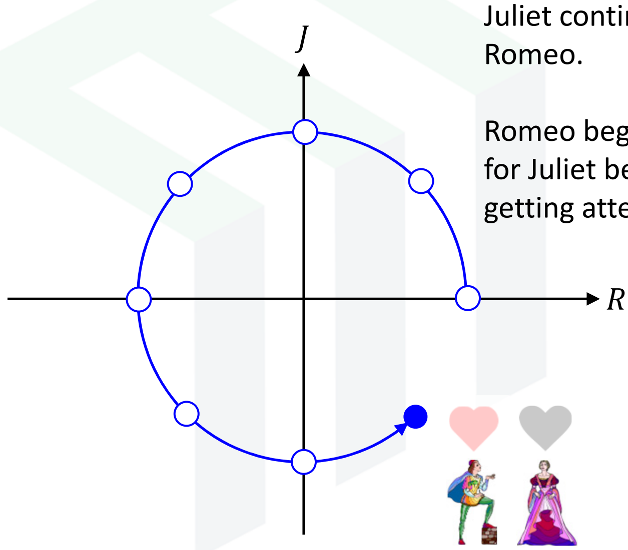
A Love Story



Juliet continues to dislike Romeo.

Romeo begins to express love for Juliet because he is not getting attention from her.

EMPossible

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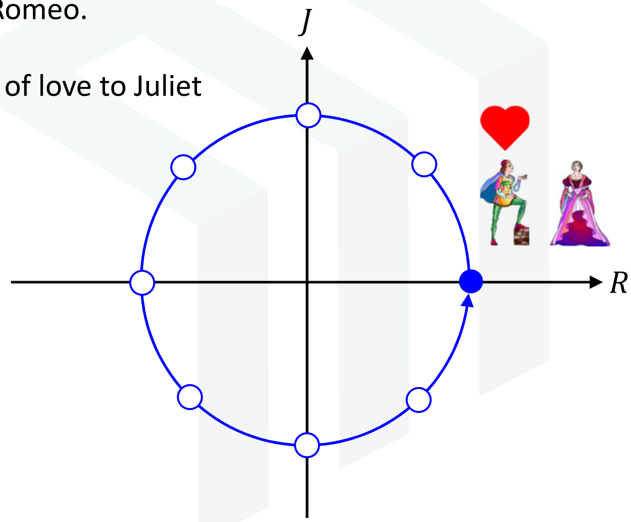
A Love Story

Juliet is indifferent to Romeo.

Romeo expresses a lot of love to Juliet to win her over.



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Mathematical Solution to Romeo & Juliet (1 of 2)

$$\frac{dR}{dt} = -J$$

Differentiate with respect to time t.

$$\frac{d^2R}{dt^2} = -\frac{dJ}{dt}$$

Substitute Juliet's equation into this.

$$\frac{d^2R}{dt^2} = -R$$

$$\frac{dJ}{dt} = R$$

Differentiate with respect to time t.

$$\frac{d^2J}{dt^2} = \frac{dR}{dt}$$

Substitute Romeo's equation into this.

$$\frac{d^2J}{dt^2} = -J$$

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Mathematical Solution to Romeo & Juliet (1 of 2)

$$\frac{dR}{dt} = -J$$

↓ Differentiate with respect to time t.

$$\frac{d^2R}{dt^2} = -\frac{dJ}{dt}$$

↓ Substitute Juliet's equation into this.

$$\frac{d^2R}{dt^2} + R = 0$$

Final differential equation

$$\frac{dJ}{dt} = R$$

↓ Differentiate with respect to time t.

$$\frac{d^2J}{dt^2} = \frac{dR}{dt}$$

↓ Substitute Romeo's equation into this.

$$\frac{d^2J}{dt^2} + J = 0$$

Final differential equation

Same equation
Same solution

EMPossible

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Mathematical Solution to Romeo & Juliet (2 of 2)

$$\frac{d^2R}{dt^2} + R = 0$$

↓ Solve

$$R(t) = A\cos t + B\sin t$$

↓ Apply boundary conditions

$$R(t) = A\cos t$$

$$\frac{d^2J}{dt^2} + J = 0$$

↓ Solve

$$J(t) = C\cos t + D\sin t$$

↓ Apply boundary conditions

$$J(t) = D\sin t$$

EMPossible

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Comparison to Electromagnetics

$$\frac{dR}{dt} = -J$$

$$\vec{H} \qquad \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\frac{dJ}{dt} = +R$$

$$\vec{E} \qquad \nabla \times \vec{H} = +\varepsilon \frac{\partial \vec{E}}{\partial t}$$

EMPossible

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