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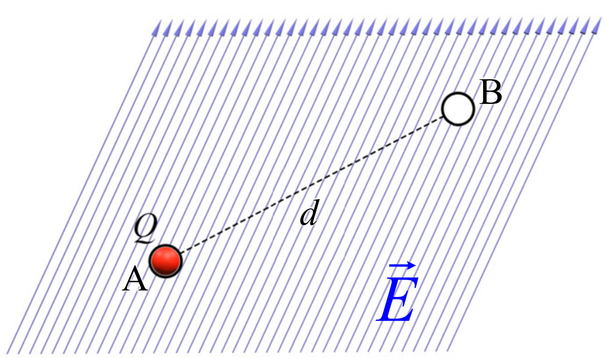
Electromagnetics:
Electromagnetic Field Theory

Energy in Electrical Potential

1

Recall Potential Difference



Recall the relation between potential difference, work, and charge.

$$V_{\mathrm{AB}} = V_{\mathrm{B}} - V_{\mathrm{A}} = \frac{W}{Q}$$

Therefore, the work it takes to move charge Q from A to B is

$$W = QV_{\mathrm{AB}}$$

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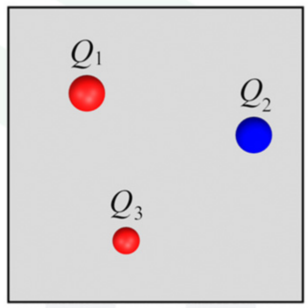
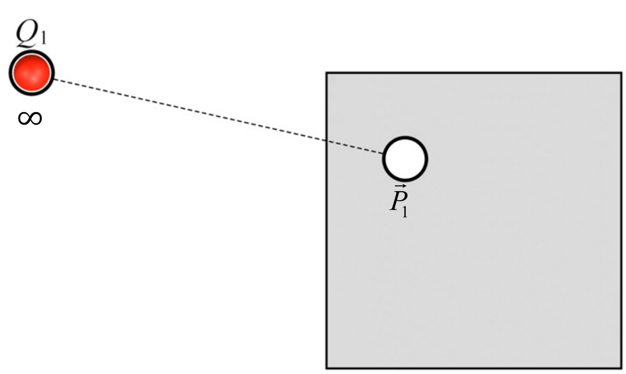
Energy in an Ensemble of Charges

An ensemble of charges contains energy because the charges are putting a force on each other and so they have the potential to do work.



The energy contained in the ensemble will be determined by calculating how much energy it took to assemble it.

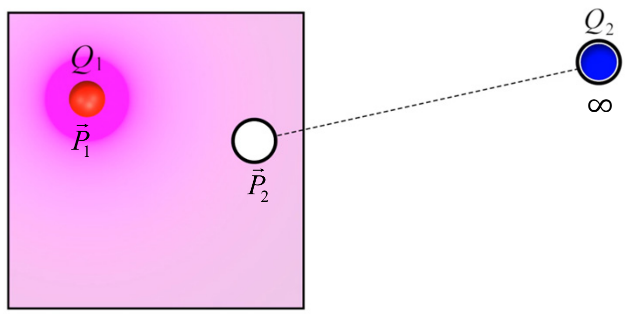
Point Charge #1



No other charges are present, so placing Q_1 at $\vec{P}_1$ takes no work.

$$W_1 = 0$$

Point Charge #2



Placing Q_2 at $\vec{P}_2$ takes work because charge Q_1 is present.

$$W_2 = Q_2 V_{21}$$

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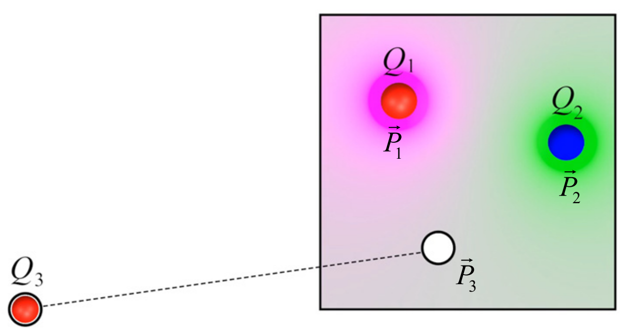
Point Charge #3



Placing Q_3 at $\vec{P}_3$ takes work because charges Q_1 and Q_2 are present.

$$W_3 = Q_3 V_{31} + Q_3 V_{32}$$

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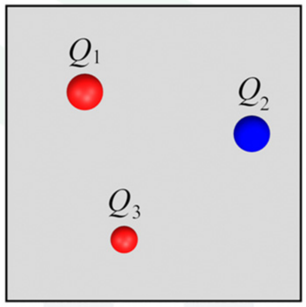
Total Work So Far

The total work placing all three charges is



$$W = W_1 + W_2 + W_3$$
$$= 0 + Q_2 V_{21} + Q_3 \left(V_{31} + V_{32} \right)$$

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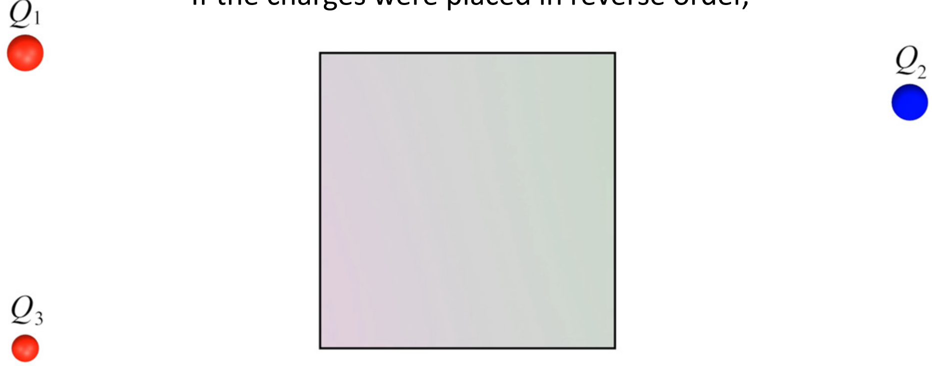
Assembly in Reverse Order

If the charges were placed in reverse order,



$$W = W_3 + W_2 + W_1$$
$$= 0 + Q_2 V_{23} + Q_1 \left(V_{12} + V_{13} \right)$$

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Add Both Approaches

$$W = 0 + Q_2 V_{21} + Q_3 \left(V_{31} + V_{32} \right)$$

Equation obtained by placing Q_1, then Q_2, and then Q_3.

$$W = 0 + Q_2 V_{23} + Q_1 \left(V_{12} + V_{13} \right)$$

Equation obtained by placing Q_3, then Q_2, and then Q_1.

$$2W = 0 + Q_2 V_{21} + Q_3 \left(V_{31} + V_{32} \right) + 0 + Q_2 V_{23} + Q_1 \left(V_{12} + V_{13} \right)$$

Add the two equations above.

$$2W = Q_1 \underbrace{\left(V_{12} + V_{13} \right)}_{V_1} + Q_2 \underbrace{\left(V_{21} + V_{23} \right)}_{V_2} + Q_3 \underbrace{\left(V_{31} + V_{32} \right)}_{V_3}$$

Total potentials →

$$2W = Q_1 V_1 + Q_2 V_2 + Q_3 V_3$$

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Final Expression

$$2W = Q_1 V_1 + Q_2 V_2 + Q_3 V_3$$

$$W = \frac{1}{2} Q_1 V_1 + \frac{1}{2} Q_2 V_2 + \frac{1}{2} Q_3 V_3$$

Solve for W.

It is straightforward to generalize this for any number of charges.

$$W = \frac{1}{2} \sum_{i=1}^{N} Q_i V_i \qquad \text{(joules)}$$

Q_i charge of ith charge

V_i total potential felt by ith charge, not including potential due to ith charge

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Energy in Charge Distributions

Point Charge

Q

Charge

$Q \ (\mathrm{C})$

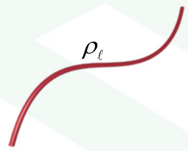
Total Charge

$$Q_{\text{Total}} = \sum_{i=1}^{N} Q_i$$

Total Energy

$$W = \frac{1}{2}\sum_{i=1}^{N} Q_i V_i$$

Line Charge



Line Charge Density

$\rho_\ell \ (\mathrm{C/m})$

Total Charge

$$Q_{\text{Total}} = \int_\ell \rho_\ell \, d\ell \cong \rho_\ell L$$

Total Energy

$$W = \frac{1}{2}\int_L \rho_\ell V \, d\ell$$

Sheet Charge

Surface Charge Density

$\rho_s \ \left(\mathrm{C/m^2}\right)$

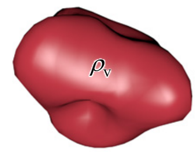
Total Charge

$$Q_{\text{Total}} = \iint_S \rho_s \, ds \cong \rho_s S$$

Total Energy

$$W = \frac{1}{2}\iint_s \rho_s V \, ds$$

Volume Charge



Volume Charge Density

$\rho_v \ \left(\mathrm{C/m^3}\right)$

Total Charge

$$Q_{\text{Total}} = \iiint_v \rho_v \, dv = \rho_v V$$

Total Energy

$$W = \frac{1}{2}\iiint_v \rho_v V \, dv$$

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