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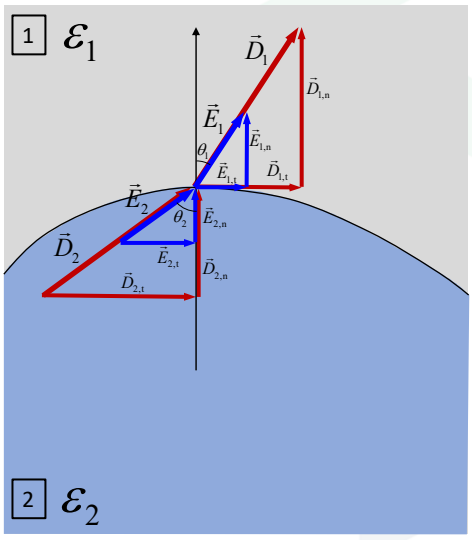
Electromagnetics:
Electromagnetic Field Theory

Electrostatic Refraction

1

Analysis Setup



A single equation is desired that relates θ_1, θ_2, ε_1, and ε_2 without any field quantities in the equation.

Given the angles θ_1 and θ_2, the electric field can be written in terms of its components as

$$\vec{E}_1 = E_{1,t}\hat{a}_t + E_{1,n}\hat{a}_n$$
$$= (E_1 \sin\theta_1)\hat{a}_t + (E_1 \cos\theta_1)\hat{a}_n$$

$$\vec{E}_2 = E_{2,t}\hat{a}_t + E_{2,n}\hat{a}_n$$
$$= (E_2 \sin\theta_2)\hat{a}_t + (E_2 \cos\theta_2)\hat{a}_n$$

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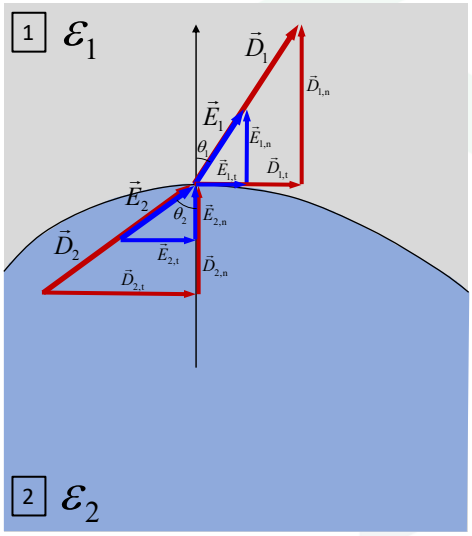
Derivation of Refraction Law



Apply the boundary conditions for tangential components.

$$E_{1,t} = E_{2,t}$$

$$E_1 \sin\theta_1 = E_2 \sin\theta_2$$

Apply the boundary conditions for normal components.

$$\varepsilon_1 E_{1,n} = \varepsilon_2 E_{2,n}$$

$$\varepsilon_1 E_1 \cos\theta_1 = \varepsilon_2 E_2 \cos\theta_2$$

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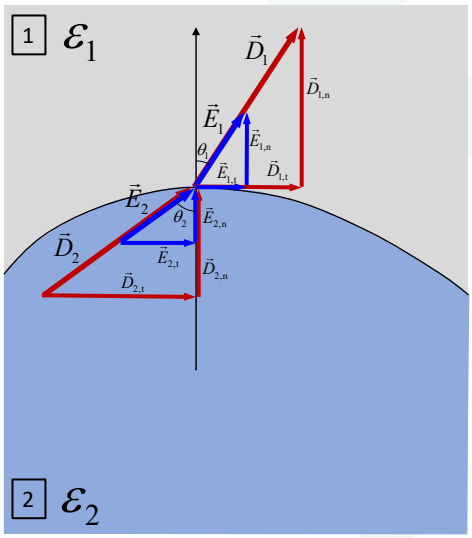
Derivation of Refraction Law



The two equations were

$$E_1 \sin\theta_1 = E_2 \sin\theta_2$$

$$\varepsilon_1 E_1 \cos\theta_1 = \varepsilon_2 E_2 \cos\theta_2$$

Divide these equations to get

$$\frac{E_1 \sin\theta_1}{\varepsilon_1 E_1 \cos\theta_1} = \frac{E_2 \sin\theta_2}{\varepsilon_2 E_2 \cos\theta_2}$$

Simplify to get the law of refraction.

$$\boxed{\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}}$$

This is NOT Snell's law.

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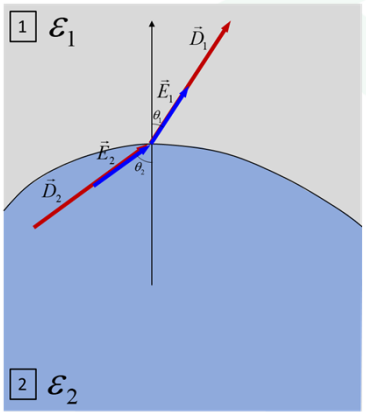
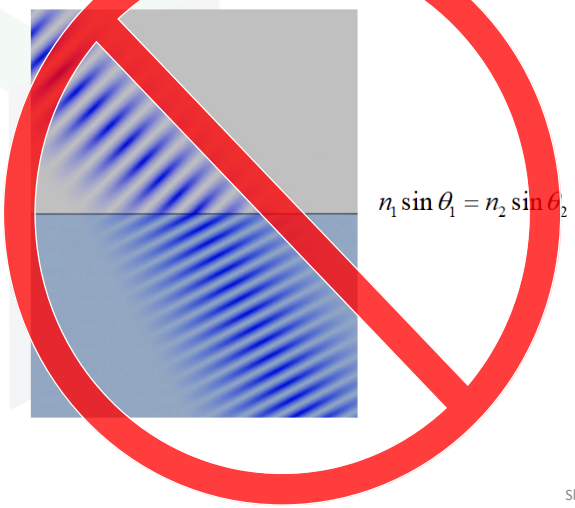
Law of Refraction Vs. Snell's Law of Refraction

Electrostatic Refraction Law



$$\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

Snell's Law of Refraction



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

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