Electromagnetics:
Electromagnetic Field Theory

Examples of Electrostatic Boundary Conditions

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Example #1

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Example #1

Let there be an interface between two semi-infinite media in the x-y plane. The dielectric constant of the first medium is 2.0 and the dielectric constant of the second medium is 4.4.

1. Given that the electric flux density in medium 1 is $\vec{D}_1 = 2.1\hat{a}_x + 0.7\hat{a}_y + 1.5\hat{a}_z$ C/m², calculate the electric flux density in medium 2, $\vec{D}_2$.
2. Calculate the angle $\vec{D}_1$ makes with the interface.
3. Using the law of refraction, calculate the angle $\vec{D}_2$ makes with the interface.

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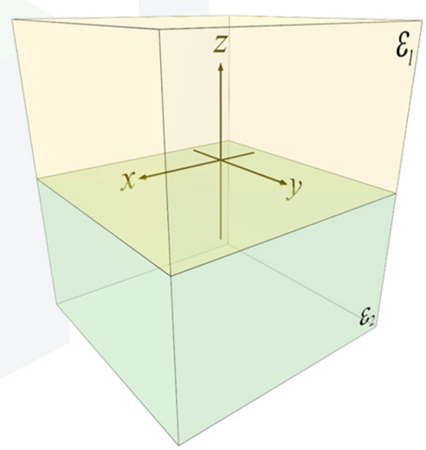
Example #1 – Problem Setup

Start by visualizing the problem and setting up the coordinates.



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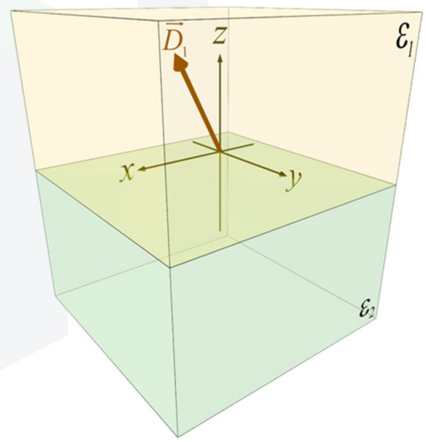
Example #1 – Problem Setup

Start by visualizing the problem and setting up the coordinates.

Plot $\vec{D}_1$.

$$\vec{D}_1 = 2.1\hat{a}_x + 0.7\hat{a}_y + 1.5\hat{a}_z \quad \text{C/m}^2$$



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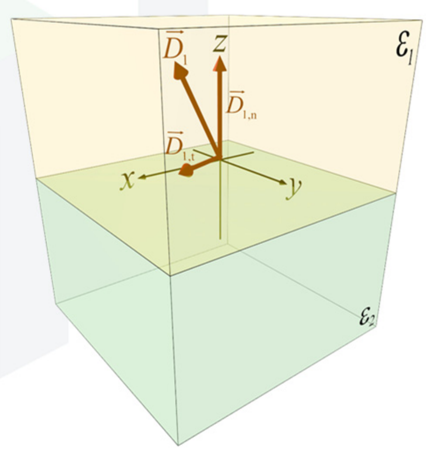
Example #1 – Part 1

Separate $\vec{D}_1$ into tangential and normal components.

$$\vec{D}_1 = \underbrace{2.1\hat{a}_x + 0.7\hat{a}_y}_{\text{Tangential}} + \underbrace{1.5\hat{a}_z}_{\text{Normal}} \quad \text{C/m}^2$$

Tangential: $\vec{D}_{1,\mathrm{t}} = 2.1\hat{a}_x + 0.7\hat{a}_y \quad \text{C/m}^2$

Normal: $\vec{D}_{1,\mathrm{n}} = 1.5\hat{a}_z \quad \text{C/m}^2$



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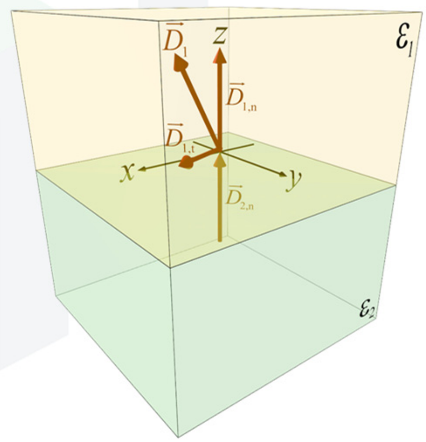
Example #1 – Part 1

Apply boundary condition for normal component.

$$\vec{D}_{1,\text{n}} = \vec{D}_{2,\text{n}}$$

$$1.5\hat{a}_z \ \ \text{C/m}^2 = \vec{D}_{2,\text{n}}$$



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Example #1 – Part 1

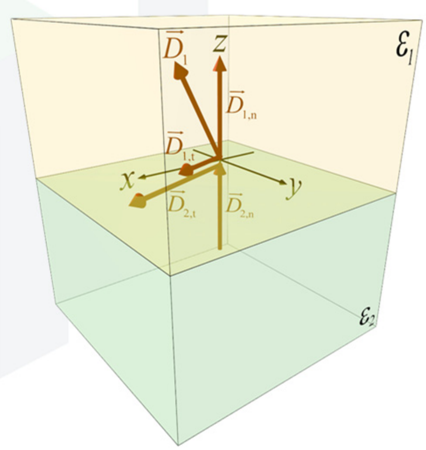
Apply boundary condition for tangential component.

$$\frac{\vec{D}_{1,\text{t}}}{\varepsilon_1} = \frac{\vec{D}_{2,\text{t}}}{\varepsilon_2}$$

$$\vec{D}_{2,\text{t}} = \frac{\varepsilon_2}{\varepsilon_1}\vec{D}_{1,\text{t}}$$

$$\vec{D}_{2,\text{t}} = \frac{4.4}{2.0}\left(2.1\hat{a}_x + 0.7\hat{a}_y \ \ \text{C/m}^2\right)$$

$$\vec{D}_{2,\text{t}} = 4.62\hat{a}_x + 1.54\hat{a}_y \ \ \text{C/m}^2$$



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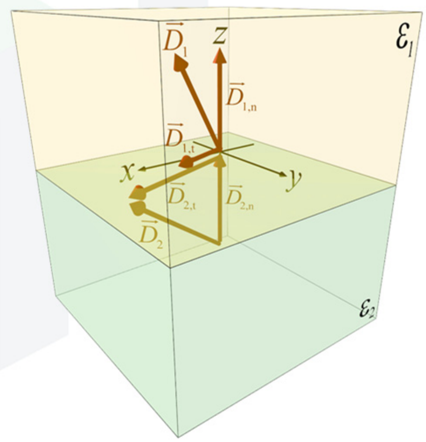
Example #1 – Part 1

Gather both components to get overall $\vec{D}_2$.

$$\vec{D}_2 = \vec{D}_{2,\mathrm{t}} + \vec{D}_{2,\mathrm{n}}$$

$$\vec{D}_2 = \left(4.62\hat{a}_x + 1.54\hat{a}_y \ \ \mathrm{C/m^2}\right) + \left(1.5\hat{a}_z \ \ \mathrm{C/m^2}\right)$$

$$\boxed{\vec{D}_2 = 4.62\hat{a}_x + 1.54\hat{a}_y + 1.5\hat{a}_z \ \ \mathrm{C/m^2}}$$



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Example #1 – Part 2

Calculate the angle θ_1 of $\vec{D}_1$.

Recall the property of dot products.

$$\vec{A} \bullet \vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta_{AB}$$

Calculate θ_1 by letting

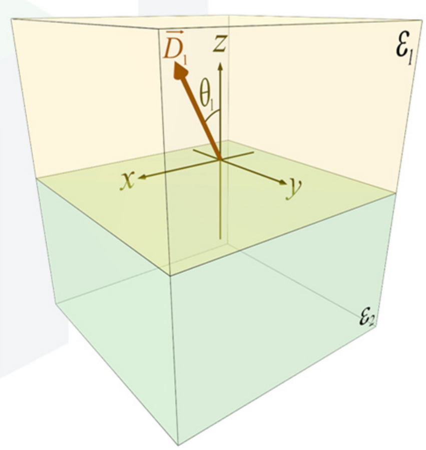
$$\vec{A} = \vec{D}_1$$

$$\vec{B} = \hat{a}_z$$

$$\theta_{AB} = \theta_1$$

The dot product becomes

$$\vec{D}_1 \bullet \hat{a}_z = \left|\vec{D}_1\right|\left|\hat{a}_z\right|\cos\theta_1$$



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Example #1 – Part 2

Continued…

Solve the dot product equation for θ_1.

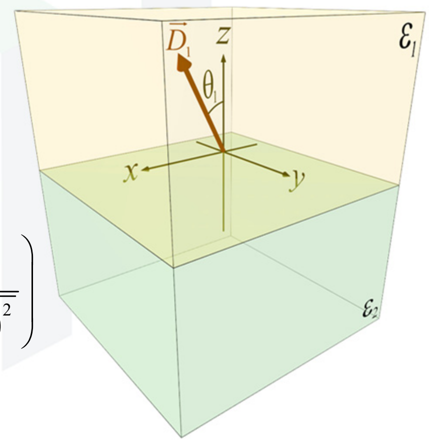
$$\vec{D}_1 \bullet \hat{a}_z = \left|\vec{D}_1\right|\left|\hat{a}_z\right|\cos\theta_1$$

$$D_z = \left|\vec{D}_1\right|\cos\theta_1$$

$$\theta_1 = \cos^{-1}\left(\frac{D_z}{\left|\vec{D}_1\right|}\right)$$

$$\theta_1 = \cos^{-1}\left(\frac{1.5}{\sqrt{(2.1)^2+(0.7)^2+(1.5)^2}}\right)$$

$$\boxed{\theta_1 = 55.9°}$$



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Example #1 – Part 2

Calculate the angle θ_2 of $\vec{D}_2$.

The law of refraction is

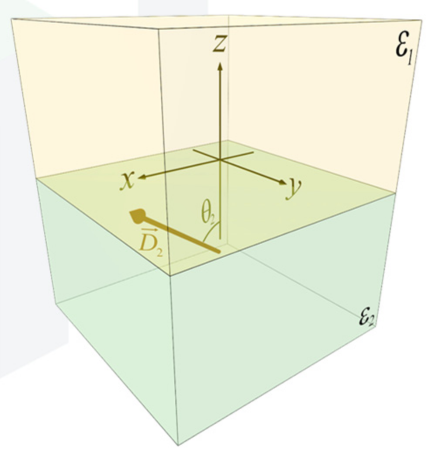
$$\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

Solving this for θ_2 gives

$$\theta_2 = \tan^{-1}\left(\frac{\varepsilon_2}{\varepsilon_1}\tan\theta_1\right)$$

$$\theta_2 = \tan^{-1}\left(\frac{4.4}{2.0}\tan 55.9°\right)$$

$$\boxed{\theta_2 = 72.9°}$$



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