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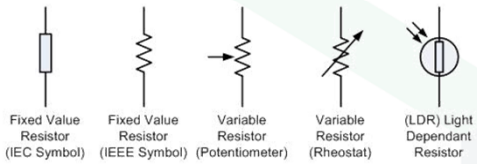
Electromagnetics:
Electromagnetic Field Theory

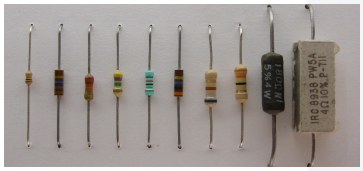
Analyzing Resistors

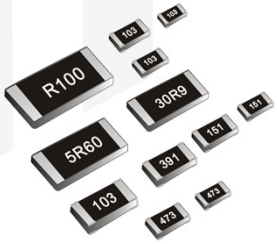
1

What is a Resistor?

A resistor is a passive two-terminal electrical component that is only partially conductive so as to limit current flow.







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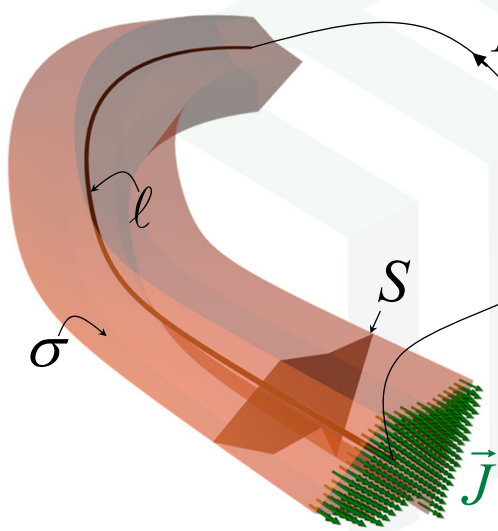
2

Analysis Setup



$$R = \frac{V}{I} = ?$$

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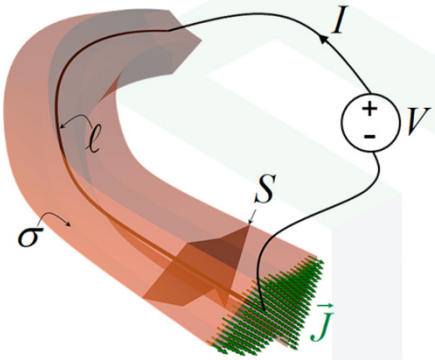
3

Derivation of Resistance for Uniform Conductivity



Ohm's Law

$$J = \sigma E$$

Electric Field Intensity

$$E = \frac{V}{\ell}$$

Electric Current Density

$$J = \frac{I}{S} = \sigma E = \sigma \frac{V}{\ell}$$

$$\boxed{R = \frac{\ell}{\sigma S} = \frac{\rho \ell}{S}}$$

$$R = \frac{V}{I} = \frac{\ell}{\sigma S} \leftarrow \frac{I}{S} = \sigma \frac{V}{\ell}$$

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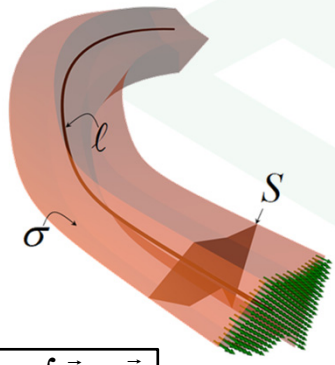
Derivation of Resistance for Nonuniform Conductivity



Now an electromagnetic analysis must be performed to derive V and I.

Voltage across conductor

$$V = -\int_{\ell} \vec{E} \bullet d\vec{\ell}$$

Current through conductor

$$I = \iint_{S} \vec{J} \bullet d\vec{s} = \iint_{S} \sigma \vec{E} \bullet d\vec{s}$$

$$R = \frac{V}{I} = \frac{-\int_{\ell} \vec{E} \bullet d\vec{\ell}}{\iint_{S} \sigma \vec{E} \bullet d\vec{s}}$$

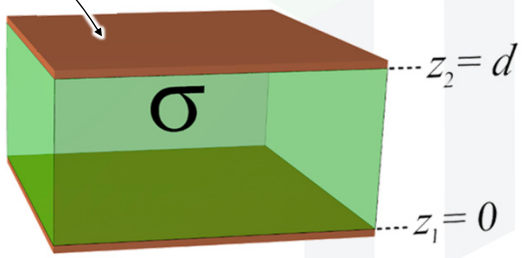
Recipe for Analyzing Resistors

1. Choose a convenient coordinate system.
2. Assume V_0 as the potential difference across the terminals of the conductor.
3. Calculate electric potential V by solving Laplace's equation $\nabla^2 V = 0$.
4. Calculate $\vec{E}$ using $\vec{E} = -\nabla V$.
5. Calculate I from $I = \iint_{S} \sigma \vec{E} \bullet d\vec{s}$.
6. Calculate R using $R = V_0 / I$.

Note: The final equation for R should not contain V_0 or I. Use this as a self-check.

The Parallel Plate Resistor

$$R = \frac{d}{\sigma S}$$



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