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Electromagnetics:

Electromagnetic Field Theory

The Biot-Savart Law

(Bee-oh-suh-vahr)

1

Origin of Biot-Savart Law

The Bio-Savart law is derived from Ampere's circuit law.

$$\nabla \times \vec{H} = \vec{J}$$

$$\vec{H} = \int_L \frac{I d\vec{\ell} \times \hat{a}_R}{4\pi R^2}$$

Line Current

$$\vec{H} = \iint_S \frac{\vec{K} ds \times \hat{a}_R}{4\pi R^2}$$

Surface Current

$$\vec{H} = \iiint_V \frac{\vec{J} dv \times \hat{a}_R}{4\pi R^2}$$

Volume Current

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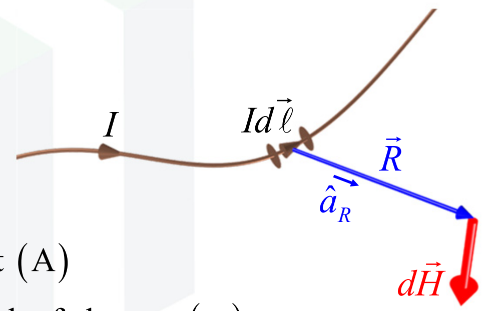
Biot-Savart Law

The Bio-Savart law is used to calculate the differential magnetic field $d\vec{H}$ due to a differential current element $Id\vec{\ell}$.

$$d\vec{H} = \frac{Id\vec{\ell} \times \hat{a}_R}{4\pi R^2}$$



$I \equiv$ total current in element (A)

$d\vec{\ell} \equiv$ vector differential length of element (m)

$R \equiv$ distance from current element (m)

$\hat{a}_R \equiv$ unit vector in radial direction (no units)

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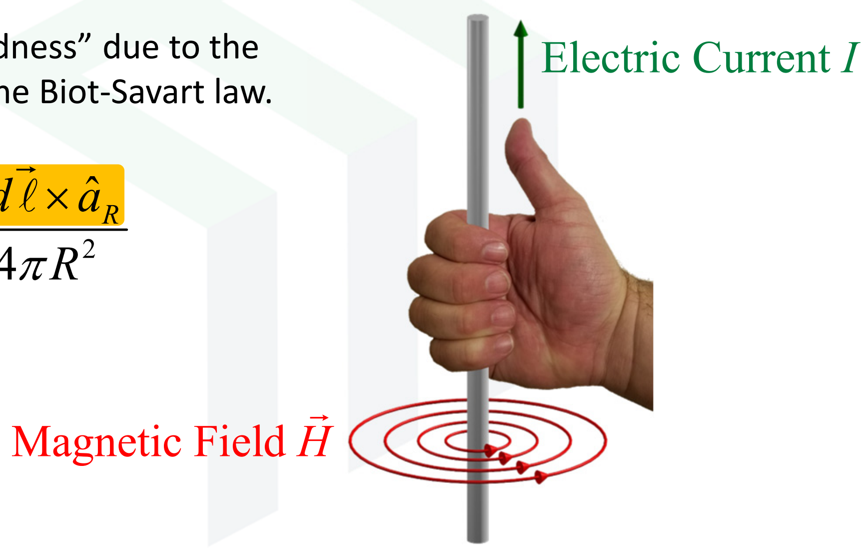
Right-Hand Rule

There is a "handedness" due to the cross-product in the Biot-Savart law.

$$d\vec{H} = \frac{Id\vec{\ell} \times \hat{a}_R}{4\pi R^2}$$



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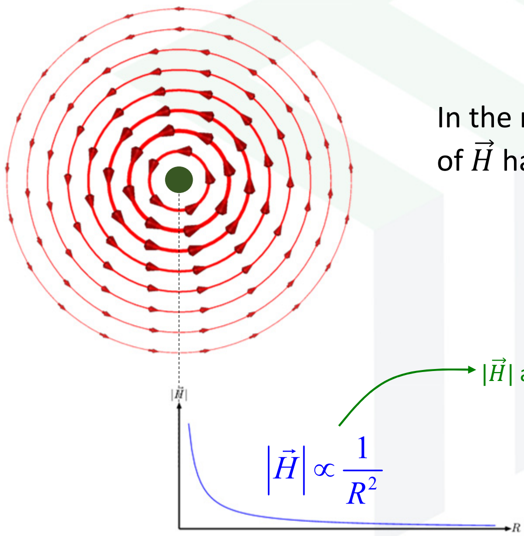
Radial Dependence

In the radial direction, the magnitude of $\vec{H}$ has a a $1/R^2$ dependence.



$|\vec{H}|$ approaches infinity as R goes to zero.

$$\left|\vec{H}\right| \propto \frac{1}{R^2}$$

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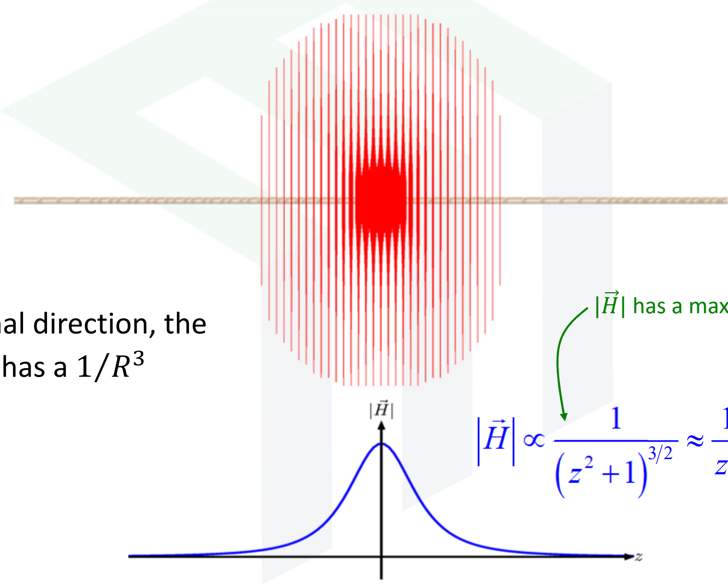
Longitudinal Dependence of Current Element $Id\vec{\ell}$



In the longitudinal direction, the magnitude of $\vec{H}$ has a $1/R^3$ dependence.

$|\vec{H}|$ has a maximum as z goes to zero.

$$\left|\vec{H}\right| \propto \frac{1}{\left(z^2+1\right)^{3/2}} \approx \frac{1}{z^3}$$

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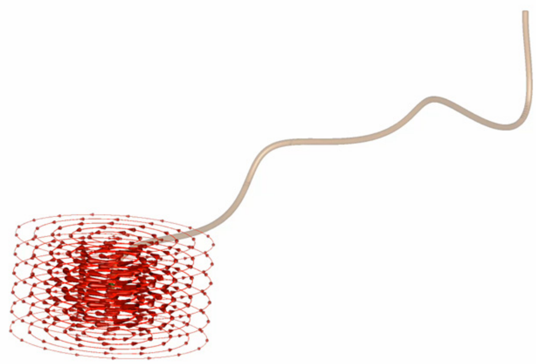
Total Magnetic Field

The total magnetic field $\vec{H}$ due to a long wire is obtained by integrating the Biot-Savart law over the length of the wire.

$$\vec{H} = \int_L d\vec{H} = \int_L \frac{I d\vec{\ell} \times \hat{a}_R}{4\pi R^2}$$



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