Electromagnetics:

Electromagnetic Field Theory

Capacitor Simulation Example

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Outline

- Comparison of analytical and numerical solutions
- How does the numerical analysis work?

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Comparison of Analytical and Numerical Solutions

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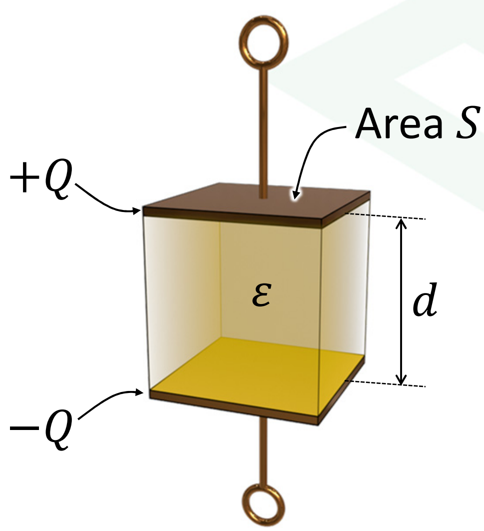
Parallel Plate Capacitor



Ignoring fringing fields, the capacitance found analytically is

$$C = \varepsilon_0 \varepsilon_r \frac{S}{d}$$

Better analytical equations exist that attempt to account for the fringing.

$$C' = C + \varepsilon_0 R \left[\ln\left(16\pi R/d\right) - 1 \right] \quad \text{Circular PPC}$$

$$C' = C + \frac{\varepsilon_0 w}{2} \left[\ln\left(32 w/d\right) - 1 \right] \quad \text{Square PPC}$$

Kirchoff's approximation

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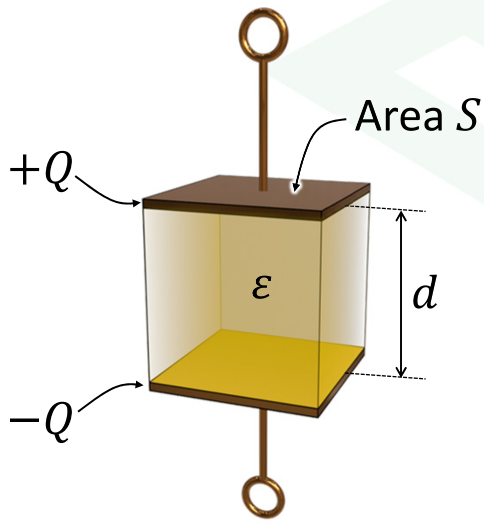
Parallel Plate Capacitor



For this example, let the plate dimensions be 1 m × 1m, separated by 1 m of air.

$$C = \left(8.854\times10^{-12}\ \tfrac{\text{F}}{\text{m}}\right)(1.0)\frac{1\ \text{m}^2}{1\ \text{m}} = 8.9\ \text{pF}$$

$$C' = 19.8\ \text{pF}$$

Is this right? To test, the device was analyzed using a more rigorous numerical method.

$$C_{\text{numerical}} = 24\ \text{pF}$$

The model predicts higher capacitance because there is energy in the fringing fields that was not accounted for in the analytical solution.

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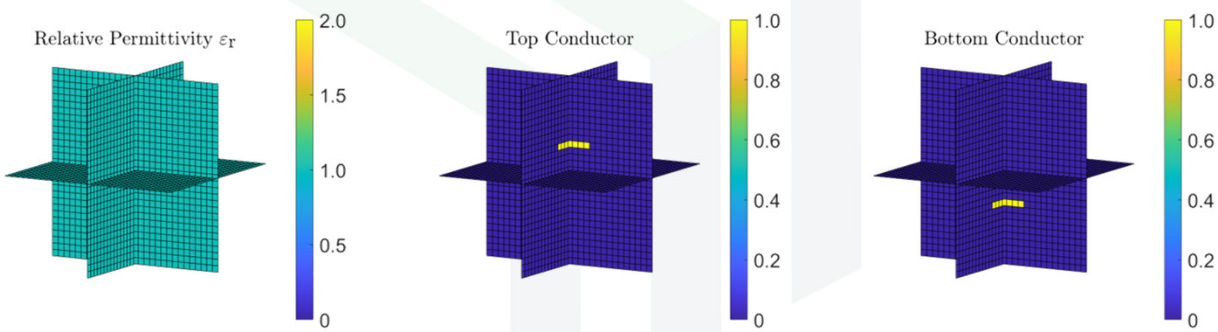
The Numerical Model (1 of 4)

Step 1 – Define capacitor by constructing three different arrays.



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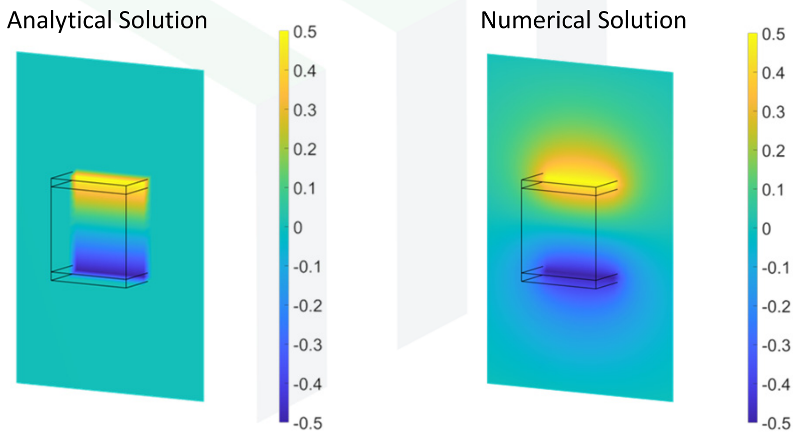
The Numerical Model (2 of 4)

Step 2 – Calculate the electric potential $V(x, y, z)$ by numerically solving the inhomogeneous Laplace's equation $\nabla \cdot [\varepsilon_r(\nabla V)] = 0$.



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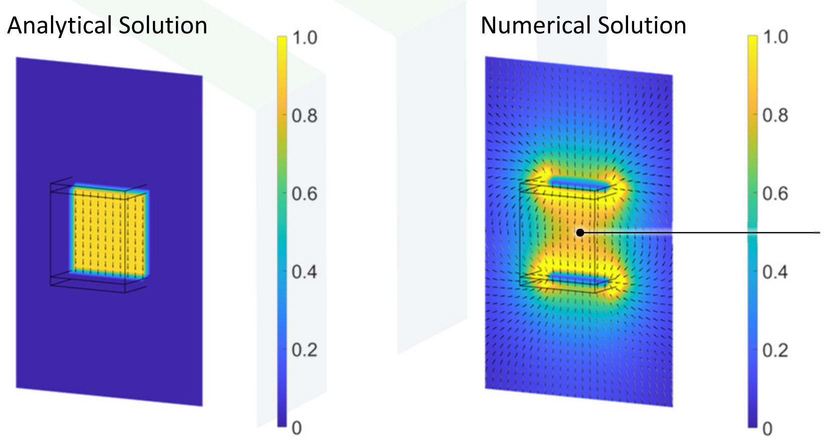
The Numerical Model (3 of 4)

Step 3 – Calculate the electric field intensity as $\vec{E}(x, y, z) = -\nabla V(x, y, z)$.



Observe the field is not uniform between the plates due to fringing.

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The Numerical Model (4 of 4)

Step 4 – Calculate the electric flux density as $\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$.

Step 5 – Calculate the total stored energy as $W = \frac{1}{2} \iiint_V \left(\vec{D} \bullet \vec{E} \right) dv$

Step 6 – Calculate capacitance as $C = \frac{Q}{V_0} = \frac{2W/V_0}{V_0} = \frac{2W}{V_0^2}$

There are two plates carrying charge Q.

$$C_{\text{analytical}} = 8.9\ \text{pF} \qquad C_{\text{numerical}} = 24.1\ \text{pF}$$

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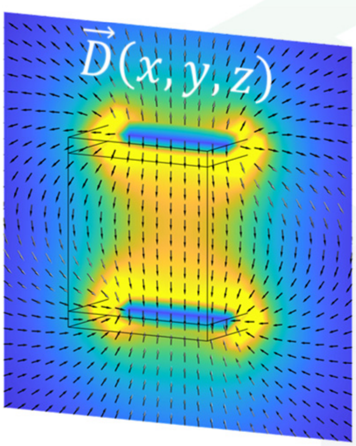
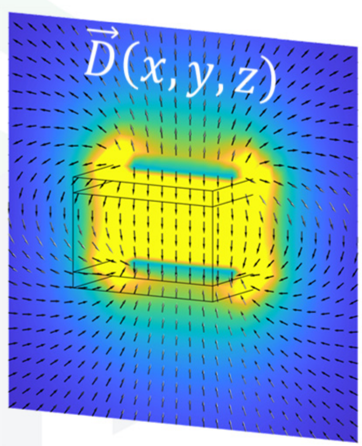
Effect of Separation d

$w/d = 1$
172% error





$w/d = 2$
63% error

As w becomes much larger than d, the field within the gap is more uniform and a greater portion of energy resides between the plates. The simple analytical equation that ignores fringing is more accurate because it is more consistent with the actual physics.

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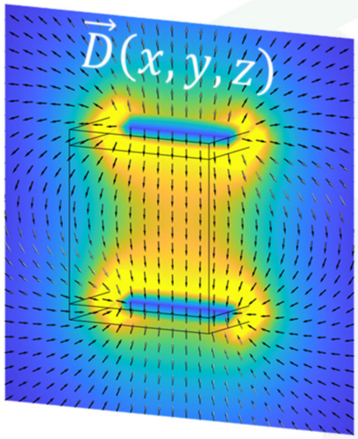
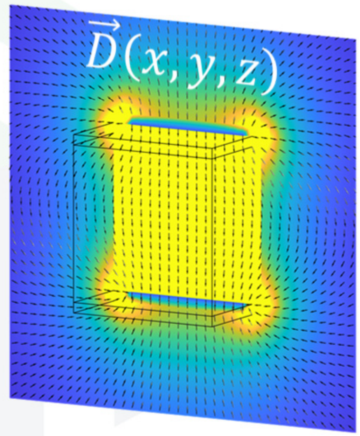
Effect of Dielectric Constant ε_r

$\varepsilon_r = 1.0$
172% error





$\varepsilon_r = 12.0$
25% error

As dielectric constant ε_r becomes larger, a greater fraction of energy resides between the plates and the analytical solution that ignores the fringing becomes more accurate.

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How Does the Numerical Analysis Work?

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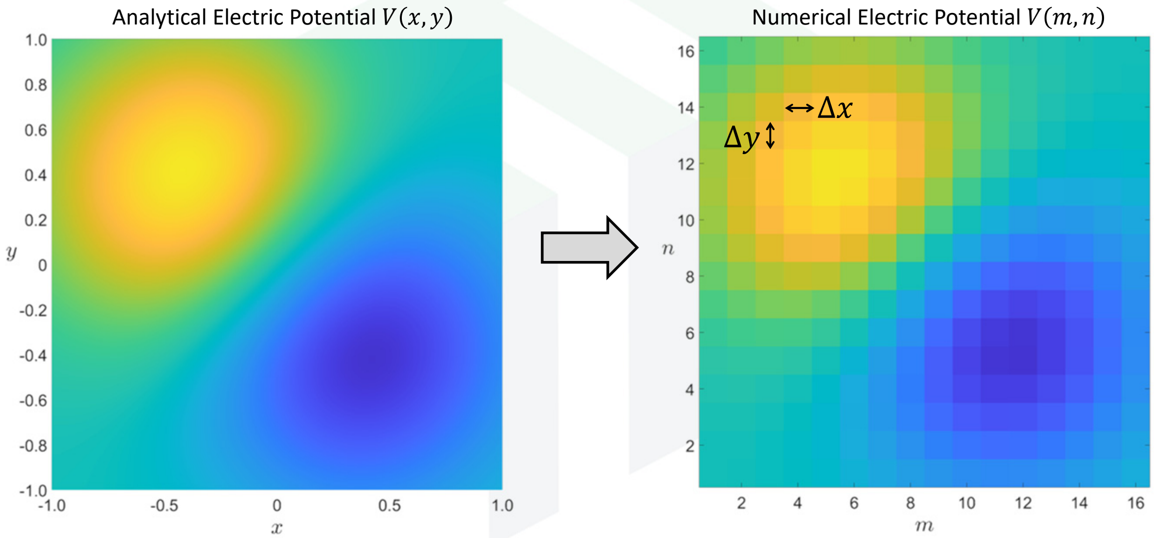
Make Functions Discrete



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Finite-Difference Form of Laplace's Equation

Approximate Laplace's equation using finite-differences (or finite elements, etc.)

$$\nabla^2 V = 0$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{V(p+1,q,r) - 2V(p,q,r) + V(p-1,q,r)}{(\Delta x)^2} + \frac{V(p,q+1,r) - 2V(p,q,r) + V(p,q-1,r)}{(\Delta y)^2} + \frac{V(p,q,r+1) - 2V(p,q,r) + V(p,q,r-1)}{(\Delta z)^2} = 0$$

Collect common electric potential terms.

$$\left[\frac{2}{(\Delta x)^2} + \frac{2}{(\Delta y)^2} + \frac{2}{(\Delta z)^2}\right] V(p,q,r) - \frac{1}{(\Delta x)^2} V(p+1,q,r) - \frac{1}{(\Delta x)^2} V(p-1,q,r) - \frac{1}{(\Delta y)^2} V(p,q+1,r) - \frac{1}{(\Delta y)^2} V(p,q-1,r) - \frac{1}{(\Delta z)^2} V(p,q,r+1) - \frac{1}{(\Delta z)^2} V(p,q,r-1) = 0$$

This equation must be satisfied at each point in the array.

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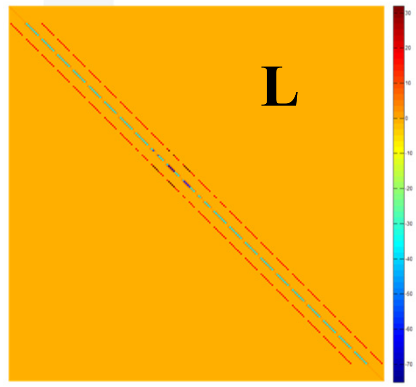
Form a Single Matrix Equation

Write the finite-difference equation at each point on the grid. This large set of equations can be written in matrix form as

$$\mathbf{Lv} = \mathbf{0}$$

$$\mathbf{v} = \begin{bmatrix} V(1,1,1) \\ V(2,1,1) \\ V(3,1,1) \\ \vdots \\ V(N_x, N_y, N_z) \end{bmatrix}$$



This equation is not yet solvable because

$$\mathbf{v} = \mathbf{L}^{-1}\mathbf{0} = \mathbf{0}$$

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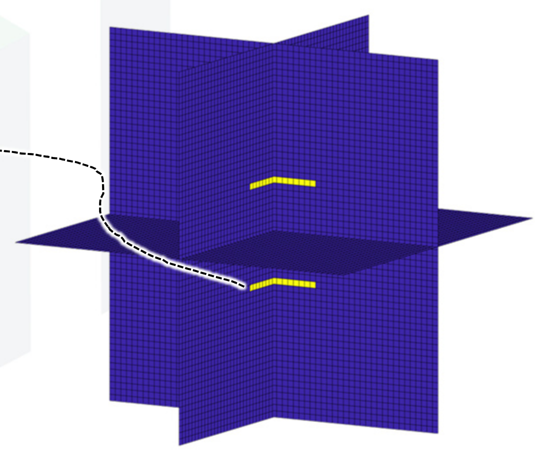
Apply a Voltage V_{applied} Across the Device

An excitation has to be incorporated by enforcing the known potentials.

$$\underbrace{\begin{bmatrix} (\#) & (\#) & (\#) & \cdots & (\#) & (\#) & (\#) \\ (\#) & (\#) & (\#) & \cdots & (\#) & (\#) & (\#) \\ & & \ddots & & \iddots & & \\ 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ & & \iddots & & \ddots & & \\ (\#) & (\#) & (\#) & \cdots & (\#) & (\#) & (\#) \\ (\#) & (\#) & (\#) & \cdots & (\#) & (\#) & (\#) \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_m^{\text{metal}} \\ \vdots \\ V_{N_x N_y - 1} \\ V_{N_x N_y} \end{bmatrix}}_{\mathbf{v}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ V_{\text{applied}} \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{b}}$$



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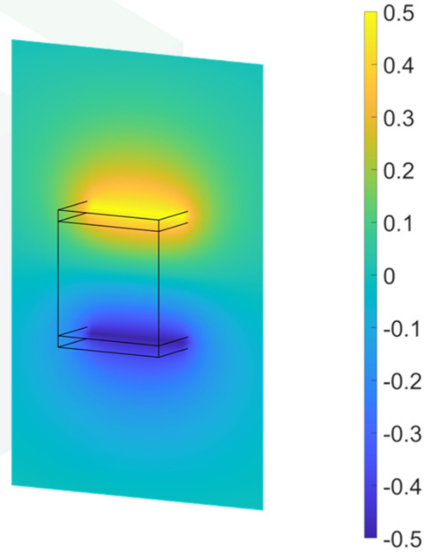
Solve Matrix Equation to Calculate Electric Potential

Calculate the electric potential

$$\mathbf{v} = \mathbf{L}^{-1}\mathbf{b}$$



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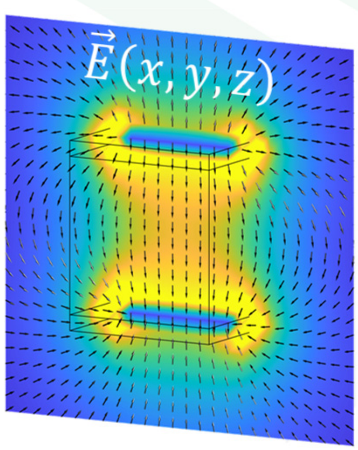
Calculate Electric Field Intensity $\vec{E}$ from Potential

Calculate the electric field intensity using $\vec{E} = -\nabla V$.



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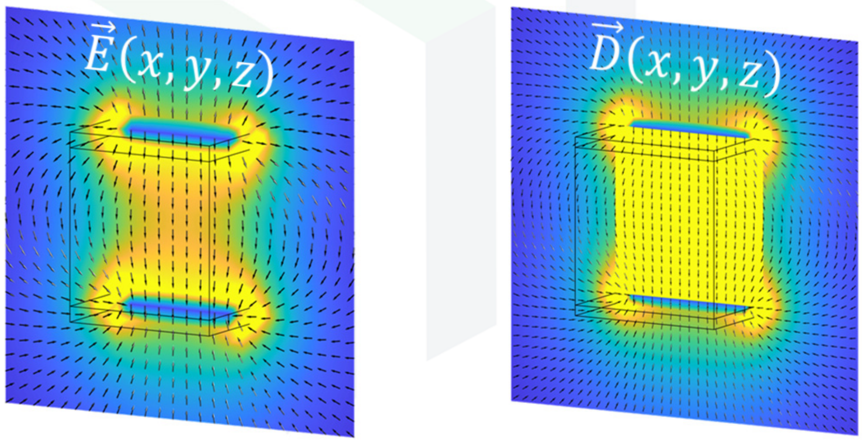
Calculate Electric Field Intensity $\vec{E}$ from Potential

Calculate the electric flux density using $\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$.



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Calculate Capacitance C

$$C = \varepsilon_0 \iiint_{\text{grid}} \left(\vec{D} \bullet \vec{E} \right) dv$$

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C = D.'*E*(e0*dx*dy*dz);
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