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Electromagnetics:
Electromagnetic Field Theory

Example 2 – Force Between Current Elements

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Example #2 – Force Between Two Wires

Two parallel wires carrying currents I_1 and I_2 are a distance s apart.

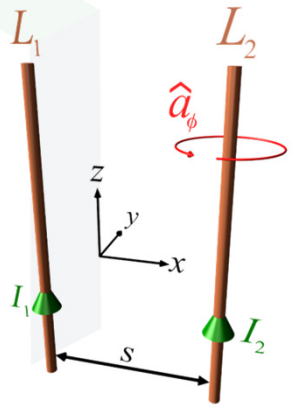
What is the force per meter between these two wires?

Solution

Solve this by calculating the magnetic field induced by the second wire, and then use this result to calculate the force on the first wire.

Using the infinite wire approximation, the magnetic field by the second wire is

$$\vec{H}_2 = \frac{I_2}{2\pi\rho}\hat{a}_\phi$$



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Example #2 – Force Between Two Wires

Use the constitutive relation to find $\vec{B}$ around the second wire.

$$\vec{H}_2 = \frac{I_2}{2\pi\rho}\hat{a}_\phi \quad \rightarrow \quad \frac{\vec{B}_2}{\mu} = \frac{I_2}{2\pi\rho}\hat{a}_\phi$$

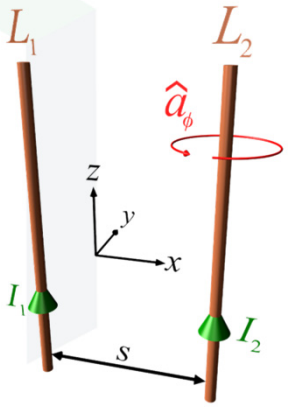
$$\rightarrow \quad \vec{B}_2 = \frac{\mu I_2}{2\pi\rho}\hat{a}_\phi$$

The force this magnetic flux puts on the first wire is

$$\vec{F}_1 = \int_{L_1}\left(I_1 d\vec{\ell}_1\right)\times\vec{B}_2 = \int_0^L \left(I_1 dz\hat{a}_z\right)\times\left(\frac{\mu I_2}{2\pi\rho}\hat{a}_\phi\right)$$

The coordinate systems have been mixed here.

At the first wire $\hat{a}_\phi = -\hat{a}_y$ and $\rho = s$.



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Example #2 – Force Between Two Wires

The equation for force becomes

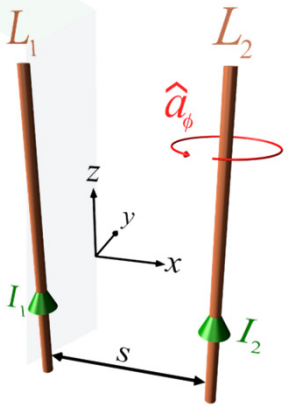
$$\vec{F}_1 = \int_0^L \left(I_1 dz\hat{a}_z\right)\times\left(-\frac{\mu I_2}{2\pi s}\hat{a}_y\right)$$

$$= \frac{\mu I_1 I_2}{2\pi s}\int_0^L \left(\hat{a}_z\right)\times\left(-\hat{a}_y\right)dz$$

$$= \frac{\mu I_1 I_2}{2\pi s}\int_0^L \left(\hat{a}_x\right)dz$$

$$= \frac{\mu I_1 I_2}{2\pi s}\hat{a}_x\int_0^L dz$$

$$= \frac{\mu I_1 I_2}{2\pi s}\hat{a}_x L$$



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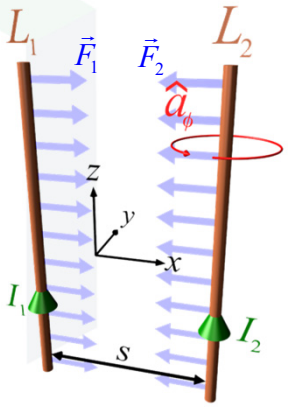
Example #2 – Force Between Two Wires

The force per unit length is

$$\vec{F}_1 = \frac{\mu I_1 I_2}{2\pi s}\hat{a}_x L$$

$$\boxed{\frac{\vec{F}_1}{L} = \frac{\mu I_1 I_2}{2\pi s}\hat{a}_x}$$



Observe that the wires are being attracted toward each other. This is called the *pinch effect*.