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Electromagnetics:
Electromagnetic Field Theory

Examples 3 & 4 – Get a Feel for the Numbers

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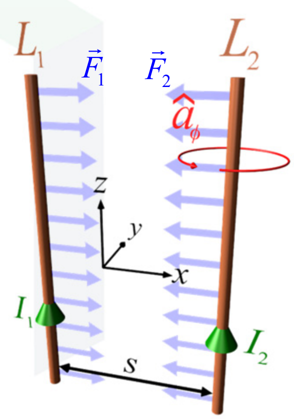
Example #3 – The Numbers

If the wires both carry 1 A, are 1 m long, and are 1 m apart, what is the total force between them?



Approximate Solution

Plug these numbers into the equation derived in the previous video.

$$\vec{F}_1 = \frac{\mu_0 \mu_r I_1 I_2 L}{2\pi s}\hat{a}_x$$

$$= \frac{\left(1.2566\times10^{-6}\ \tfrac{\text{H}}{\text{m}}\right)(1.0)(1\ \text{A})(1\ \text{A})(1\ \text{m})}{2\pi(1\ \text{m})}\hat{a}_x$$

$$= \boxed{2.0\times10^{-7}\hat{a}_x\ \text{N} = 200\hat{a}_x\ \text{nN}}$$

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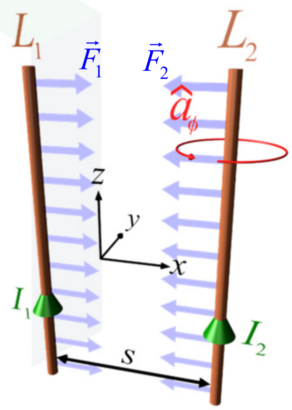
Example #4 – Exact Numbers

If the wires both carry 1 A, are 1 m long, and are 1 m apart, what is the total force between them?



Exact Solution

The rigorous equation to calculate the force between these two wires is

$$\vec{F}_1 = \frac{\mu I_1 I_2}{4\pi} \int_{L_1} \int_{L_2} \frac{d\vec{\ell}_1 \times d\vec{\ell}_2 \times \hat{a}_{21}}{R_{21}^2}$$

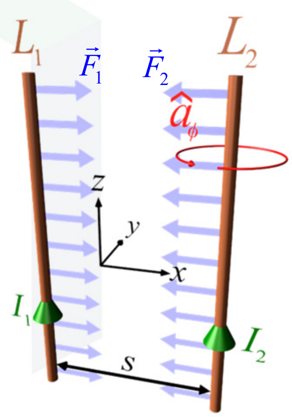
$$= \frac{\mu I_1 I_2}{4\pi} \int_{L_1} \int_{L_2} d\vec{\ell}_1 \times d\vec{\ell}_2 \times \frac{\vec{R}_{21}}{\left|\vec{R}_{21}\right|^3}$$

Example #4 – Exact Numbers

Determine expressions for each term in the force equation.

$$\vec{F}_1 = \frac{\mu I_1 I_2}{4\pi} \int_{L_1} \int_{L_2} d\vec{\ell}_1 \times d\vec{\ell}_2 \times \frac{\vec{R}_{21}}{\left|\vec{R}_{21}\right|^3}$$

$$\mu = \mu_0$$
$$I_1 = 1\text{ A}$$
$$I_2 = 1\text{ A}$$
$$d\vec{\ell}_1 = dz_1 \hat{a}_z$$
$$d\vec{\ell}_2 = dz_2 \hat{a}_z$$
$$\vec{R}_{21} = -s\hat{a}_x + (z_2 - z_1)\hat{a}_z$$
$$\frac{\vec{R}_{21}}{\left|\vec{R}_{21}\right|^3} = \frac{-s\hat{a}_x + (z_2 - z_1)\hat{a}_z}{\left[s^2 + (z_2 - z_1)^2\right]^{3/2}}$$



Example #4 – Exact Numbers

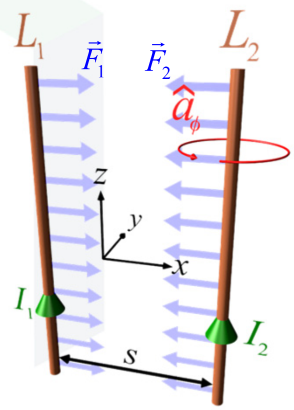
The force equation becomes

$$\vec{F}_1 = \frac{\mu I_1 I_2}{4\pi} \int_{L_1} \int_{L_2} d\vec{\ell}_1 \times d\vec{\ell}_2 \times \frac{\vec{R}_{21}}{\left|\vec{R}_{21}\right|^3}$$

$$= \frac{\mu I_1 I_2}{4\pi} \int_0^L \int_0^L dz_1 \hat{a}_z \times \left(dz_2 \hat{a}_z \times \frac{-s\hat{a}_x + (z_2 - z_1)\hat{a}_z}{\left[s^2 + (z_2 - z_1)^2\right]^{3/2}} \right)$$

$$= \frac{\mu I_1 I_2}{4\pi} \int_0^L \int_0^L dz_1 \hat{a}_z \times \left(\frac{-s dz_2}{\left[s^2 + (z_2 - z_1)^2\right]^{3/2}} \hat{a}_y \right)$$

$$= \frac{\mu s I_1 I_2}{4\pi} \hat{a}_x \int_0^L \int_0^L \frac{dz_1 dz_2}{\left[s^2 + (z_2 - z_1)^2\right]^{3/2}}$$



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Example #4 – Exact Numbers

As $L \rightarrow \infty$, the answer converges to the approximate result from the last example.

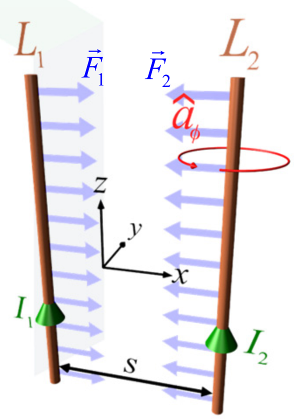
$$\vec{F}_1 = \frac{\mu_0 \mu_r I_1 I_2 L}{2\pi s} \hat{a}_x \rightarrow (200 \text{ nN}) \hat{a}_x$$

Performing the integration gives the exact result of

$$\boxed{\vec{F}_1 = (82.8 \text{ nN}) \hat{a}_x}$$

If the first wire were 1 m long and the second was 10 m long, the force would be

$$\vec{F}_1 = (196 \text{ nN}) \hat{a}_x$$



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