Electromagnetics:

Electromagnetic Field Theory

Current Distributions

1

"Point" Current

Magnetic Field

$$d\vec{H} = \frac{Id\vec{\ell} \times \hat{a}_R}{4\pi R^2}$$

$Id\vec{\ell}$ Differential Current (A·m)

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Line Current

Magnetic Field

$$\vec{H} = \int_L \frac{Id\vec{\ell} \times \hat{a}_R}{4\pi R^2}$$

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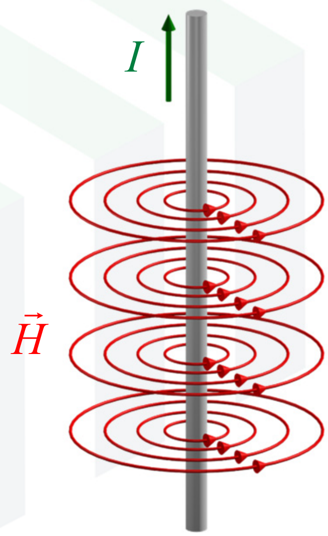
Magnetic Field Around an Infinite Line Current

Magnetic Field

$$\vec{H} = \int_L \frac{Id\vec{\ell} \times \hat{a}_R}{4\pi R^2}$$

$$= \frac{I}{2\pi\rho}\hat{a}_\phi$$



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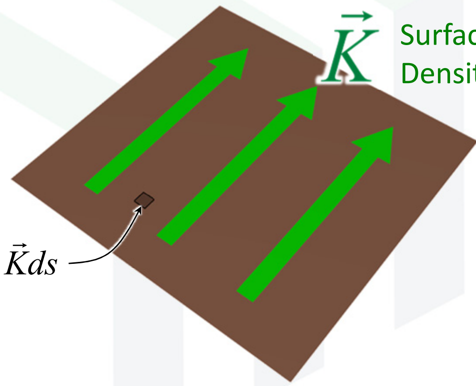
Surface Current

Magnetic Field

$$\vec{H} = \iint_S \frac{\vec{K} ds \times \hat{a}_R}{4\pi R^2}$$



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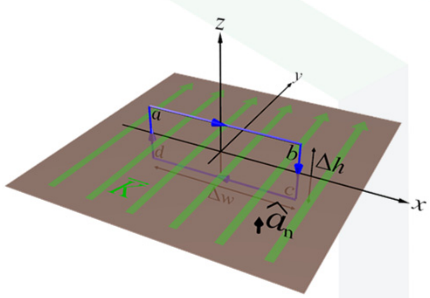
Magnetic Field Due to Surface Current



The magnetic field $\vec{H}$ can be determined by applying Ampere's circuit law and integrating around a closed-line that encompasses some surface current.

$$I = \oint_L \vec{H} \bullet d\vec{\ell}$$

This separates into six integrals.

$$I = \int_a^b \vec{H} \bullet d\vec{\ell} + \int_b^0 \vec{H} \bullet d\vec{\ell} + \int_0^c \vec{H} \bullet d\vec{\ell} + \int_c^d \vec{H} \bullet d\vec{\ell} + \int_d^0 \vec{H} \bullet d\vec{\ell} + \int_0^a \vec{H} \bullet d\vec{\ell}$$

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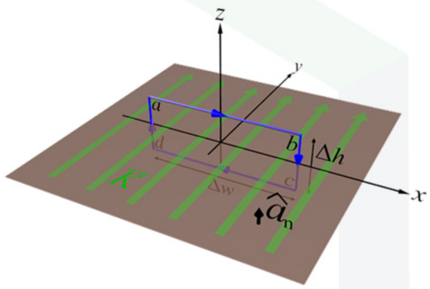
Magnetic Field Due to Surface Current

$$I = \int_a^b \vec{H} \bullet d\vec{\ell} + \int_b^0 \vec{H} \bullet d\vec{\ell} + \int_0^c \vec{H} \bullet d\vec{\ell} + \int_c^d \vec{H} \bullet d\vec{\ell} + \int_d^0 \vec{H} \bullet d\vec{\ell} + \int_0^a \vec{H} \bullet d\vec{\ell}$$

$$I = H_{x,\text{top}}\Delta w - \cancel{H_{z,\text{top}}\frac{\Delta h}{2}} - \cancel{H_{z,\text{bot}}\frac{\Delta h}{2}} - H_{x,\text{bot}}\Delta w + \cancel{H_{z,\text{bot}}\frac{\Delta h}{2}} + \cancel{H_{z,\text{top}}\frac{\Delta h}{2}}$$



In the limit as $\Delta h \rightarrow 0$,

$$I = H_{x,\text{top}}\Delta w - H_{x,\text{bot}}\Delta w$$

Due to symmetry $|H_{x,\text{top}}| = |H_{x,\text{bot}}|$

If $H_{x,\text{top}} = H_{x,\text{bot}}$, then $I = 0$. → ???

It is concluded that

$$H_x = H_{x,\text{top}} = -H_{x,\text{bot}}$$

The equation becomes

$$I = H_x\Delta w + H_x\Delta w = 2H_x\Delta w$$

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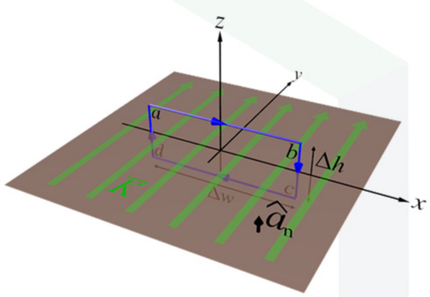
Magnetic Field Due to Surface Current



The total current I is related to the surface current density $\vec{K}$ as

$$I = K_y\Delta w$$

The equation becomes

$$K_y\Delta w = 2H_x\Delta w$$

$$K_y = 2H_x$$

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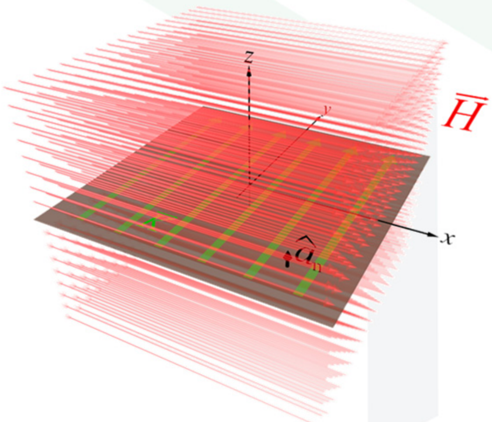
Magnetic Field Due to Surface Current



The total current I is related to the surface current density $\vec{K}$ as

$$I = K_y \Delta w$$

The equation becomes

$$K_y \Delta w = 2H_x \Delta w$$

$$K_y = 2H_x$$

In vector form, this is

$$\boxed{\vec{H} = \frac{1}{2}\vec{K} \times \hat{a}_n}$$

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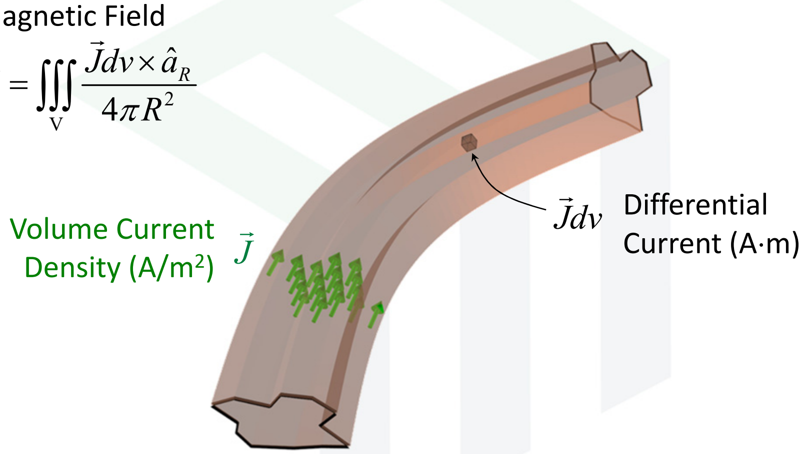
Volume Current

Magnetic Field

$$\vec{H} = \iiint_V \frac{\vec{J}dv \times \hat{a}_R}{4\pi R^2}$$



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Recipe to Calculate Field Around Current Distributions

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Recipe for Solving Problems

1. Draw the problem and label with all dimensions and parameters.
2. Choose a coordinate system that makes the math easiest.
3. Write the general equation.

	Point	Line	Surface	Volume
$\vec{H}_{\text{total}}$	$d\vec{H} = \dfrac{Id\vec{\ell} \times \hat{a}_R}{4\pi R^2}$	$\vec{H} = \displaystyle\int_L \dfrac{Id\vec{\ell} \times \hat{a}_R}{4\pi R^2}$	$\vec{H} = \displaystyle\iint_S \dfrac{\vec{K}ds \times \hat{a}_R}{4\pi R^2}$	$\vec{H} = \displaystyle\iiint_V \dfrac{\vec{J}dv \times \hat{a}_R}{4\pi R^2}$

4. Write expressions for each term in the integral.
5. Choose limits of integration.
6. Solve the integral.
7. Interpret the result.

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