

# Magnetostatic Fields & Currents

### Governing Eqs.

Conditions  $\nabla \times \vec{H} = \vec{J}$

- $\omega = 0$   $\nabla \cdot \vec{B} = 0$
- Size  $\ll \lambda$   $\vec{B} = [\mu] \vec{H}$

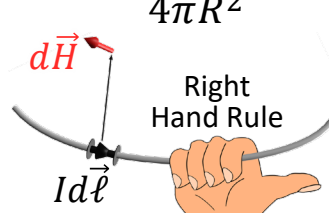
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### Lorentz Force Law

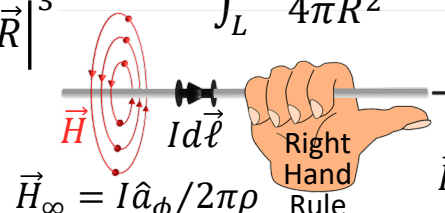
$$\vec{F} = Q\vec{E} + Q(\vec{u} \times \vec{B})$$

### Magnetic Fields Around Current Elements

#### Biot-Savart Law

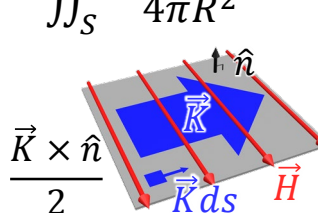
$$d\vec{H} = \frac{Id\vec{\ell} \times \hat{a}_R}{4\pi R^2} = \frac{Id\vec{\ell} \times \vec{R}}{4\pi |\vec{R}|^3}$$


#### Line Current

$$\vec{H} = \int_L \frac{Id\vec{\ell} \times \hat{a}_R}{4\pi R^2}$$


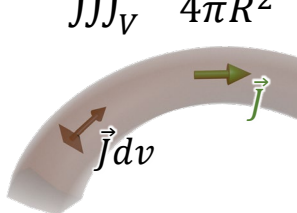
$\vec{H}_\infty = I\hat{a}_\phi / 2\pi\rho$

#### Surface Current

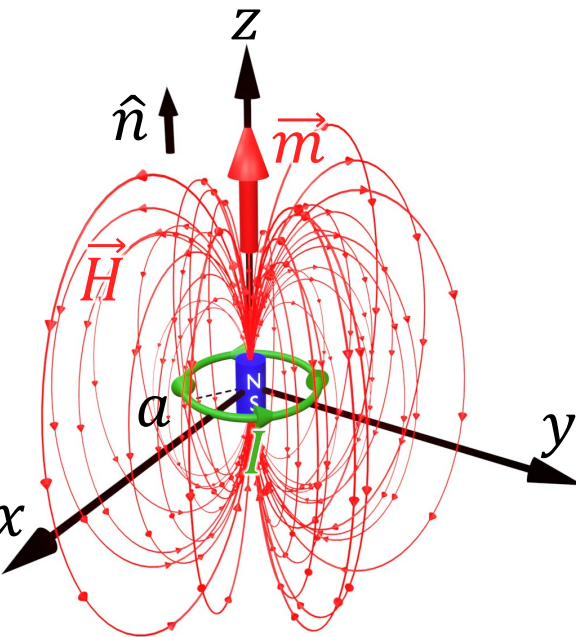
$$\vec{H} = \iint_S \frac{\vec{K} ds \times \hat{a}_R}{4\pi R^2}$$


$\vec{H}_\infty = \frac{\vec{K} \times \hat{n}}{2}$

#### Volume Current

$$\vec{H} = \iiint_V \frac{\vec{J} dv \times \hat{a}_R}{4\pi R^2}$$


### Magnetic Dipole



Magnetic Field

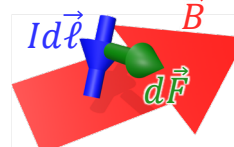
$$\vec{H} = \frac{|\vec{m}|}{4\pi r^3} (2 \cos \theta \hat{a}_r + \sin \theta \hat{a}_\theta)$$

Magnetic Dipole Moment

$$\vec{m} = \pi a^2 I \hat{a}_n$$

### Force Magnetic Fields Put on Current Elements

#### Differential Current

$$d\vec{F} = Id\vec{\ell} \times \vec{B}$$


#### Line Current

$$\vec{F} = \int_L Id\vec{\ell} \times \vec{B}$$

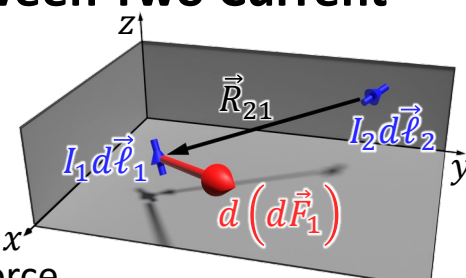
#### Surface Current

$$\vec{H} = \iint_S \vec{K} ds \times \vec{B}$$

#### Volume Current

$$\vec{H} = \iiint_V \vec{J} dv \times \vec{B}$$

### Force Between Two Current Elements



Differential Force

$$d(d\vec{F}_1) = \frac{\mu}{4\pi} \frac{(I_1 d\vec{\ell}_1) \times (I_2 d\vec{\ell}_2) \times \hat{a}_{21}}{R_{21}^2}$$

Total Force

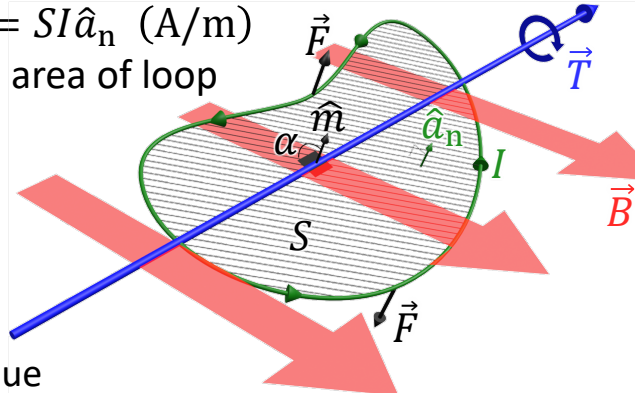
$$\vec{F}_1 = \frac{\mu I_1 I_2}{4\pi} \int_{L_1} \int_{L_2} \frac{d\vec{\ell}_1 \times d\vec{\ell}_2 \times \hat{a}_{21}}{R_{21}^2}$$

### Magnetic Torque & Moment

Magnetic Dipole Moment

$$\vec{m} = SI\hat{a}_n \text{ (A/m)}$$

S is area of loop



Torque

$$\vec{T} = \vec{m} \times \vec{B}$$

Acts to reduce  $\alpha$  and put  $\vec{m}$  and  $\vec{B}$  in same direction.

Angle Dependence

$$|\vec{T}| = BIS \sin \alpha$$