Trigonometric Identities

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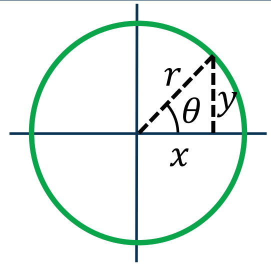
Definitions & Relations

$x^2 + y^2 = r^2$



$\sin\theta = \frac{y}{r} = \frac{\text{opp}}{\text{hyp}}$ $\cos\theta = \frac{x}{r} = \frac{\text{adj}}{\text{hyp}}$ $\tan\theta = \frac{y}{x} = \frac{\text{opp}}{\text{adj}} = \frac{\sin\theta}{\cos\theta}$

$\csc\theta = 1/\sin\theta$ $\sec\theta = 1/\cos\theta$ $\cot\theta = 1/\tan\theta$

$\sin\theta = -\sin(\theta \pm \pi) = \cos(\pi/2 - \theta)$ $\cot\theta = \tan(\pi/2 - \theta)$

$\cos\theta = -\cos(\theta \pm \pi) = \sin(\pi/2 - \theta)$ $\tan\theta = -\tan(\pi - \theta) = \cot(\pi/2 - \theta)$

$\csc\theta = \sec(\pi/2 - \theta)$ $\sec\theta = \csc(\pi/2 - \theta)$

Periodicity: $\sin\theta = \sin(\theta \pm 2\pi m)$ $\cos\theta = \cos(\theta \pm 2\pi m)$ $\tan\theta = \tan(\theta \pm \pi m)$

Negative Angles: $\sin(-\theta) = -\sin(\theta)$ $\cos(-\theta) = \cos(\theta)$ $\tan(-\theta) = -\tan(\theta)$

Identities for General Triangles

Law of Sines

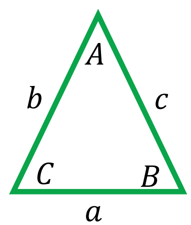
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Law of Cosines

$c^2 = a^2 + b^2 - 2ab\cos C$

Law of Tangents

$\frac{a+b}{a-b} = \frac{\tan\left[\frac{1}{2}(A+B)\right]}{\tan\left[\frac{1}{2}(A-B)\right]}$



Pythagorean Theorem: $\sin^2\theta + \cos^2\theta = 1$

Sum & Difference Formulas

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Product-Sum Identities

$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

Product Identities

$\sin\alpha\sin\beta = \frac{\cos(\alpha+\beta) - \cos(\alpha-\beta)}{2}$

$\cos\alpha\cos\beta = \frac{\cos(\alpha+\beta) + \cos(\alpha-\beta)}{2}$

$\sin\alpha\cos\beta = \frac{\sin(\alpha+\beta) + \sin(\alpha-\beta)}{2}$

Half, Double & Triple Angle Formulas

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$ $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$ $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$
$= 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta$

$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

$\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$

$\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$

$\tan(3\theta) = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

Inverse Identities

$\sin(\cos^{-1}\alpha) = \sqrt{1-\alpha^2}$

$\cos(\sin^{-1}\alpha) = \sqrt{1-\alpha^2}$

$\tan(\sin^{-1}\alpha) = \frac{\alpha}{\sqrt{1-\alpha^2}}$

$\sin(\tan^{-1}\alpha) = \frac{\alpha}{\sqrt{1+\alpha^2}}$

$\cos(\tan^{-1}\alpha) = \frac{1}{\sqrt{1-\alpha^2}}$

$\tan(\cos^{-1}\alpha) = \frac{\sqrt{1-\alpha^2}}{\alpha}$

Complex Exponentials

$e^{i\theta} = \cos\theta + i\sin\theta$

$ie^{i\theta} = -e^{i(\theta-\pi/2)}$

$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$

$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$

Trig Vales at Special Angles

	0°	30°	45°	60°	90°
$\sin\theta$	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞