

Trigonometric Identities

Definitions & Relations

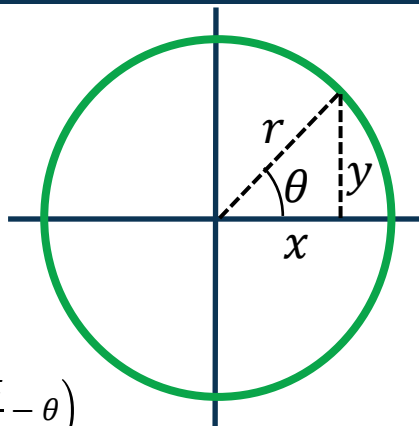
$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{\sin \theta}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\sin \theta = -\sin(\theta \pm \pi) = \cos\left(\frac{\pi}{2} - \theta\right)$$

$$\cot \theta = \tan\left(\frac{\pi}{2} - \theta\right) \quad \cos \theta = -\cos(\theta \pm \pi) = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\csc \theta = \sec\left(\frac{\pi}{2} - \theta\right) \quad \sec \theta = \csc\left(\frac{\pi}{2} - \theta\right) \quad \tan \theta = -\tan(\pi - \theta) = \cot\left(\frac{\pi}{2} - \theta\right)$$



Half, Double & Triple Angle Formulas

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin \theta}$$

$$\sin(2\theta) = 2\sin \theta \cos \theta$$

$$\begin{aligned} \cos(2\theta) &= \cos^2 \theta - \sin^2 \theta \\ &= 2\cos^2 \theta - 1 \\ &= 1 - 2\sin^2 \theta \end{aligned}$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin(3\theta) = 3\sin \theta - 4\sin^3 \theta$$

$$\cos(3\theta) = 4\cos^3 \theta - 3\cos \theta$$

$$\tan(3\theta) = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$$

Periodicity

$$\sin \theta = \sin(\theta \pm 2\pi m) \quad \cos \theta = \cos(\theta \pm 2\pi m) \quad \tan \theta = \tan(\theta \pm \pi m)$$

Negative Angles

$$\sin(-\theta) = -\sin(\theta) \quad \cos(-\theta) = \cos(\theta) \quad \tan(-\theta) = -\tan(\theta)$$

Sum & Difference Formulas

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

Product-Sum Identities

$$\sin \alpha + \sin \beta = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin \alpha - \sin \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha + \cos \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha - \cos \beta = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

Product Identities

$$\sin \alpha \sin \beta = \frac{\cos(\alpha + \beta) - \cos(\alpha - \beta)}{2}$$

$$\cos \alpha \cos \beta = \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{2}$$

$$\sin \alpha \cos \beta = \frac{\sin(\alpha + \beta) + \sin(\alpha - \beta)}{2}$$

Inverse Identities

$$\sin(\cos^{-1} \alpha) = \sqrt{1 - \alpha^2}$$

$$\cos(\sin^{-1} \alpha) = \sqrt{1 - \alpha^2}$$

$$\tan(\sin^{-1} \alpha) = \frac{\alpha}{\sqrt{1 - \alpha^2}}$$

$$\sin(\tan^{-1} \alpha) = \frac{\alpha}{\sqrt{1 + \alpha^2}}$$

$$\cos(\tan^{-1} \alpha) = \frac{1}{\sqrt{1 + \alpha^2}}$$

$$\tan(\cos^{-1} \alpha) = \frac{\sqrt{1 - \alpha^2}}{\alpha}$$

Complex Exponentials

$$\begin{aligned} e^{i\theta} &= \cos \theta + i \sin \theta \\ ie^{i\theta} &= -e^{i(\theta - \pi/2)} \end{aligned}$$

$$\begin{aligned} \sin \theta &= \frac{e^{i\theta} - e^{-i\theta}}{2i} \\ \cos \theta &= \frac{e^{i\theta} + e^{-i\theta}}{2} \end{aligned}$$

Trig Vales at Special Angles

	0°	30°	45°	60°	90°
sin θ	0	1/2	1/√2	√3/2	1
cos θ	1	√3/2	1/√2	1/2	0
tan θ	0	1/√3	1	√3	∞