EMPossible

Reading

Textbooks: *Numerical Methods for Engineers*, 7th Ed.
Steven C. Chapra & Raymond P. Canale
McGraw Hill

Electromagnetic & Photonic Simulation for the Beginner:
Finite-Difference Frequency-Domain in MATLAB
Raymond C. Rumpf
Artech House

Assignment: Chapra – Read Chapters 3 and 4
Rumpf – Read Chapter 1

Problem #1: Significant Digits

How many significant digits do each of the following numbers have?

a. 64.12345
b. 0.0000173
c. 189,000
d. 6.02×10^{23}
e. 500.001×10^{-12}

Problem #2: Error

While driving along a long desert highway, some professor other than your professor sees a mirage ahead. He understands that he is observing *total internal reflection* (TIR). Light passing from the colder air into the hotter air near the road is 100% reflected, giving the mirror-like appearance of the mirage. He knows that TIR can only occur above a certain angle of incidence when light passes from one medium into another that has a lower refractive index. He wonders what change in refractive index Δn the hot air experiences. Hmmm…

This special angle is called the *critical angle* θ_c. He derives an equation for it in his head from Snell's law while his wife stares at him strangely. The equation he comes up with is

$$\theta_c = \sin^{-1}\left(n_{hot}/n_{cold}\right)$$

where n_{hot} is the refractive index of the hot air and n_{cold} is the refractive index of the cold air. The professor estimates the edge of the mirage is 150 m $\pm$ 50 m in front of the car and his head is 5 ft $\pm$ 1 ft above the ground. From this triangle, he estimates the critical angle θ_c by asking his kind, patient, and brilliant wife to use the calculator on her phone to perform the following calculation for him while he drives. Continuing to look at him strangely, his wife performs the calculation.

$$\theta_c = 90° - \tan^{-1}\left(5 \text{ ft}/150 \text{ m}\right) = 89.42°$$

The professor does not know the refractive index of air exactly, but knows it is very close to 1. So, he expresses the two refractive indices as

$$n_{hot} = 1 \qquad n_{cold} = 1 + \Delta n$$

Substituting these into the first equation and solving for Δn gave him

$$\Delta n = 1/\sin \theta_c - 1$$

He again asks his kind, patient, and brilliant wife to run this last calculation. Shaking her head as she calculates, she gets

$$\Delta n = 1/\sin 89.42° - 1 = 5.2 \times 10^{-5}$$

Calculate the uncertainty of this crude calculation of Δn.

Problem #3: Build a Dartboard on a Cartesian Grid

The standard English dartboard is shown below. It is composed of the numbers 1 to 20 arranged randomly as wedges in a pie so each wedge spans 18°. The inner bullseye is worth 50 points while the outer bullseye is worth 25 points. Within the outermost ring the number score is doubled. Inside the middle ring, the number score is tripled. The number scores in each of these regions are provided in the figure. The approximate dimensions of the radii describing these regions are also provided.

$r_1 = 14.2$ mm
$r_2 = 31.8$ mm
$r_3 = 184.1$ mm
$r_4 = 203.2$ mm
$r_5 = 304.8$ mm
$R = 425.5$ mm
$r_6 = 323.8$ mm
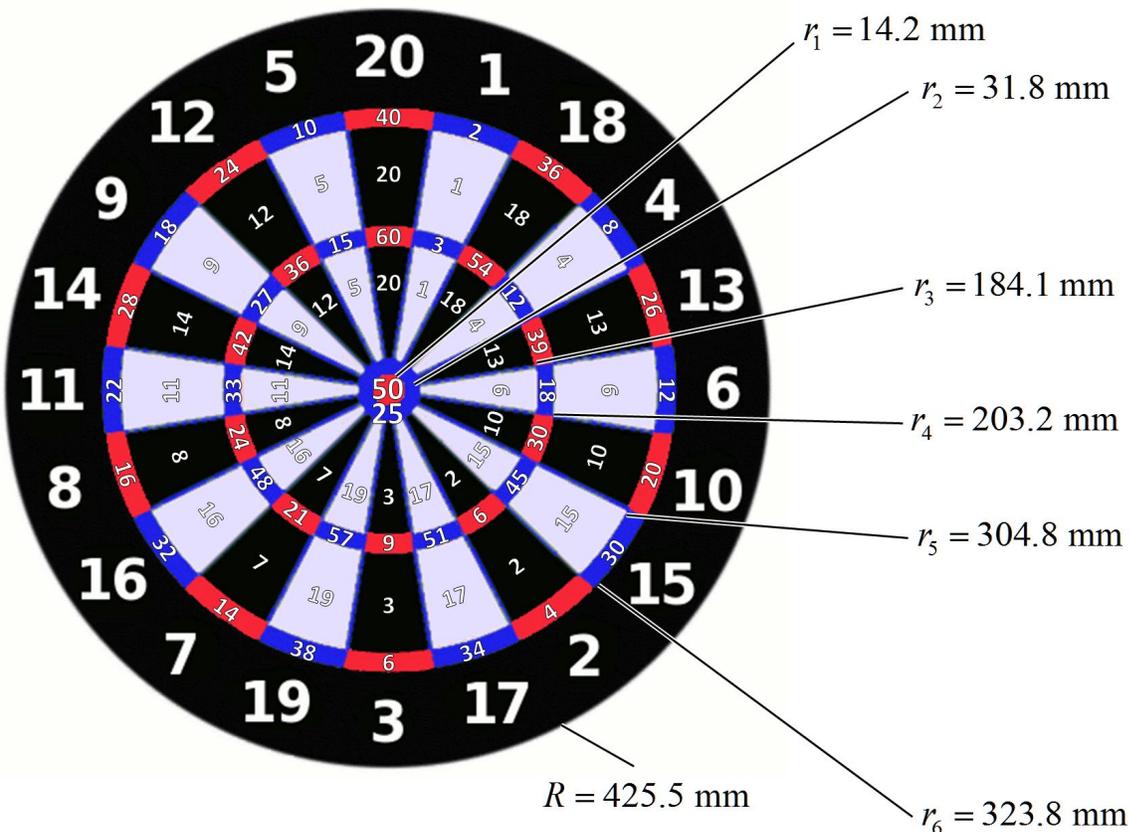
Write a MATLAB program to build a standard English dartboard onto a Cartesian grid with 512×512 points. This should be a single array with values assigned to each point in a manner that describes the scores on dartboard. Visualize the data in your array to a figure window and include the image in your homework solution. Remember to include your MATLAB code in an Appendix.

Hints:

1. Use `meshgrid()`.
2. Use `atan2()`.
3. Your data should look like the figure to the right.
4. Do not use `for` loops to fill in pixels in your grid.
5. Aside from the dashboard, calculating the grid, and graphics, the code provided in the solution uses only 12 lines of code to build dartboard into the array.