Reading

Textbooks: *Numerical Methods for Engineers*, 7th Ed.
Steven C. Chapra & Raymond P. Canale, McGraw Hill

Electromagnetic & Photonic Simulation for the Beginner: Finite-Difference Frequency-Domain in MATLAB
Raymond C. Rumpf, Artech House

Website: *Numerical Methods for Engineers*, 7th Ed.

Assignment: Chapra – Read Chapter 23
Rumpf – Read Chapter 3, pp. 71-82
Website – Topic 6, Lectures 6f to 6k

Problem #1: Finite-Difference Approximations

Using the polynomial method, derive expressions for the following finite-difference approximations. You may do this by hand or use MATLAB in any way that you wish. Show all your work. Report your equations in standard math format using Equation Editor, LaTex, or whatever tool you wish. Do not report your equations using pure text or ASCII art style. Your finite-difference equations must be written in a way that conveys the location where the derivatives are being evaluated. They must also be written in terms of the given parameters. An example solution of a second-order derivative of $f(x)$ with respect to x, evaluated at the second point is

$$\frac{df_2}{dx^2} \approx \frac{f_1 - 2f_2 + f_3}{(\Delta x)^2}$$

Part a

$\frac{df(x)}{dx}$ at the midpoint between two points

Part b

$\frac{df(x)}{dx}$ at the midpoint between three points

Part c

$\frac{d^2 f(t)}{dt^2}$ at the midpoint between four points

Part d

$\frac{d^2 f(t)}{dt^2}$ at the second of four points

Part e

$\frac{d^4 f(\theta)}{d\theta^4}$ at midpoint between the second and third of five points

Problem #2: Numerical Differentiation

It is desired to numerically differentiate the $\text{sinc}\,\theta$ function in the interval $-5 \leq \theta \leq 5$. Recall the sinc function is defined as

$$\text{sinc}\,\theta = \begin{cases} \sin(\pi\theta)/\pi\theta & \theta \neq 0 \\ 1 & \theta = 0 \end{cases}$$

Part a

Derive all the finite-difference approximations you will need across the entire grid if all your finite-difference approximations are to use 5 points to calculate a second derivative. Remember to derive special finite-difference approximations for the points at the boundaries! Report your answers using professional math formatting.

Part b

Using the finite-difference approximations you derived in Part a of this problem, write a MATLAB program to numerically calculate $d^2\,\text{sinc}\,\theta/d\theta^2$. Use 200 points within the specified interval. DO NOT CALCULATE ANYTHING OUTSIDE OF THE SPECIFIED INTERVAL or points will be deducted from your assignment. On the same plot, show the original sinc function and its numerical second derivative across the entire interval. Be sure your plot is of professional quality. Complete and sign a graphics checklist and attach it to this homework.