Reading

Textbook: *Electromagnetic & Photonic Simulation for the Beginner: Finite-Difference Frequency-Domain in MATLAB*
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Website: https://empossible.net/academics/emp4301_5301/

Assignment: Read Chapter 3, pp. 82-85
Website, Topic 7, Lectures 7a to 7d

Problem #1: 1D Derivative Matrices

Write a MATLAB function to construct derivative matrices for first and second derivatives on a one-dimensional grid along the x-axis. Incorporate the following equations for the finite-difference approximations that use Dirichlet boundary conditions.

$$\frac{df_i}{dx} \cong \begin{cases} \dfrac{f_{i+1}}{2\Delta x} & i=1 \\ \dfrac{f_{i+1}-f_{i-1}}{2\Delta x} & 2 \le i \le N-1 \\ -\dfrac{f_{i-1}}{2\Delta x} & i=N \end{cases} \qquad \frac{d^2 f_i}{dx^2} \cong \begin{cases} \dfrac{f_{i+1}-2f_i}{(\Delta x)^2} & i=1 \\ \dfrac{f_{i+1}-2f_i+f_{i-1}}{(\Delta x)^2} & 2 \le i \le N-1 \\ \dfrac{-2f_i+f_{i-1}}{(\Delta x)^2} & i=N \end{cases}$$

Use the following header exactly for your function. Be sure your function can construct any size matrix defined by `Nx`. Save your function with the filename `fdder1d.m`. For this problem only, provide your code up front in your homework, not in an appendix. To verify your function, duplicate the results provided in the benchmarking document.

```
function [DX,DX2] = fdder1d(Nx,dx)
% FDDER1D       One-Dimensional Finite-Difference Derivative Operators
%               Using Dirichlet Boundary Conditions
%
% [DX,DX2] = fdder1d(Nx,dx);
%
% Nx is the size the grid.
%
% dx is the grid resolution.
%
% DX is a square matrix of size Nx-by-Nx that calculates a first
% derivative.
%
% DX2 is a square matrix of size Nx-by-Nx that calculates a second
% derivative.
```

Problem #2: Benchmark First Derivative Matrix $[D_x]$

Write a MATLAB script that calculates the first derivative of the $\cos(\theta)$ function in the interval $0 \leq \theta \leq 2\pi$. Your program should store the function in a column vector and use the first-order derivative matrix $[D_x]$ from Problem #1 to perform the numerical differentiation. This is accomplished simply by premultiplying the column vector by the derivative matrix. Your program should not modify the derivative matrix produced by `fdder1d()` in any way. Use 100 points for your one-dimensional grid. Plot all points of your function along with the first derivative on the same plot. Label the curves and be sure the figure is clean, clear, and professional. Explain any anomalies that may be present in your results. Include a signed graphics checklist at the end of your assignment.

Problem #3: Benchmark Second Derivative Matrix $[D_x^2]$

Write a MATLAB script that calculates the second derivative of the $\cos(\theta)$ function in the interval $0 \leq \theta \leq 2\pi$. Your program should store the function in a column vector and use the second-order derivative matrix $[D_x^2]$ from Problem #1 to perform the numerical differentiation. This is accomplished simply by premultiplying the column vector by the derivative matrix. Your program should not modify the derivative matrix produced by `fdder1d()` in any way. Use 100 points for your one-dimensional grid. Plot all points of your function along with the second derivative on the same plot. Label the curves and be sure the figure is clean, clear, and professional. Explain any anomalies that may be present in your results. Include a signed graphics checklist at the end of your assignment.