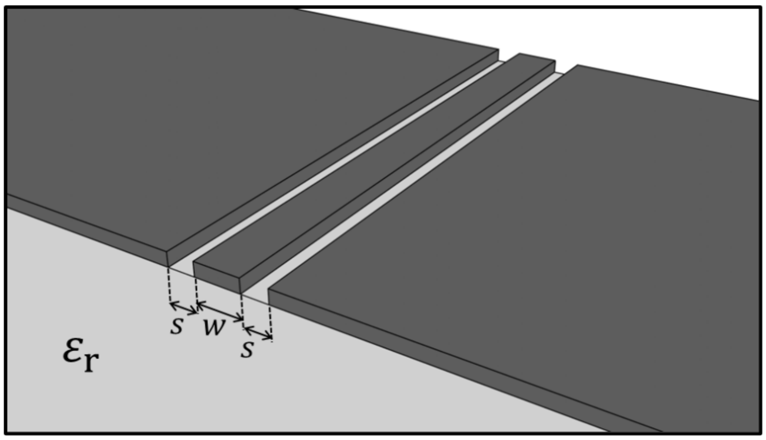
Problem Statement

A coplanar waveguide (CW) is a planar transmission line commonly used in printed circuit boards (PCBs). The consist of a signal line surrounded by two semi-infinite grounds. The parameters describing the CW are shown in the figure below and consist of the line width w, separation length s, and dielectric constant ε_r of the substrate. The superstrate is air, which has a dielectric constant of 1.0. The grounds, the superstrate, and the substrate are all considered to be semi-infinite.



Over the next two homework assignments, you will develop a program to analyze this transmission line using a two-dimensional finite-difference method and calculate the transmission line parameters including the distributed capacitance C, distributed inductance L and characteristic impedance Z_0.

In this assignment, Homework 10a, you will program the `tlder()` function that builds the derivative matrices. You will also build the four arrays that describe the coplanar transmission line. In the next assignment, Homework 10b, you will write the code in MATLAB that analyzes the coplanar transmission line using a two-dimensional finite-difference method.

Problem #1: Derivative Matrices by Hand

For each of the following cases, construct the four derivative matrices $\mathbf{D}_x^v$, $\mathbf{D}_y^v$, $\mathbf{D}_x^e$ and $\mathbf{D}_y^e$ by hand. The derivative matrices should use Dirichlet boundary conditions. You may type or write the matrices by hand.

a) `Nx = 2` and `Ny = 3`
b) `Nx = 4` and `Ny = 2`
c) `Nx = 2` and `Ny = 5`
d) `Nx = 1` and `Ny = 3`
e) `Nx = 2` and `Ny = 4`

For each case (a)-(e) above:

1. Make a sketch of the grid in the style of the grid depicted in Problem #2 of this assignment.
2. Write the matrix equation $\mathbf{Dv} = \mathbf{v}'$, where $\mathbf{D}$ is a large blank matrix to be determined, $\mathbf{v}$ is a column vector of the function values and $\mathbf{v}'$ is a column vector containing the correct finite-difference expressions.

$$\begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \frac{1}{\Delta x}\begin{bmatrix} v_2 - v_1 \\ v_3 - v_2 \\ v_4 - v_3 \\ 0 - v_4 \end{bmatrix}$$

3. Fill in the large matrix $\mathbf{D}$ so that the matrix multiplication $\mathbf{Dv}$ will give $\mathbf{v}'$.

$$\frac{1}{\Delta x}\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \frac{1}{\Delta x}\begin{bmatrix} v_2 - v_1 \\ v_3 - v_2 \\ v_4 - v_3 \\ 0 - v_4 \end{bmatrix}$$

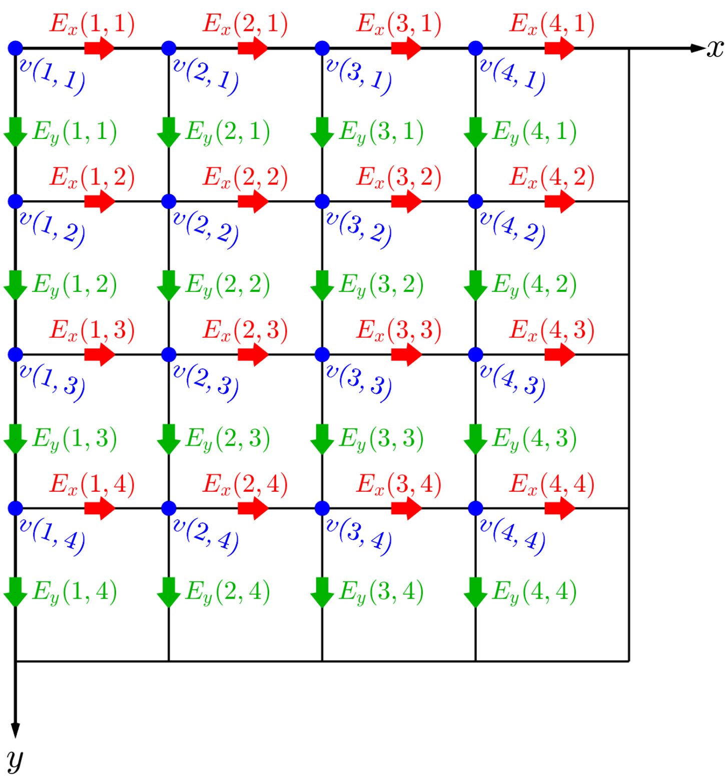
4. Extract the derivative matrix from this equation.

$$\mathbf{D} = \frac{1}{\Delta x}\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

5. Perform Steps 2-4 for $\mathbf{D}_x^v$, $\mathbf{D}_y^v$, $\mathbf{D}_x^e$ and $\mathbf{D}_y^e$.

Problem #2: `tlder()`

Write a MATLAB function to construct the four derivative matrices for first-order derivatives on a two-dimensional staggered grid of <u>any</u> size. These are $\mathbf{D}_x^v$, $\mathbf{D}_y^v$, $\mathbf{D}_x^e$ and $\mathbf{D}_y^e$. Include the MATLAB header <u>exactly</u> as provided in the Appendix for this function. Be sure your function can handle any size grid and can handle the special cases of `Nx=1` and/or `Ny=1`. Use only sparse matrices throughout your entire function. <u>Never</u> at any point in your code should you generate full matrices. Use Dirichlet boundary conditions at all edges of the grid. The 2D grid along with the staggered functions $V(x,y)$ and $\vec{E}(x,y)$ are shown in the figure below. Only the $\vec{E}$ field components are shown, but D_x exists at the same points as E_x, and D_y exists at the same points as E_y.

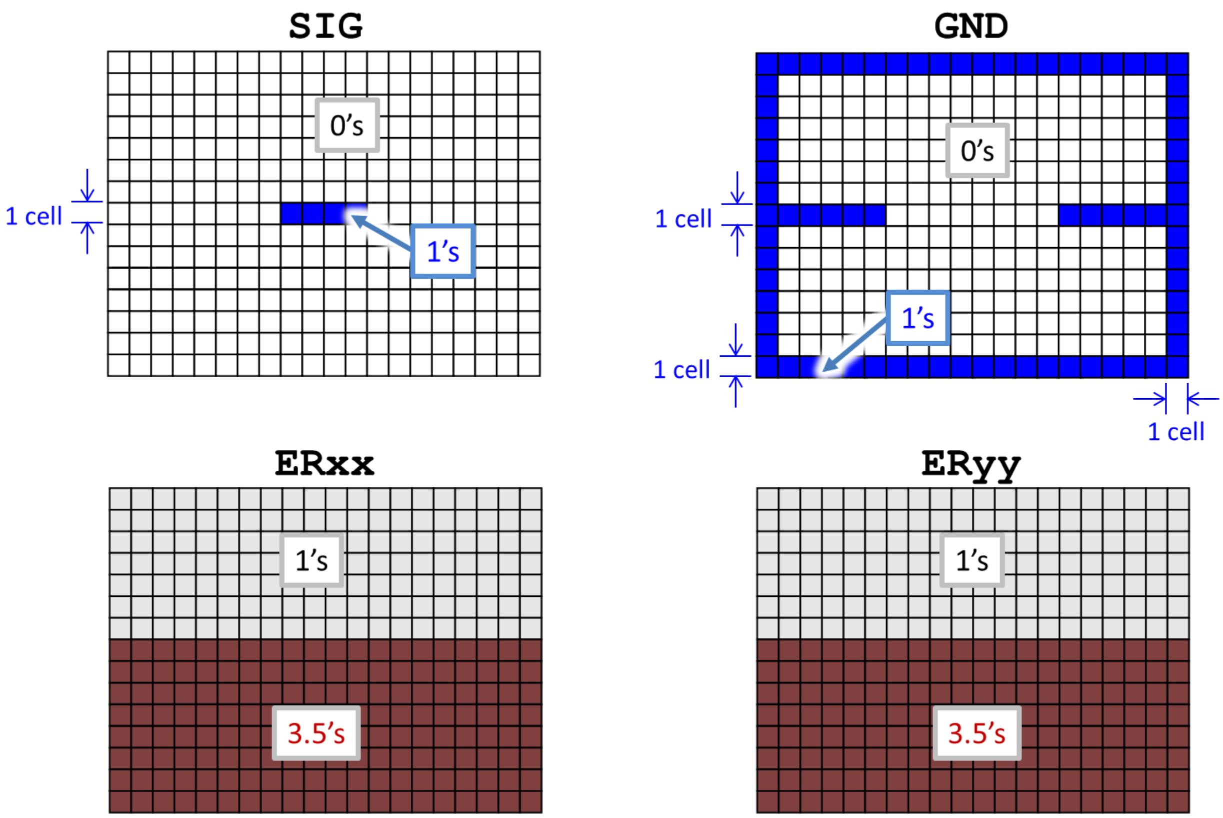


Download and run the `test_tlder()` function from the course website and provide the verification code it reports in your homework. Install `test_tlder()` in your MATLAB working directory and then type "`>> test_tlder`" at the commend prompt.

Save your function with the name `tlder.m`. Consider duplicating the results in the benchmarking document in order to test your function. There is also "`stlder.p`" available on the course website. It works exactly like `stlder()` except it will only generate derivative matrices for small grids. You may use this for further troubleshooting.

Problem #3: MATLAB Program to Build a Microstrip

Write a program in MATLAB to build four arrays (`SIG`, `GND`, `ERxx`, and `ERyy`) that describe the coplanar transmission line. The conductors should be only one grid cell thick as shown below. Do not yet include any code that would begin to analyze this problem. Start your program with the exact header provided in the appendix of this homework. Your arrays should look something like the following, but with many more points. Use $w = 2.5$ mm, $s = 1.0$ mm, and $\varepsilon_r = 3.5$. Visualize your arrays to ensure correctness.



Hints:

1. Your code to build these arrays will not use `tlder()`. This function will not be used again until Homework #10b.
2. Your graphics should be clear and convey the numerical values inserted into your arrays.
3. You are not simply drawing a picture of a microstrip transmission line. You are building values into arrays that represent your microstrip. These arrays will be used in Homework #10b to analyze the transmission line using a two-dimensional finite-difference method.
4. Think about how to accurately represent the substrate thickness. Remember, the grounds are not part of the substrate and overwrite any dielectric.

Appendix A: Header for tlder()

```
function [DVX,DVY,DEX,DEY] = tlder(NS,RES)
% TLDER      Construct Matrix Derivative Operators on a 2D Grid
%
% [DVX,DVY,DEX,DEY] = tlder(NS,RES);
%
% INPUT ARGUMENTS
% ================
% NS         [Nx Ny] grid size
% RES        [dx dy] grid resolution of the 1X grid
%
% Note:  For normalized grid coordinates, you may need to use
%        [DVX,DVY,DEX,DEY] = tlder(NS,k0*RES);
%
% OUTPUT ARGUMENTS
% ================
% DVX, DVY      Matrices that calculate derivatives for V
% DEX, DEY      Matrices that calculate derivatives for E and D
%
% EXAMPLE CODE TO CALL TLDER
% ============================
% The following program generates the four derivative matrices for a
% grid that is 4 cells wide and 5 cells tall and where the cell size
% is 0.1 wide and 0.2 tall.
%
% % DEFINE GRID
% Nx = 4;
% Ny = 5;
% dx = 0.1;
% dy = 0.2;
%
% % BUILD DERIVATIVE MATRICES
% NS  = [Nx Ny];
% RES = [dx dy];
% [DVX,DVY,DEX,DEY] = tlder(NS,RES);
```

Appendix B: Header for Problem #3

```
% HW10_Prob3.m
%
% This MATLAB program models a coplanar transmission line in the
% electrostatic approximation using the finite-difference method.
%
% Instructor:
% Prof. Raymond C. Rumpf
%
% Course Information:
% Computational Methods

% INITIALIZE MATLAB
close all;
clc;
clear all;

% UNITS
millimeters = 1;
meters      = 1e3 * millimeters;

% CONSTANTS
u0 = 1.2566370614e-6 * 1/meters;
e0 = 8.8541878176e-12 * 1/meters;
c0 = 299792458 * meters;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% STEP 1 -- DASHBOARD
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% TRANSMISSION LINE PARAMETERS
er = 3.5;                   %dielectric constant of substrate
w  = 2.5 * millimeters;     %thickness of substrate
s  = 1 * millimeters;       %width of signal line

% GRID PARAMETERS
Sx = 20 * millimeters;      %physical size of grid along x
Sy = 20 * millimeters;      %physical size of grid along y
Nx = 100;                   %number of grid points along x
Ny = 100;                   %number of grid points along y
```