Electromagnetics:

**Electromagnetic Field Theory**

# Introduction to the Rectangular Metal Waveguide

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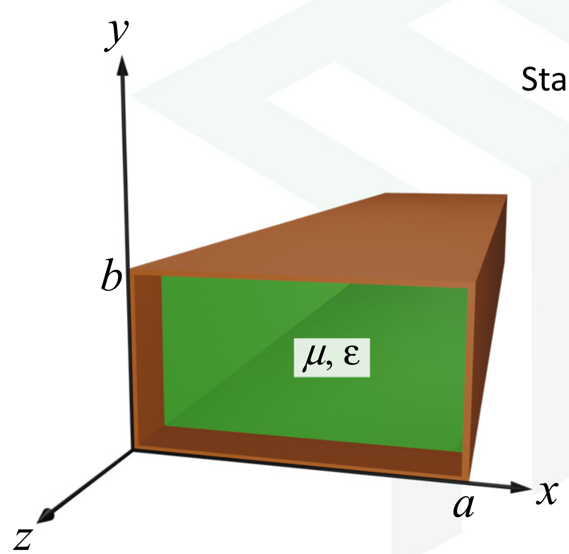
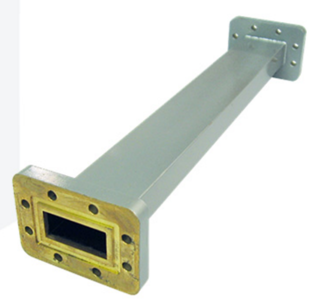
## Geometry of Rectangular Waveguide




Standard size convention: $a \geq b$





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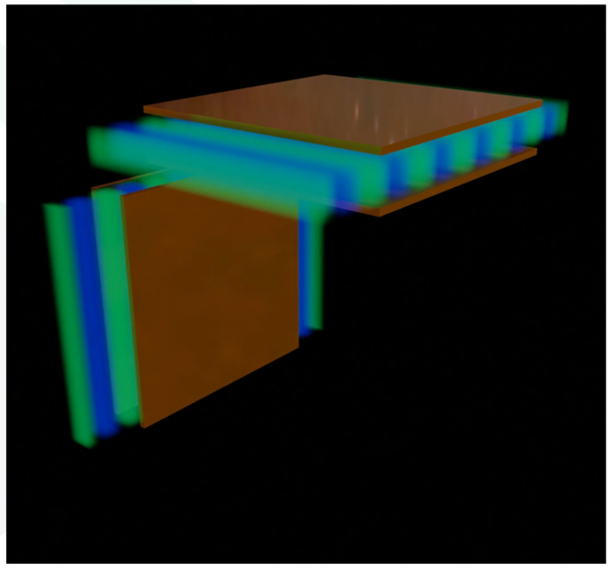
# Analysis of Rectangular Waveguide

A rectangular metal waveguide is analyzed, and operates, analogous to each axis being a parallel plate waveguide.

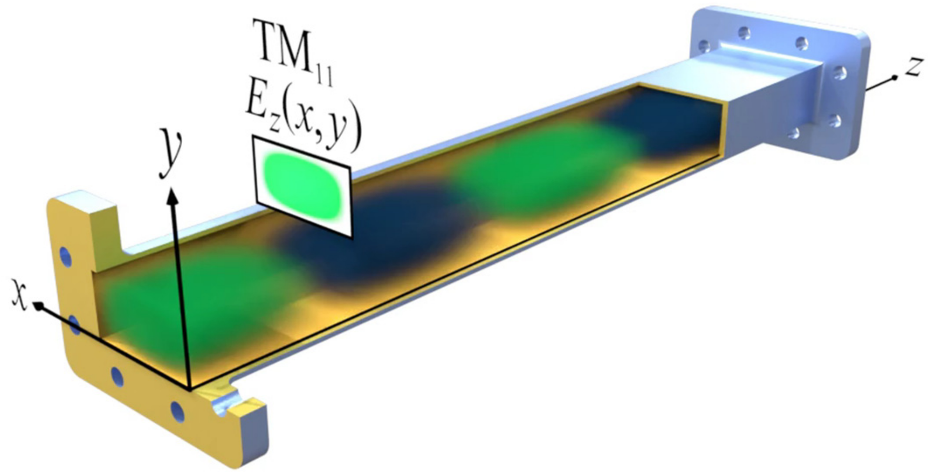




# Notes on the Rectangular Waveguide

- Most classic waveguide example
- Some of the first waveguides used for microwaves
- Not a transmission line because it has only one conductor
- Does not support a TEM mode
- Exhibits a low-frequency cutoff below which no waves will propagate

# Visualization of $E_z$ for the $\text{TM}_{11}$ Mode

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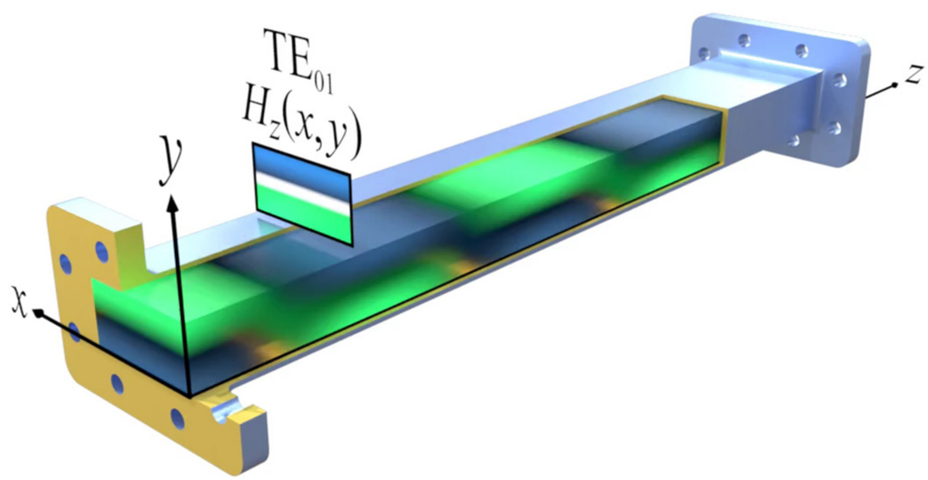
# Visualization of $H_z$ for the $\text{TE}_{01}$ Mode

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# Summary of Rectangular Metal Waveguide

| Parameter | $\text{TE}_{mn}$<br>$m = n = 0$ not allowed | $\text{TM}_{mn}$<br>$m \neq 0$ and $n \neq 0$ |
|---|---|---|
| $k$ | $\omega/\sqrt{\mu\varepsilon}$ | |
| $k_\text{c}$ | $\sqrt{(m\pi/a)^2 + (m\pi/b)^2}$ | |
| $\beta$ | $\sqrt{k^2 - k_\text{c}^2}$ | |
| $\lambda_\text{c}$ | $2\pi/k_\text{c} = 2d/n$ | |
| $\lambda_\text{g}$ | $2\pi/\beta_{mn}$ | |
| $v_\text{phase}$ | $\omega/\beta_{mn}$ | |
| $\alpha_\text{d}$ | $k^2 \tan\delta/2\beta_{mn}$ | |
| $E_x$ | $\frac{j\omega\mu n\pi}{k_\text{c}^2 b} A_{mn} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_{mn}z}$ | $-\frac{j\beta_{mn} m\pi}{k_\text{c}^2 a} B_{mn} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_{mn}z}$ |
| $E_y$ | $-\frac{j\omega\mu m\pi}{k_\text{c}^2 a} A_{mn} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_{mn}z}$ | $-\frac{j\beta_{mn} n\pi}{k_\text{c}^2 b} B_{mn} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_{mn}z}$ |
| $E_z$ | 0 | $B_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_{mn}z}$ |
| $H_x$ | $-\frac{j\beta_{mn} m\pi}{k_\text{c}^2 a} A_{mn} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_{mn}z}$ | $\frac{j\omega\varepsilon n\pi}{k_\text{c}^2 b} B_{mn} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_{mn}z}$ |
| $H_y$ | $\frac{j\beta_{mn} n\pi}{k_\text{c}^2 b} A_{mn} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_{mn}z}$ | $-\frac{j\omega\varepsilon m\pi}{k_\text{c}^2 a} B_{mn} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta_{mn}z}$ |
| $H_z$ | $A_{mn} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_{mn}z}$ | 0 |
| $Z_0$ | $k\eta/\beta_{mn}$ | $\beta_{mn}\eta/k$ |

$$\omega\mu = k_0 \eta_0 \mu_\text{r}$$

$$\omega\varepsilon = \frac{k_0}{\eta_0} \varepsilon_\text{r}$$

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# The Fundamental Mode

The fundamental mode is the mode which has the lowest cutoff frequency. It will be shown in later lectures that this is either the $\text{TE}_{10}$ or the $\text{TM}_{11}$ mode.

$$f_{\text{c,TE}} = \frac{1}{2\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{1}{a}\right)^2}$$

$$f_{\text{c,TM}} = \frac{1}{2\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$$

It can be observed that the $\text{TE}_{10}$ mode will always have the lowest cutoff frequency.

It is concluded that the $\text{TE}_{10}$ mode is the fundamental mode of the waveguide.

This is also called the dominant mode. When multiple modes are excited, usually most of the power ends up in the fundamental mode.

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# Key Points

- The rectangular waveguide is not a transmission line because it has less than two conductors.
- When filled with a homogeneous dielectric, the rectangular waveguide supports TE and TM modes, but not TEM.
- The cutoff frequencies for $\text{TE}_{mn}$ and $\text{TM}_{mn}$ modes are the same.
- The $\text{TE}_{00}$ mode does not exist.
- For $\text{TM}_{mn}$ modes, $m \neq 0$ and $n \neq 0$ .
- The $\text{TE}_{10}$ is the dominant mode because the $\text{TM}_{10}$ mode does not exist.
- Phase velocity of the modes exceeds the vacuum speed of light.

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