EMPossible

Electromagnetics:
Electromagnetic Field Theory

Slab Waveguide Analysis Setup

1

Solution Categories

Hybrid Mode Analysis

$E_{0,z} \neq 0$ and $H_{0,z} \neq 0$

Can analyze any waveguide.
Usually very complicated.

- No simplifications to the analysis are possible.
- Solutions contain all six field components

TEM Analysis

$E_{0,z} = H_{0,z} = 0$

- Must be a transmission line,
- and must have a homogeneous dielectric

- **Transverse electromagnetic (TEM) mode**
- Both electric and magnetic fields are transverse to the direction of propagation.

TE & TM Analysis

$E_{0,z} = 0$ or $H_{0,z} = 0$, but not both

- Homogeneous dielectric,
- or has a dielectric uniform in one direction

- **Transverse electric (TE) mode**
- $E_{0,z} = 0$ so the electric field is completely transverse to the direction of propagation.
- **Transverse magnetic (TM) mode**
- $H_{0,z} = 0$ so the magnetic field is completely transverse to the direction of propagation.

Slab Waveguide Analysis

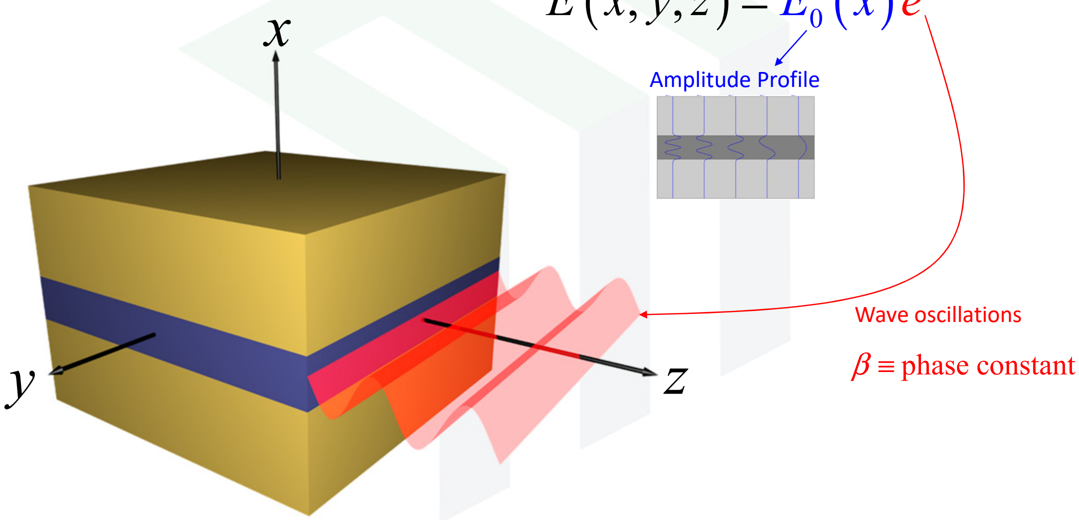
- Material properties uniform and infinite along one direction

- Maxwell's equations simplify and decouple into two independent sets of modes.

2

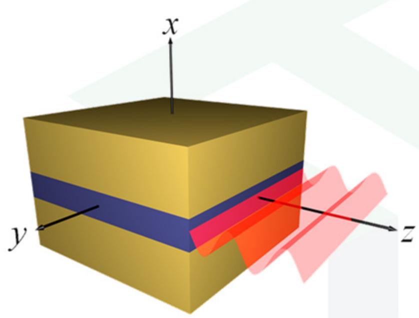
Geometry and Solution

$$\vec{E}(x,y,z) = \vec{E}_0(x)\, e^{-j\beta z}$$



EMPossible

Origin of TE and TM Modes (1 of 2)



Given this geometry

$$\frac{\partial}{\partial y} = 0$$

$$\cancel{\frac{\partial E_{0,z}}{\partial y}} + j\beta E_{0,y} = -j\omega\mu H_{0,x} \qquad \text{Eq. (1a)}$$

$$-j\beta E_{0,x} - \frac{\partial E_{0,z}}{\partial x} = -j\omega\mu H_{0,y} \qquad \text{Eq. (1b)}$$

$$\frac{\partial E_{0,y}}{\partial x} - \cancel{\frac{\partial E_{0,x}}{\partial y}} = -j\omega\mu H_{0,z} \qquad \text{Eq. (1c)}$$

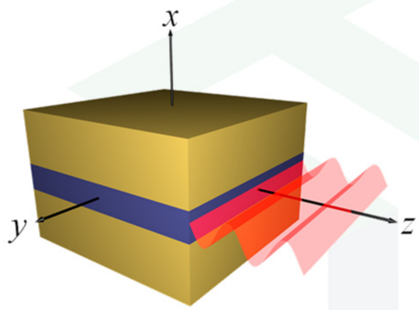
$$\cancel{\frac{\partial H_{0,z}}{\partial y}} + j\beta H_{0,y} = j\omega\varepsilon E_{0,x} \qquad \text{Eq. (1d)}$$

$$-j\beta H_{0,x} - \frac{\partial H_{0,z}}{\partial x} = j\omega\varepsilon E_{0,y} \qquad \text{Eq. (1e)}$$

$$\frac{\partial H_{0,y}}{\partial x} - \cancel{\frac{\partial H_{0,x}}{\partial y}} = j\omega\varepsilon E_{0,z} \qquad \text{Eq. (1f)}$$

EMPossible

Origin of TE and TM Modes (1 of 2)



Given this geometry

$$\frac{\partial}{\partial y} = 0$$

$$j\beta E_{0,y} = -j\omega\mu H_{0,x} \quad \text{Eq. (1a)}$$

$$-j\beta E_{0,x} - \frac{\partial E_{0,z}}{\partial x} = -j\omega\mu H_{0,y} \quad \text{Eq. (1b)}$$

$$\frac{\partial E_{0,y}}{\partial x} = -j\omega\mu H_{0,z} \quad \text{Eq. (1c)}$$

$$j\beta H_{0,y} = j\omega\varepsilon E_{0,x} \quad \text{Eq. (1d)}$$

$$-j\beta H_{0,x} - \frac{\partial H_{0,z}}{\partial x} = j\omega\varepsilon E_{0,y} \quad \text{Eq. (1e)}$$

$$\frac{\partial H_{0,y}}{\partial x} = j\omega\varepsilon E_{0,z} \quad \text{Eq. (1f)}$$

EMPossible Slide 5

5

Origin of TE and TM Modes (2 of 2)

Maxwell's equations have decoupled into two independent sets of equations.

TE Mode (i.e. $E_{0,z} = 0$)

TM Mode (i.e. $H_{0,z} = 0$)

$$j\beta E_{0,y} = -j\omega\mu H_{0,x} \quad \text{Eq. (2a)}$$

$$-j\beta E_{0,x} - \frac{\partial E_{0,z}}{\partial x} = -j\omega\mu H_{0,y} \quad \text{Eq. (2b)}$$

$$\frac{\partial E_{0,y}}{\partial x} = -j\omega\mu H_{0,z} \quad \text{Eq. (2c)}$$

$$j\beta H_{0,y} = j\omega\varepsilon E_{0,x} \quad \text{Eq. (2d)}$$

$$-j\beta H_{0,x} - \frac{\partial H_{0,z}}{\partial x} = j\omega\varepsilon E_{0,y} \quad \text{Eq. (2e)}$$

$$\frac{\partial H_{0,y}}{\partial x} = j\omega\varepsilon E_{0,z} \quad \text{Eq. (2f)}$$

EMPossible Slide 6

6

Origin of TE and TM Modes (2 of 2)

Maxwell's equations have decoupled into two independent sets of equations.

TE Mode (i.e. $E_{0,z} = 0$)

$$-j\beta H_{0,x} - \frac{\partial H_{0,z}}{\partial x} = j\omega\varepsilon E_{0,y} \qquad \text{Eq. (3a)}$$

$$j\beta E_{0,y} = -j\omega\mu H_{0,x} \qquad \text{Eq. (3b)}$$

$$\frac{\partial E_{0,y}}{\partial x} = -j\omega\mu H_{0,z} \qquad \text{Eq. (3c)}$$

TM Mode (i.e. $H_{0,z} = 0$)

$$-j\beta E_{0,x} - \frac{\partial E_{0,z}}{\partial x} = -j\omega\mu H_{0,y} \qquad \text{Eq. (3d)}$$

$$j\beta H_{0,y} = j\omega\varepsilon E_{0,x} \qquad \text{Eq. (3e)}$$

$$\frac{\partial H_{0,y}}{\partial x} = j\omega\varepsilon E_{0,z} \qquad \text{Eq. (3f)}$$

EMPossible

Slide 7

7

TE Wave Equation

Solve Eq. (3b) for $H_{0,x}$ and solve Eq. (3c) for $H_{0,z}$.

TE Mode (i.e. $E_{0,z} = 0$)

$$-j\beta H_{0,x} - \frac{\partial H_{0,z}}{\partial x} = j\omega\varepsilon E_{0,y} \qquad \text{Eq. (3a)}$$

$$j\beta E_{0,y} = -j\omega\mu H_{0,x} \qquad \text{Eq. (3b)} \quad \longrightarrow \quad H_{0,x} = -\frac{\beta}{\omega\mu} E_{0,y} \qquad \text{Eq. (4a)}$$

$$\frac{\partial E_{0,y}}{\partial x} = -j\omega\mu H_{0,z} \qquad \text{Eq. (3c)} \quad \longrightarrow \quad H_{0,z} = -\frac{1}{j\omega\mu}\frac{\partial E_{0,y}}{\partial x} \qquad \text{Eq. (4b)}$$

Substitute Eq. (4a) and (4b) into Eq. (3a) to obtain an equation that only contains $E_{0,y}$.

$$-j\beta H_{0,x} - \frac{\partial H_{0,z}}{\partial x} = j\omega\varepsilon E_{0,y} \quad \Rightarrow \quad -j\beta\left(-\frac{\beta}{\omega\mu}E_{0,y}\right) - \frac{\partial}{\partial x}\left(-\frac{1}{j\omega\mu}\frac{\partial E_{0,y}}{\partial x}\right) = j\omega\varepsilon E_{0,y} \quad \Rightarrow \quad \boxed{\mu\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial E_{0,y}}{\partial x}\right) + k_c^2 E_{0,y} = 0}$$

EMPossible

Slide 8

8

TM Wave Equation

Solve Eq. (3e) for $E_{0,x}$ and solve Eq. (3f) for $E_{0,z}$.

TM Mode (i.e. $H_{0,z} = 0$)

$$-j\beta E_{0,x} - \frac{\partial E_{0,z}}{\partial x} = -j\omega\mu H_{0,y} \qquad \text{Eq. (3d)}$$

$$j\beta H_{0,y} = j\omega\varepsilon E_{0,x} \qquad \text{Eq. (3e)} \longrightarrow E_{0,x} = \frac{\beta}{\omega\varepsilon} H_{0,y} \qquad \text{Eq. (5a)}$$

$$\frac{\partial H_{0,y}}{\partial x} = j\omega\varepsilon E_{0,z} \qquad \text{Eq. (3f)} \longrightarrow E_{0,z} = \frac{1}{j\omega\varepsilon}\frac{\partial H_{0,y}}{\partial x} \qquad \text{Eq. (5b)}$$

Substitute Eq. (5a) and (5b) into Eq. (3d) to obtain an equation that only contains $H_{0,y}$.

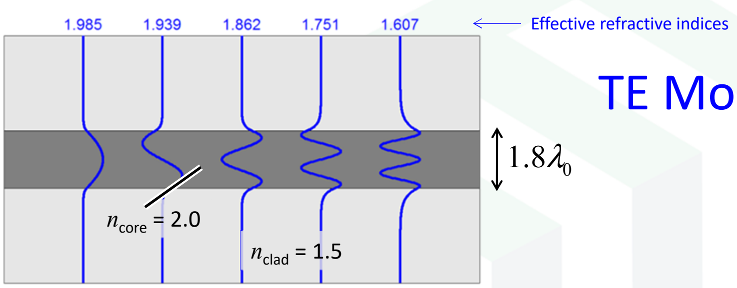
$$-j\beta E_{0,x} - \frac{\partial E_{0,z}}{\partial x} = -j\omega\mu H_{0,y} \;\Rightarrow\; -j\beta\left(\frac{\beta}{\omega\varepsilon} H_{0,y}\right) - \frac{\partial}{\partial x}\left(\frac{1}{j\omega\varepsilon}\frac{\partial H_{0,y}}{\partial x}\right) = -j\omega\mu H_{0,y} \;\Rightarrow\; \boxed{\varepsilon\frac{\partial}{\partial x}\left(\frac{1}{\varepsilon}\frac{\partial H_{0,y}}{\partial x}\right) + k_c^2 H_{0,y} = 0}$$

EMPossible

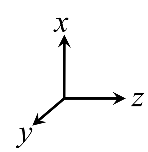
Slide 9

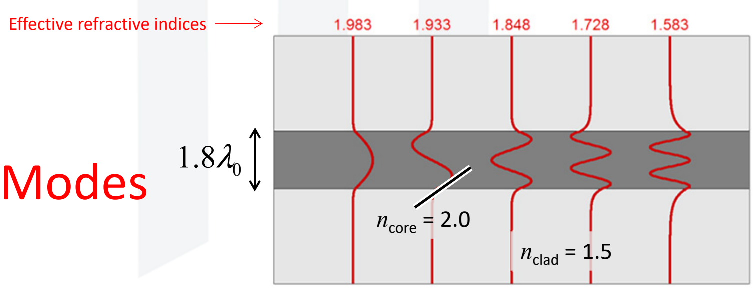
9

Typical Modes in a Slab Waveguide



TE Modes





TM Modes

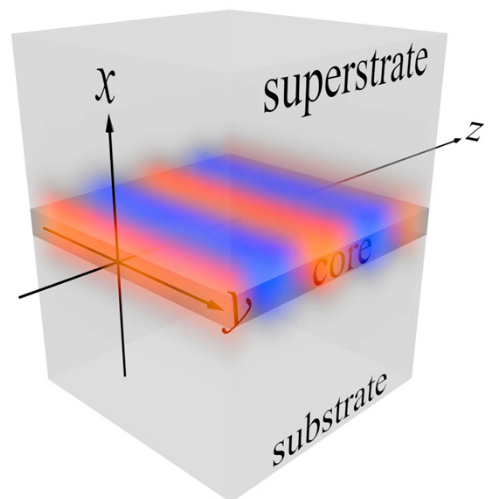
EMPossible

Slide 10

10

Remarks About Slab Waveguide Analysis

- Waves are confined in only one transverse direction.
- Waves are free to spread out in the uniform transverse direction
- Propagation within the slab can be restricted to the z direction without loss of generality.



EMPossible