Electromagnetics:
Electromagnetic Field Theory

TE Analysis of the Rectangular Metal Waveguide

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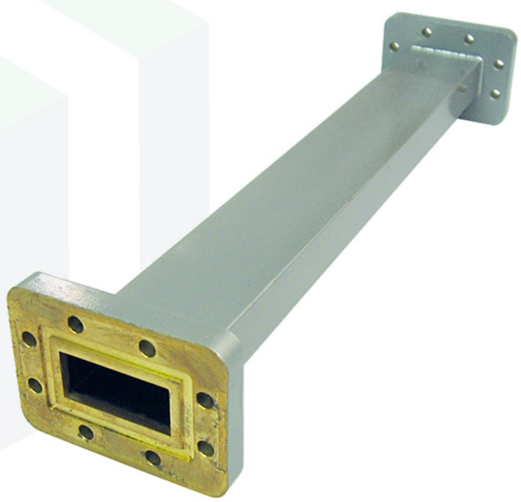
Lecture Outline

- TE Analysis
- Analysis of the TE Solution
- Visualization of Modes
- Conclusions



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TE Analysis

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Recall TE Analysis

The governing equation for TE analysis is

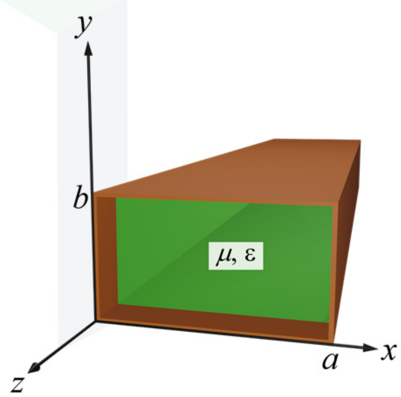
$$\frac{\partial^2 H_{0,z}}{\partial x^2} + \frac{\partial^2 H_{0,z}}{\partial y^2} - k_c^2 H_{0,z} = 0 \qquad k_c^2 = k^2 - \beta^2$$

After a solution is obtained, the remaining field components are calculated according to

$$H_{0,x} = -\frac{j\beta}{k_c^2}\frac{\partial H_{0,z}}{\partial x} \qquad E_{0,x} = -\frac{j\omega\mu}{k_c^2}\frac{\partial H_{0,z}}{\partial y}$$

$$H_{0,y} = -\frac{j\beta}{k_c^2}\frac{\partial H_{0,z}}{\partial y} \qquad E_{0,y} = \frac{j\omega\mu}{k_c^2}\frac{\partial H_{0,z}}{\partial x}$$

$$E_{0,z} = 0$$



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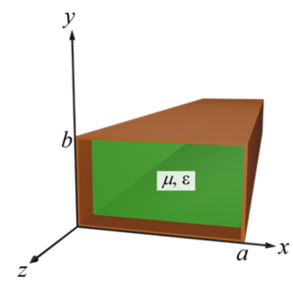
General Form of the Solution

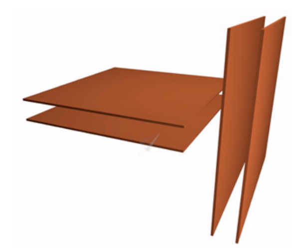
From the geometry of the waveguide, the general form of the solution can be immediately written as



$$H_z(x,y,z) = H_{0,z}(x,y)e^{-j\beta z}$$

Viewing the rectangular waveguide as the combination of two parallel plate waveguides, apply separation of variables to write $H_{0,z}(x,y)$ as the product of two functions.

$$H_{0,z}(x,y) = X(x)Y(y)$$



Separation of Variables (1 of 3)

The solution is written as the product of two 1D functions, $X(x)$ and $Y(y)$. Substitute this solution back into the differential equation.

$H_{0,z}(x,y) = X(x)Y(y)$

$$\frac{\partial^2 H_{0,z}}{\partial x^2} + \frac{\partial^2 H_{0,z}}{\partial y^2} - k_c^2 H_{0,z} = 0$$

$$\Downarrow$$

$$\frac{\partial^2 XY}{\partial x^2} + \frac{\partial^2 XY}{\partial y^2} - k_c^2 XY = 0$$

To be compact, drop the (x) and (y) notation.

$$\frac{\partial^2 X}{\partial x^2} Y + X\frac{\partial^2 Y}{\partial y^2} - k_c^2 XY = 0$$

Move $Y(y)$ out of the $\partial^2/\partial x^2$ operation and $X(x)$ out of the $\partial^2/\partial y^2$ operation.

$$\frac{1}{X}\frac{d^2 X}{dx^2} + \frac{1}{Y}\frac{d^2 Y}{dy^2} - k_c^2 = 0$$

Divide both sides by XY. The derivatives become ordinary because $X(x)$ and $Y(y)$ have only one independent variable each.

Separation of Variables (2 of 3)

First, attention is focused on the x-dependence in the differential equation.

$$\frac{1}{X}\frac{d^2X}{dx^2}+\underbrace{\frac{1}{Y}\frac{d^2Y}{dy^2}-k_c^2}_{-k_x^2}=0$$

This definition of k_x lets the differential equation be written as a wave equation.

$$\frac{d^2X}{dx^2}-k_x^2X=0$$

Second, attention is focused on the y-dependence in the differential equation.

$$\underbrace{\frac{1}{X}\frac{d^2X}{dx^2}-k_c^2}_{-k_y^2}+\frac{1}{Y}\frac{d^2Y}{dy^2}=0$$

This definition of k_y lets the differential equation be written as a wave equation.

$$\frac{d^2Y}{dy^2}-k_y^2Y=0$$

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Separation of Variables (3 of 3)

If all of this is correct, then it should be possible to add the two new differential equations together to get the original differential equation.

$$\frac{d^2X}{dx^2}-k_x^2X=0$$

$$\frac{d^2Y}{dy^2}-k_y^2Y=0$$

$$\Rightarrow$$

$$\frac{1}{X}\frac{d^2X}{dx^2}-k_x^2=0$$

$$+\ \frac{1}{Y}\frac{d^2Y}{dy^2}-k_y^2=0$$

$$\frac{1}{X}\frac{d^2X}{dx^2}+\frac{1}{Y}\frac{d^2Y}{dy^2}-k_x^2-k_y^2=0$$

The original differential equation is obtained if

$$k_c^2=k_x^2+k_y^2$$

Original differential equation

$$\frac{1}{X}\frac{d^2X}{dx^2}+\frac{1}{Y}\frac{d^2Y}{dy^2}-k_c^2=0$$

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General Solution

There are now two differential equations to solve.

$$\frac{d^2X}{dx^2} - k_x^2 X = 0 \qquad \frac{d^2Y}{dy^2} - k_y^2 Y = 0$$

These are essentially the same differential equation so their solution has the same general form.

$$\frac{d^2X}{dx^2} - k_x^2 X = 0 \;\rightarrow\; X(x) = A\cos(k_x x) + B\sin(k_x x)$$ PP waveguide along x

$$\frac{d^2Y}{dy^2} - k_y^2 Y = 0 \;\rightarrow\; Y(y) = C\cos(k_y y) + D\sin(k_y y)$$ PP waveguide along y

The overall solution is the product of $X(x)$ and $Y(y)$.

$$H_{0,z}(x,y) = X(x)Y(y) = \left[A\cos(k_x x) + B\sin(k_x x)\right]\left[C\cos(k_y y) + D\sin(k_y y)\right]$$

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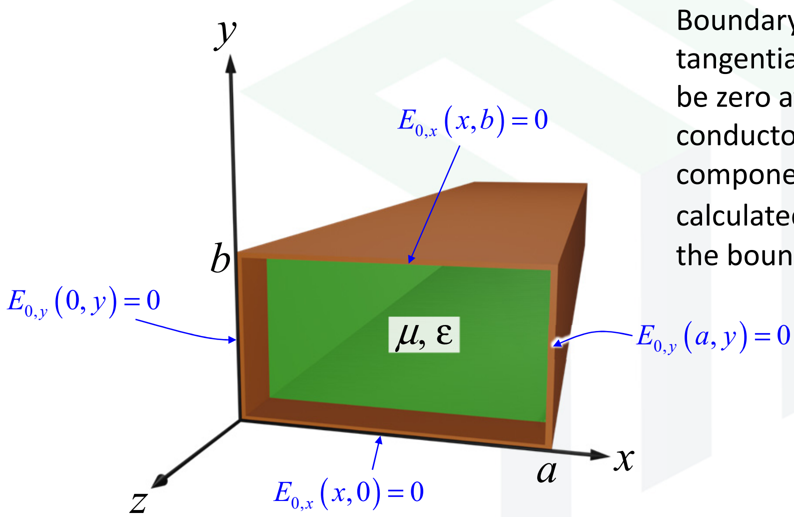
Electromagnetic Boundary Conditions



Boundary conditions require that the tangential component of the electric field be zero at the boundary with a perfect conductor. Unfortunately, electric field components $E_{0,x}$ and $E_{0,y}$ have to be calculated from the solution $H_{0,z}$ to apply the boundary conditions.

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$E_{0,x}$ and $E_{0,y}$

In order to apply the boundary conditions, the electric field components $E_{0,x}$ and $E_{0,y}$ must be derived from the expression for $H_{0,z}$.

$$E_{0,x}(x,y) = -\frac{j\omega\mu}{k_c^2}\frac{\partial H_{0,z}}{\partial y}$$
$$= -\frac{j\omega\mu}{k_c^2}\frac{\partial}{\partial y}\left[A\cos(k_x x)+B\sin(k_x x)\right]\left[C\cos(k_y y)+D\sin(k_y y)\right]$$
$$= -\frac{j\omega\mu}{k_c^2}k_y\left[A\cos(k_x x)+B\sin(k_x x)\right]\left[-C\sin(k_y y)+D\cos(k_y y)\right]$$

$$E_{0,y}(x,y) = \frac{j\omega\mu}{k_c^2}\frac{\partial H_{0,z}}{\partial x}$$
$$= \frac{j\omega\mu}{k_c^2}\frac{\partial}{\partial x}\left[A\cos(k_x x)+B\sin(k_x x)\right]\left[C\cos(k_y y)+D\sin(k_y y)\right]$$
$$= \frac{j\omega\mu}{k_c^2}k_x\left[-A\sin(k_x x)+B\cos(k_x x)\right]\left[C\cos(k_y y)+D\sin(k_y y)\right]$$

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Apply Boundary Conditions (1 of 2)

At the $x = 0$ boundary,

$$0 = E_{0,y}(0,y)$$
$$= \frac{j\omega\mu}{k_c^2}k_x\left[-\cancel{A\sin(0)}+B\cos(0)\right]\left[C\cos(k_y y)+D\sin(k_y y)\right]$$
$$= \frac{j\omega\mu}{k_c^2}k_x[B]\left[C\cos(k_y y)+D\sin(k_y y)\right] \longrightarrow B=0$$

At the $x = a$ boundary,

$$0 = E_{0,y}(a,y)$$
$$= \frac{j\omega\mu}{k_c^2}k_x\left[-A\sin(k_x a)\right]\left[C\cos(k_y y)+D\sin(k_y y)\right]$$

$A = 0$ leads to a trivial solution. It must be the $\sin(k_x a)$ term that enforces the BC.

$$0 = \sin(k_x a) \quad \rightarrow \quad k_x a = m\pi \quad m = 0,1,2,\ldots$$

Valid choices of m will be considered later.

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Apply Boundary Conditions (2 of 2)

At the $y = 0$ boundary,

$$\begin{aligned} 0 &= E_{0,x}(x,0) \\ &= -\frac{j\omega\mu}{k_c^2} k_y \left[A\cos(k_x x) + B\sin(k_x x) \right] \left[-\cancel{C\sin(0)} + D\cos(0) \right] \\ &= -\frac{j\omega\mu}{k_c^2} k_y \left[A\cos(k_x x) + B\sin(k_x x) \right] [D] \longrightarrow D = 0 \end{aligned}$$

At the $y = b$ boundary,

$$\begin{aligned} 0 &= E_{0,x}(x,b) \\ &= -\frac{j\omega\mu}{k_c^2} k_y \left[A\cos(k_x x) + B\sin(k_x x) \right] \left[-C\sin(k_y b) \right] \end{aligned}$$

$C = 0$ leads to a trivial solution. It must be the $\sin(k_y b)$ term that enforces the BC.

$$0 = \sin(k_y b) \quad \rightarrow \quad k_y b = n\pi \quad n = 0,1,2,\ldots$$

Valid choices for n will be considered later.

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Revised Solution for $H_{0,z}$

It was determined that $B = D = 0$ so the expression for $H_{0,z}$ becomes

$$H_{0,z}(x,y) = \boxed{AC}\cos(k_x x)\cos(k_y y)$$

The product AC is written as a single constant A_{mn}.

$$H_{0,z}(x,y) = A_{mn}\cos(k_x x)\cos(k_y y)$$

Also, recall the conditions for k_x and k_y.

$$k_x a = m\pi \quad \rightarrow \quad k_x = \frac{m\pi}{a} \qquad\qquad k_y b = n\pi \quad \rightarrow \quad k_y = \frac{n\pi}{b}$$

$$H_{0,z}(x,y) = A_{mn}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

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Entire Solution (1 of 2)

The final expression for $H_{0,z}$ is

$$H_{0,z}(x,y) = A_{mn}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) \qquad E_{0,z}(x,y) = 0$$

From this, the other field components are

$$E_{0,x}(x,y) = \frac{j\omega\mu n\pi}{k_c^2 b} A_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$E_{0,y}(x,y) = -\frac{j\omega\mu m\pi}{k_c^2 a} A_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

$$H_{0,x}(x,y) = \frac{j\beta_{mn} m\pi}{k_c^2 a} A_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

$$H_{0,y}(x,y) = \frac{j\beta_{mn} n\pi}{k_c^2 b} A_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

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Entire Solution (2 of 2)

The overall electric and magnetic fields at any position are

$$E_x(x,y,z) = \frac{j\omega\mu n\pi}{k_c^2 b} A_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta_{mn}z}$$

$$E_y(x,y,z) = -\frac{j\omega\mu m\pi}{k_c^2 a} A_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta_{mn}z}$$

$$E_z(x,y,z) = 0$$

$$H_x(x,y,z) = \frac{j\beta_{mn} m\pi}{k_c^2 a} A_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta_{mn}z}$$

$$H_y(x,y,z) = \frac{j\beta_{mn} n\pi}{k_c^2 b} A_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta_{mn}z}$$

$$H_z(x,y,z) = A_{mn}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta_{mn}z}$$

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Valid Values of m and n for TE Modes

$$E_{0,x}(x,y) = \frac{j\omega\mu n\pi}{k_c^2 b} A_{mn} \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$E_{0,y}(x,y) = -\frac{j\omega\mu m\pi}{k_c^2 a} A_{mn} \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

$$E_{0,z}(x,y) = 0$$

$$H_{0,x}(x,y) = \frac{j\beta_{mn} m\pi}{k_c^2 a} A_{mn} \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

$$H_{0,y}(x,y) = \frac{j\beta_{mn} n\pi}{k_c^2 b} A_{mn} \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$H_{0,z}(x,y) = A_{mn} \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

For TE modes, the only choice of m and n that leads to a trivial solution is when both are zero at the same time ($m = n = 0$).

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Analysis of the TE Solution

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Phase Constant, β

Recall the cutoff wave number

$$k_c^2 = k_x^2 + k_y^2$$

After analyzing the boundary conditions, this expression can be written as

$$k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

The phase constant β is therefore

$$k_c^2 = k^2 - \beta^2$$

$$\beta^2 = k^2 - k_c^2$$

$$\boxed{\beta_{mn} = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}}$$

Cutoff Frequency, f_c

Recall the expression for the phase constant

$$\beta_{mn} = \sqrt{k^2 - k_c^2}$$

The phase constant must be a real number for a guided mode. This requires

$$k > k_c$$

Any time $k < k_c$, the mode is cutoff and not supported by the waveguide. From this, the cutoff frequency f_c is derived to be

$$k > k_c$$

$$\omega\sqrt{\mu\varepsilon} > k_c$$

$$2\pi f_c \sqrt{\mu\varepsilon} = k_c$$

$$\boxed{f_{c,mn} = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}$$

Characteristic Impedance, Z_{TE}

The characteristic impedance Z_{TE} of the TE mode is

$$Z_{\mathrm{TE}} = \frac{E_x}{H_y} = \frac{\frac{j\omega\mu n\pi}{k_c^2 b} A_{mn} \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}}{\frac{j\beta_{mn} n\pi}{k_c^2 b} A_{mn} \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}} = \frac{\omega\mu}{\beta_{mn}} = \frac{k\eta}{\beta_{mn}}$$

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Cutoff for First-Order TE Mode (1 of 2)

The cutoff frequency for the TE_{mn} mode was found to be

$$f_{c,mn} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

What about the TE_{00} mode?

$$\mathrm{TE}_{00} \rightarrow m = n = 0$$

$$f_{c,00} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{0}{a}\right)^2 + \left(\frac{0}{b}\right)^2} = 0$$

The TE_{00} mode does not exist because it is impossible for a mode in this waveguide to have a cutoff frequency of 0 Hz.

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Cutoff for First-Order TE Mode (2 of 2)

What about the TE_{01} mode?

$$\mathrm{TE}_{01} \rightarrow m=0, n=1$$

$$f_{c,01} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{0}{a}\right)^2 + \left(\frac{1\cdot\pi}{b}\right)^2} = \frac{1}{2b\sqrt{\mu\varepsilon}}$$

What about the TE_{10} mode?

$$\mathrm{TE}_{10} \rightarrow m=1, n=0$$

$$f_{c,10} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{1\cdot\pi}{a}\right)^2 + \left(\frac{0}{b}\right)^2} = \frac{1}{2a\sqrt{\mu\varepsilon}}$$

Since $a > b$, it is concluded that the first-order TE mode is TE_{10} because it has the lowest cutoff frequency.

CAUTION: It cannot yet be said that the TE_{10} is the fundamental mode because the cutoff frequency of the TM modes has not yet been checked.

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Single Mode Operation (1 of 2)

Over what range of frequencies does a rectangular waveguide supports only a single TE mode?

$$f_{c1} < f < f_{c2}$$

<u>Low-Frequency Cutoff</u>

The lower frequency cutoff was just found.

$$f_{c1} = \frac{1}{2a\sqrt{\mu\varepsilon}}$$

<u>High-Frequency Cutoff</u>

The high-frequency cutoff is the frequency where the second-order TE mode is supported. This could be the TE_{01}, TE_{11} or TE_{20} mode. All must be considered.

$$\mathrm{TE}_{01}: \; f_{c,01} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{0}{a}\right)^2 + \left(\frac{1\cdot\pi}{b}\right)^2} = \frac{1}{2b\sqrt{\mu\varepsilon}}$$

$$\mathrm{TE}_{11}: \; f_{c,11} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2} = \frac{1}{2b\sqrt{\mu\varepsilon}}\sqrt{1+\left(\frac{b}{a}\right)^2}$$

$$\mathrm{TE}_{20}: \; f_{c,20} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{2\cdot\pi}{a}\right)^2 + \left(\frac{0}{b}\right)^2} = \frac{1}{2b\sqrt{\mu\varepsilon}}\frac{2b}{a}$$

TE_{11} will always have a higher cutoff frequency than TE_{01}.

The second-order mode depends on choice of a and b.

$$f_{c2} = \begin{cases} f_{c,01} & a \leq 2b \\ f_{c,20} & a > 2b \quad \text{(typical)} \end{cases}$$

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Single Mode Operation (2 of 2)

Bandwidth

Typical rectangular waveguides will have $a > 2b$, so

$$f_{c1} = \frac{1}{2a\sqrt{\mu\varepsilon}} \qquad f_{c2} = \frac{1}{a\sqrt{\mu\varepsilon}}$$

$$\Delta f = f_{c2} - f_{c1} = \frac{1}{a\sqrt{\mu\varepsilon}} - \frac{1}{2a\sqrt{\mu\varepsilon}} = \frac{1}{2a\sqrt{\mu\varepsilon}}$$

Fractional Bandwidth

Continuing the assumption that $a > 2b$, the fractional bandwidth can be calculated from f_{c1} and f_{c2} above as follows

$$\text{FBW} = \frac{\Delta f}{f_c} = \frac{f_{c2} - f_{c1}}{\left(f_{c2} + f_{c1}\right)/2} = 2\frac{f_{c2} - f_{c1}}{f_{c2} + f_{c1}} = 2\frac{\frac{1}{a\sqrt{\mu\varepsilon}} - \frac{1}{2a\sqrt{\mu\varepsilon}}}{\frac{1}{a\sqrt{\mu\varepsilon}} + \frac{1}{2a\sqrt{\mu\varepsilon}}} = \frac{2}{3} = 66.7\%$$

Example #1 – TE Mode Analysis (1 of 4)

Suppose there exists an air-filled rectangular waveguide with a = 3 cm and b = 2 cm.

What is the cutoff frequency of the waveguide?

$$f_{c1} = \frac{1}{2a\sqrt{\mu\varepsilon}} = \frac{c_0}{2a\sqrt{\mu_r\varepsilon_r}} = \frac{299792458 \text{ m/s}}{2(0.03 \text{ m})\sqrt{(1.0)(1.0)}} = \boxed{5.0 \text{ GHz}}$$

Over what range of frequencies is the waveguide single mode?

Observing that $a < 2b$, so the second-order mode is TE_{01}.

$$f_{c2} = \frac{1}{2b\sqrt{\mu\varepsilon}} = \frac{c_0}{2b\sqrt{\mu_r\varepsilon_r}} = \frac{299792458 \text{ m/s}}{2(0.02 \text{ m})\sqrt{(1.0)(1.0)}} = 7.5 \text{ GHz}$$

$$\boxed{5.0 \text{ GHz} < f < 7.5 \text{ GHz}}$$

Example #1 – TE Mode Analysis (2 of 4)

What is the fractional bandwidth of the waveguide?

$$\text{FBW} = (100\%)\frac{\Delta f}{f} = (200\%)\frac{f_2 - f_1}{f_2 + f_1} = (200\%)\frac{7.5 - 5.0}{7.5 + 5.0} = \boxed{40\%}$$

Plot the phase constant and effective refractive index for the first-order and second-order modes from DC up to 15 GHz.

The phase constant is calculated as:

$$\beta_{mn} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \quad \rightarrow \quad \begin{aligned} \text{TE}_{10}&: \ \beta_1 = \sqrt{\left(\frac{2\pi f}{c_0}\right)^2 - \left(\frac{\pi}{a}\right)^2} \\ \text{TE}_{01}&: \ \beta_2 = \sqrt{\left(\frac{2\pi f}{c_0}\right)^2 - \left(\frac{\pi}{b}\right)^2} \end{aligned}$$

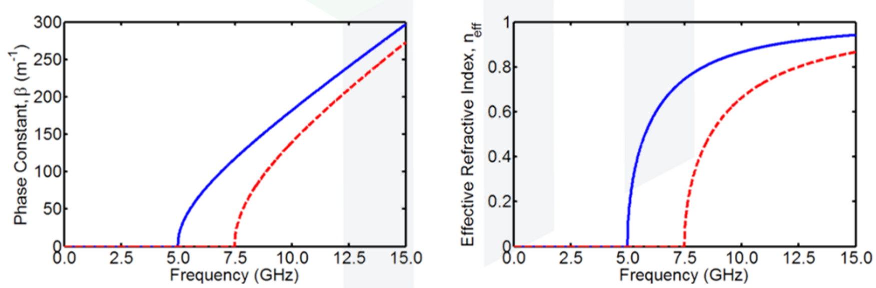
The effective refractive index is calculated as:

$$\beta = k_0 n_{\text{eff}} \quad \rightarrow \quad n_{\text{eff}} = \frac{\beta}{\frac{2\pi f}{c_0}} = \beta \frac{c_0}{2\pi f}$$

Example #1 – TE Mode Analysis (3 of 4)

Plot the phase constant and effective refractive index for the first-order and second-order modes from DC up to 15 GHz.

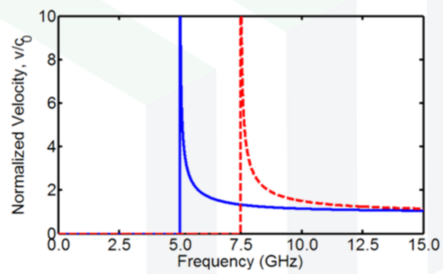
$$\beta_1 = \sqrt{\left(\frac{2\pi f}{c_0}\right)^2 - \left(\frac{\pi}{a}\right)^2} \qquad \beta_2 = \sqrt{\left(\frac{2\pi f}{c_0}\right)^2 - \left(\frac{\pi}{b}\right)^2} \qquad n_{\text{eff},1} = \beta_1 \frac{c_0}{2\pi f} \qquad n_{\text{eff},2} = \beta_2 \frac{c_0}{2\pi f}$$



Example #1 – TE Mode Analysis (4 of 4)

Plot the velocity of the modes as a function of frequency.

$$v_1 = \frac{c_0}{n_{\text{eff},1}} \qquad v_2 = \frac{c_0}{n_{\text{eff},2}}$$



Are the modes travelling faster than the speed of light?

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Visualization of the TE Modes

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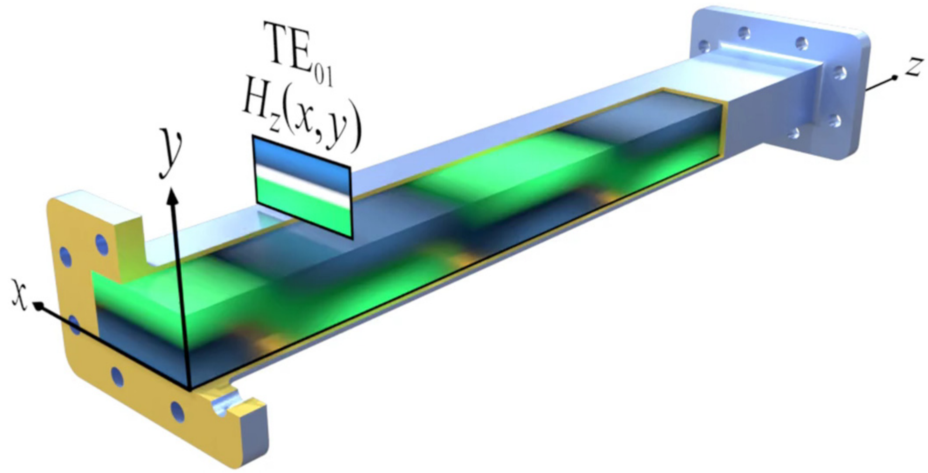
Visualization of H_z for the TE_{01} Mode



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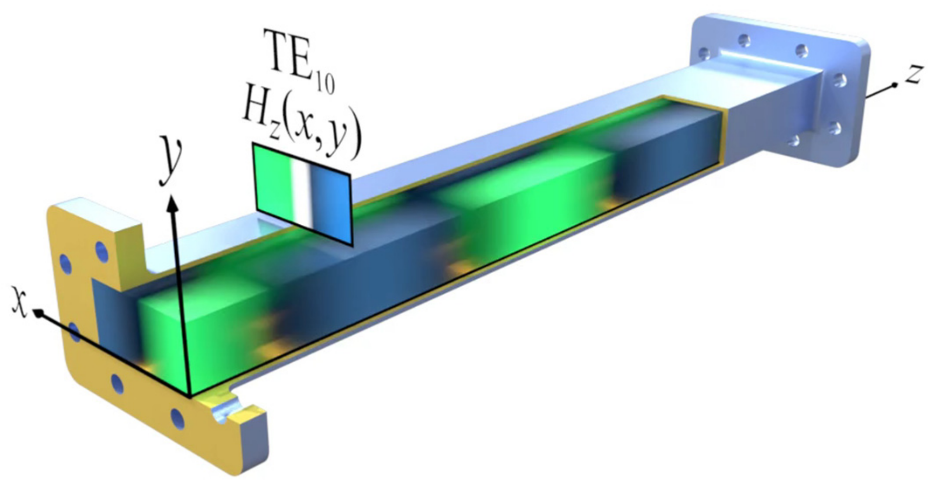
Visualization of H_z for the TE_{10} Mode



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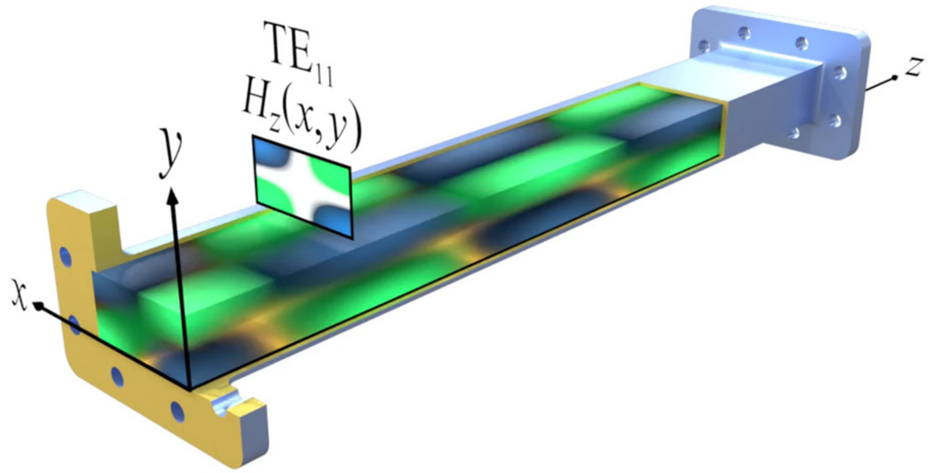
Visualization of H_z for the TE_{11} Mode



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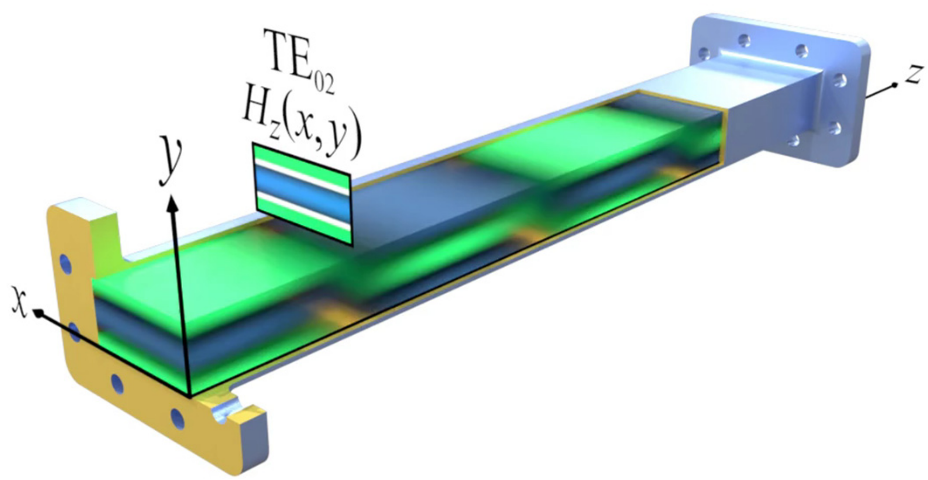
Visualization of H_z for the TE_{02} Mode



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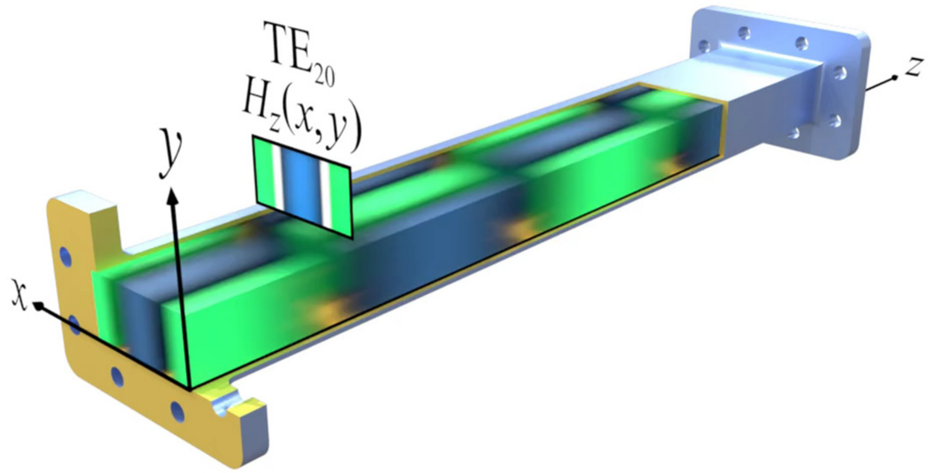
Visualization of H_z for the TE_{20} Mode



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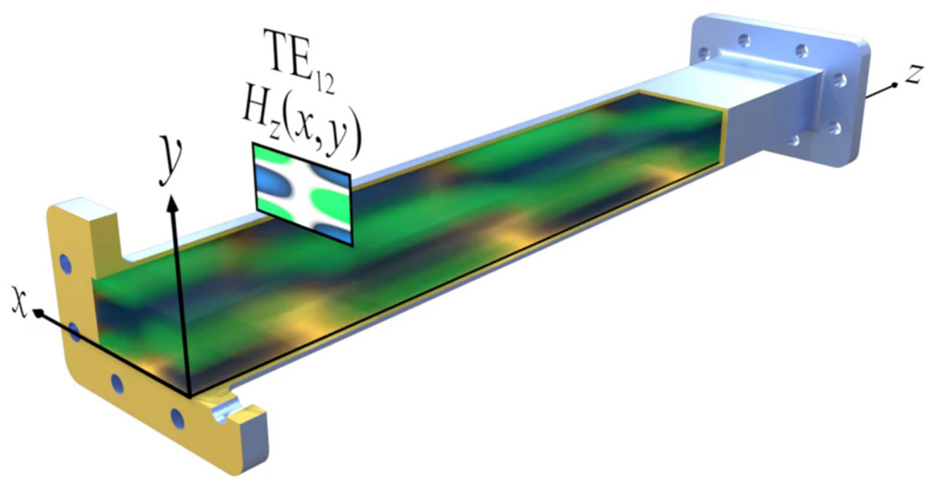
Visualization of H_z for the TE_{12} Mode



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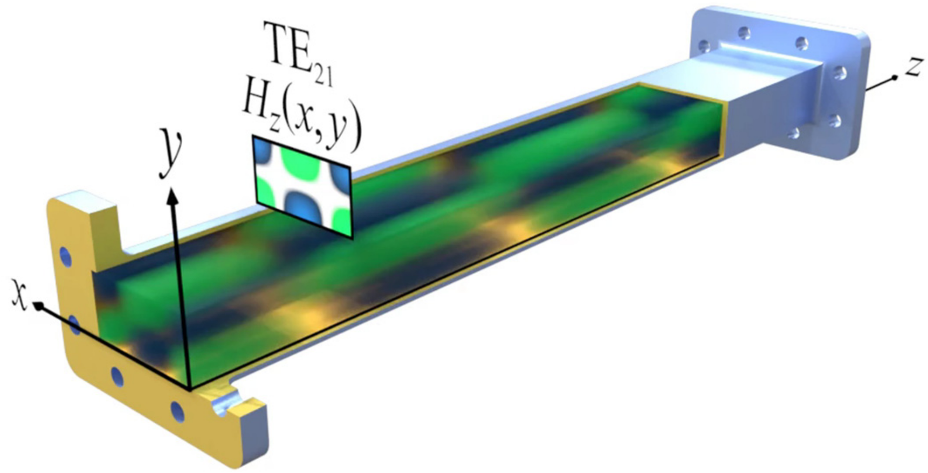
Visualization of H_z for the TE_{21} Mode



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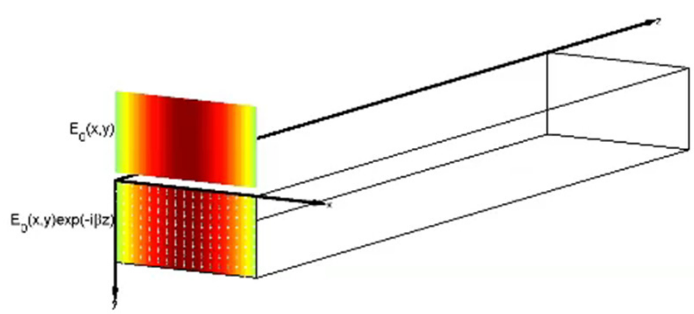
Animation of TE_{10}



Notice one bright spot along x and zero along y. ($m = 1$, $n = 0$)

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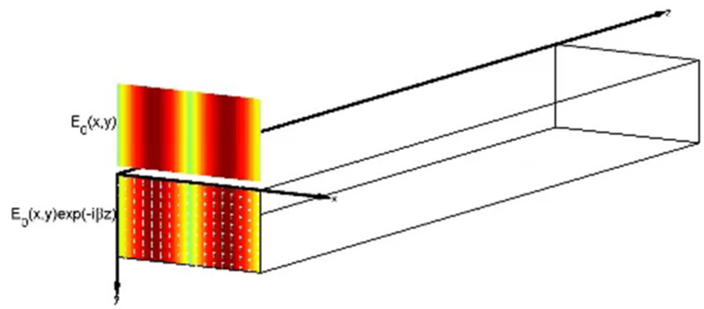
Animation of TE_{20}



Notice two bright spots along x and zero along y. ($m = 2$, $n = 0$)

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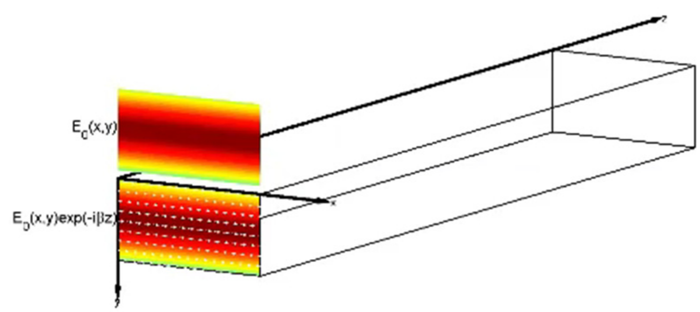
Animation of TE_{01}



Notice zero bright spots along x and one along y. ($m = 0$, $n = 1$)

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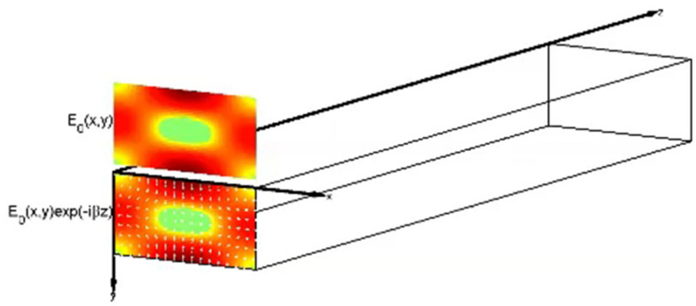
Animation of TE_{11}



Notice one bright spot along x and one along y. ($m = 1$, $n = 1$)

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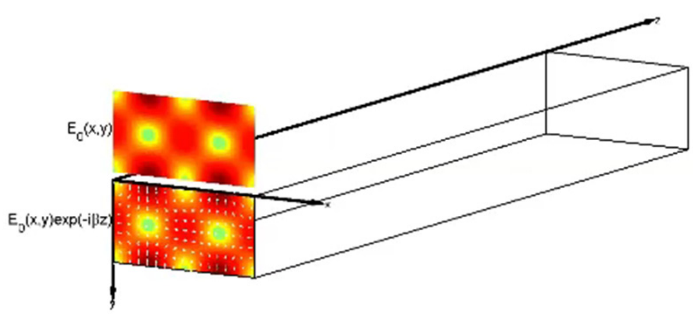
Animation of TE_{21}



Notice two bright spots along x and one along y. ($m = 2$, $n = 1$)

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Conclusion

Summary of TE Analysis

<u>Field Solution</u>

$$E_x(x,y,z) = \frac{j\omega\mu n\pi}{k_c^2 b} A_{mn} \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta_{mn}z}$$

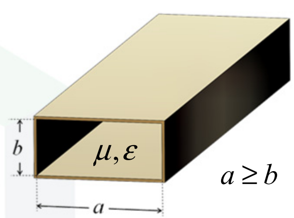
$$E_y(x,y,z) = -\frac{j\omega\mu m\pi}{k_c^2 a} A_{mn} \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta_{mn}z}$$

$$E_z(x,y,z) = 0$$

$$H_x(x,y,z) = \frac{j\beta_{mn} m\pi}{k_c^2 a} A_{mn} \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta_{mn}z}$$

$$H_y(x,y,z) = \frac{j\beta_{mn} n\pi}{k_c^2 b} A_{mn} \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta_{mn}z}$$

$$H_z(x,y,z) = A_{mn} \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-j\beta_{mn}z}$$



- TE_{00} mode does not exist
- TE_{10} is the lowest order TE mode
- Cutoff for the second-order mode is

$$f_{c2} = \begin{cases} f_{c,01} & a \le 2b \\ f_{c,20} & a > 2b \quad \text{(typical)} \end{cases}$$

<u>Phase Constant</u>

$$\beta_{mn} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

Same equation as for TM

<u>Cutoff Frequency</u>

$$f_{c,mn} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Same equation as for TM

<u>Characteristic Impedance</u>

$$Z_{\text{TE},mn} = \frac{k\eta}{\beta_{mn}}$$

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