EMPossible

Electromagnetics:
Electromagnetic Field Theory

Waveguide Analysis Setup

1

Solution Categories

Hybrid Mode Analysis

$E_{0,z} \neq 0$ and $H_{0,z} \neq 0$

Can analyze any waveguide.
Usually very complicated.

- No simplifications to the analysis are possible.
- Solutions contain all six field components.

TEM Analysis

$E_{0,z} = H_{0,z} = 0$

- Must be a transmission line,
- and must have a homogeneous dielectric

- **Transverse electromagnetic (TEM) mode**
- Both electric and magnetic fields are transverse to the direction of propagation.

TE & TM Analysis

$E_{0,z} = 0$ or $H_{0,z} = 0$, but not both

- Homogeneous dielectric,
- or has a dielectric uniform in one direction

- **Transverse electric (TE) mode**
- $E_{0,z} = 0$ so the electric field is completely transverse to the direction of propagation.
- **Transverse magnetic (TM) mode**
- $H_{0,z} = 0$ so the magnetic field is completely transverse to the direction of propagation.

Slab Waveguide Analysis

- Material properties uniform and infinite along one direction

- Maxwell's equations simplify and decouple into two independent sets of modes.

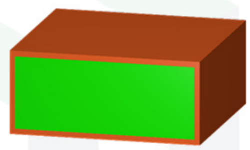
EMPossible

Slide 2

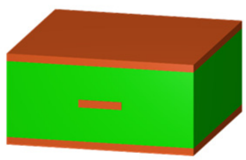
2

Existence Conditions for TEM

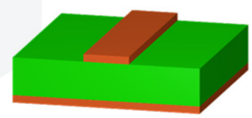
TEM modes only exist in transmission lines embedded in a homogeneous fill.



Does Not Support TEM



Supports TEM



Does Not Support TEM

EMPossible

Slide 3

3

TEM Analysis (1 of 3)

For TEM waves, $E_{0,z} = H_{0,z} = 0$. Under this condition, Maxwell's equations reduce to

$$\cancel{\frac{\partial E_{0,z}}{\partial y}} + j\beta E_{0,y} = -j\omega\mu H_{0,x} \qquad \text{Eq. (1a)}$$

$$-j\beta E_{0,x} - \cancel{\frac{\partial E_{0,z}}{\partial x}} = -j\omega\mu H_{0,y} \qquad \text{Eq. (1b)}$$

$$\frac{\partial E_{0,y}}{\partial x} - \frac{\partial E_{0,x}}{\partial y} = \cancel{-j\omega\mu H_{0,z}} \qquad \text{Eq. (1c)}$$

$$\cancel{\frac{\partial H_{0,z}}{\partial y}} + j\beta H_{0,y} = j\omega\varepsilon E_{0,x} \qquad \text{Eq. (1d)}$$

$$-j\beta H_{0,x} - \cancel{\frac{\partial H_{0,z}}{\partial x}} = j\omega\varepsilon E_{0,y} \qquad \text{Eq. (1e)}$$

$$\frac{\partial H_{0,y}}{\partial x} - \frac{\partial H_{0,x}}{\partial y} = \cancel{j\omega\varepsilon E_{0,z}} \qquad \text{Eq. (1f)}$$

⇩

$$j\beta E_{0,y} = -j\omega\mu H_{0,x} \qquad \text{Eq. (2a)}$$

$$-j\beta E_{0,x} = -j\omega\mu H_{0,y} \qquad \text{Eq. (2b)}$$

$$\frac{\partial E_{0,y}}{\partial x} - \frac{\partial E_{0,x}}{\partial y} = 0 \qquad \text{Eq. (2c)}$$

$$j\beta H_{0,y} = j\omega\varepsilon E_{0,x} \qquad \text{Eq. (2d)}$$

$$-j\beta H_{0,x} = j\omega\varepsilon E_{0,y} \qquad \text{Eq. (2e)}$$

$$\frac{\partial H_{0,y}}{\partial x} - \frac{\partial H_{0,x}}{\partial y} = 0 \qquad \text{Eq. (2f)}$$

EMPossible

Slide 4

4

TEM Analysis (2 of 3)

From the previous slide…

$$j\beta E_{0,y} = -j\omega\mu H_{0,x} \quad \text{Eq. (2a)}$$
$$-j\beta E_{0,x} = -j\omega\mu H_{0,y} \quad \text{Eq. (2b)}$$
$$\frac{\partial E_{0,y}}{\partial x} - \frac{\partial E_{0,x}}{\partial y} = 0 \quad \text{Eq. (2c)}$$
$$j\beta H_{0,y} = j\omega\varepsilon E_{0,x} \quad \text{Eq. (2d)}$$
$$-j\beta H_{0,x} = j\omega\varepsilon E_{0,y} \quad \text{Eq. (2e)}$$
$$\frac{\partial H_{0,y}}{\partial x} - \frac{\partial H_{0,x}}{\partial y} = 0 \quad \text{Eq. (2f)}$$

Solve Eq. (2d) for $H_{0,y}$.

$$H_{0,y} = \frac{\omega\varepsilon}{\beta} E_{0,x}$$

Substitute $H_{0,y}$ into Eq. (2b).

$$-j\beta E_{0,x} = -j\omega\mu\left(\frac{\omega\varepsilon}{\beta} E_{0,x}\right)$$
$$\beta^2 E_{0,x} = \omega^2 \mu\varepsilon E_{0,x}$$
$$\beta^2 E_{0,x} = k^2 E_{0,x}$$

This shows that for TEM analysis

$$\beta = k$$

TEM modes propagate at the same speed as a plane wave in the same medium.

Previously, the cutoff wave number was defined as $k_c^2 = k^2 - \beta^2$.

If $\beta = k$, then $k_c = 0$ indicating that there is no cutoff frequency for the TEM mode. This means the TEM mode is supported by DC.

EMPossible

Slide 5

5

TEM Analysis (3 of 3)

In LHI media, recall that the wave equation was

$$\nabla^2 \vec{E}_{0,xy} + k_c^2 \vec{E}_{0,xy} = 0$$

But for the TEM mode, $k_c = 0$.

$$\nabla^2 \vec{E}_{0,xy} + k_c^2 \vec{E}_{0,xy} = 0$$

$$\nabla^2 \vec{E}_{0,xy} = 0$$

The wave equation reduces to Laplace's equation from electrostatics.

EMPossible

Slide 6

6

Alternate Derivation of TEM Analysis

The TEM mode in a transmission line has no cutoff frequency ($k_c = 0$).
This means that it can be analyzed as $\omega \rightarrow 0$ and the problem reduces to an electrostatics problem.

Derivation

Maxwell's equations for electrostatics

$$\nabla \times \vec{E} = 0 \quad \text{Eq. (3a)}$$
$$\nabla \bullet \vec{D} = 0 \quad \text{Eq. (3b)}$$
$$\vec{D} = [\varepsilon]\vec{E} \quad \text{Eq. (3c)}$$
$$\vec{E} = -\nabla V \quad \text{Eq. (3d)}$$

Substitute Eq. (3c) into Eq. (3b).

$$\nabla \bullet \left([\varepsilon]\vec{E}\right) = 0 \quad \text{Eq. (4)}$$

Substitute Eq. (3d) into Eq. (4).

$$\nabla \bullet \left\{[\varepsilon](\nabla V)\right\} = 0$$

For isotropic dielectrics

$$\nabla \bullet \left[\varepsilon(\nabla V)\right] = 0$$

For homogeneous dielectrics

$$\boxed{\nabla^2 V = 0}$$

EMPossible Slide 7

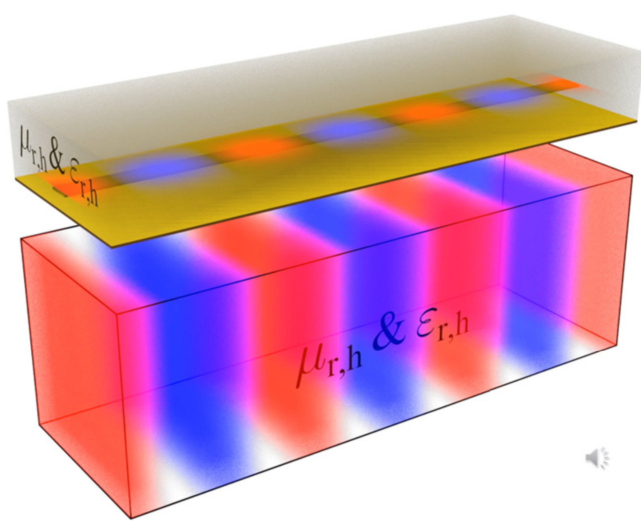
7

Speed of TEM Waves

For TEM modes, the voltage signal along the transmission line travels at the same velocity as a plane wave in the same homogeneous medium.

$$v_V = v_{\vec{E}}$$

$$\frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_h \varepsilon_h}}$$



EMPossible 8

8


Learn more about the EMProfessor:
https://raymondrumpf.com/
EMPossible
See all our education content:
https://empossible.net/
SUBSCRIBE

9