Transmission Lines

EMPossible

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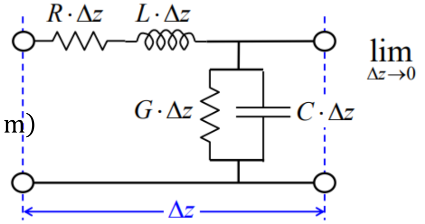
Transmission Line Model

$R \overset{\text{def}}{=}$ distributed resistance (Ω/m)

$L \overset{\text{def}}{=}$ distributed inductance (H/m)

$G \overset{\text{def}}{=}$ distributed conductance (1/Ω · m)

$C \overset{\text{def}}{=}$ distributed capacitance (F/m)



TL Equations

$$-\frac{\partial V(z,t)}{\partial z} = R \cdot I(z,t) + L\frac{\partial I(z,t)}{\partial z} \qquad -\frac{\partial I(z,t)}{\partial z} = G \cdot V(z,t) + C\frac{\partial V(z,t)}{\partial z}$$

TL Wave Equations + Solutions

$$\frac{d^2V(z)}{dz^2} - (R + j\omega L)(G + j\omega C)V(z) = 0 \qquad V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$\frac{d^2I(z)}{dz^2} - (R + j\omega L)(G + j\omega C)I(z) = 0 \qquad I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

Transmission Line Parameters

Characteristic Impedance: $Z_0 = \frac{V_0^+}{I_0^+} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_0 + jX_0$

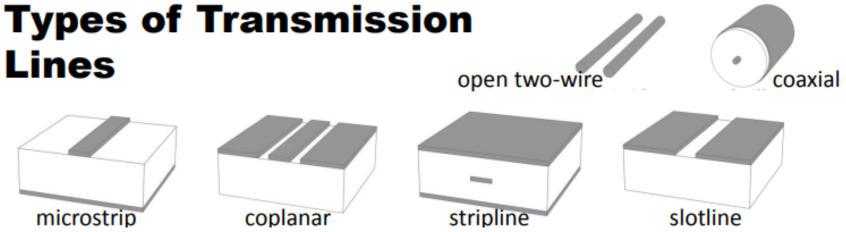
Complex Propagation Constant: $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$

$$\alpha = \sqrt{\frac{RG - \omega^2 LC + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}{2}}$$

$$\beta = \sqrt{\frac{-RG + \omega^2 LC + \sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}}{2}}$$

Lossless Lines $(R = G = 0)$:	$\alpha = 0$	$\beta = \omega\sqrt{LC}$	$Z_0 = \sqrt{\frac{L}{C}}$
Distortionless Lines $(RC = LG)$:	$\alpha = \sqrt{RG}$	$\beta = \omega\sqrt{LC}$	$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}}$

Types of Transmission Lines



Transmission Line Behavior

General	Short	Open	Matched
Load Impedance Z_L	Load Impedance $Z_L = 0$	Load Impedance $Z_L = \infty$	Load Impedance $Z_L = Z_0$
Reflection Coefficient $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$	Reflection Coefficient $\Gamma_L = -1$	Reflection Coefficient $\Gamma_L = +1$	Reflection Coefficient $\Gamma_L = 0$
Lossy Line $Z_{in}(\ell) = Z_0 \frac{Z_L + jZ_0 \tanh\gamma\ell}{Z_0 + jZ_L \tanh\gamma\ell}$	Lossy Line $Z_{in}(\ell) = Z_0 \tanh\gamma\ell$	Lossy Line $Z_{in}(\ell) = Z_0 \coth\gamma\ell$	Lossy Line $Z_{in}(\ell) = Z_0$
Lossless Line $Z_{in}(\ell) = Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell}$	Lossless Line $Z_{in}(\ell) = Z_0 \tan\beta\ell$	Lossless Line $Z_{in}(\ell)$ $= -jZ_0 \cot\beta\ell$	Lossless Line $Z_{in}(\ell) = Z_0$
Standing Waves $V_{min} = \lvert V_0^+\rvert(1 - \lvert\Gamma\rvert)$ $V_{max} = \lvert V_0^+\rvert(1 + \lvert\Gamma\rvert)$ $I_{min} = \frac{\lvert V_0^+\rvert}{Z_0}(1 - \lvert\Gamma\rvert)$ $I_{max} = \frac{\lvert V_0^+\rvert}{Z_0}(1 + \lvert\Gamma\rvert)$ $\text{VSWR} = \frac{1 + \lvert\Gamma\rvert}{1 - \lvert\Gamma\rvert}$ $\min[Z_{in}] = Z_0/\text{VSWR}$ $\max[Z_{in}] = Z_0 \cdot \text{VSWR}$	Standing Waves $V_{min} = 0$ $V_{max} = 2\lvert V_0^+\rvert$ $I_{min} = 0$ $I_{max} = 2\frac{\lvert V_0^+\rvert}{Z_0}$ $\text{VSWR} = \infty$ $\min[Z_{in}] = 0$ $\max[Z_{in}] = \infty$		Standing Waves $V_{min} = \lvert V_0^+\rvert$ $V_{max} = \lvert V_0^+\rvert(1 + \lvert\Gamma\rvert)$ $I_{min} = \frac{\lvert V_0^+\rvert}{Z_0}(1 - \lvert\Gamma\rvert)$ $I_{max} = \frac{\lvert V_0^+\rvert}{Z_0}(1 + \lvert\Gamma\rvert)$ $\text{VSWR} = 1$ $\min[Z_{in}] = Z_0$ $\max[Z_{in}] = Z_0$
Standing Waves $Z_{in,short} Z_{in,open} = Z_0^2 \qquad P_{avg} = \frac{\lvert V_0^+\rvert}{2Z_0}(1 - \lvert\Gamma\rvert^2)$			

Impedance Transformation

General Case (with Loss):

$$Z_{in}(\ell) = Z_0 \frac{Z_L + jZ_0 \tanh\gamma\ell}{Z_0 + jZ_L \tanh\gamma\ell}$$

Lossless Case:

$$Z_{in}(\ell) = Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell}$$