EMPossible

Advanced Computation:
Computational Electromagnetics

Nonuniform Grids in FDFD

1

Outline

- Finite-difference approximations on a nonuniform grid
- Geometry of a nonuniform grid
- Building devices onto a nonuniform grid
- Miscellaneous tips

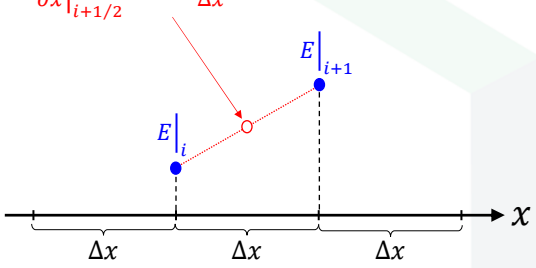
2

Finite-Difference Approximations on a Nonuniform Grid

3

Finite-Difference Approximation on a Uniform Grid

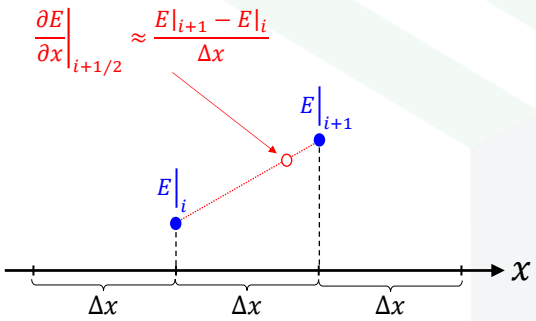
$$\left.\frac{\partial E}{\partial x}\right|_{i+1/2} \approx \frac{E|_{i+1} - E|_i}{\Delta x}$$



This finite-difference approximation has second-order accuracy because it is a central finite difference.

4

Finite-Difference Approximation on a Nonuniform Grid



While this finite-difference approximation has the same mathematical form, it only has first-order accuracy because it is no longer a central finite difference.

EMPossible

Slide 5

5

Geometry of a Nonuniform Grid

Slide 6

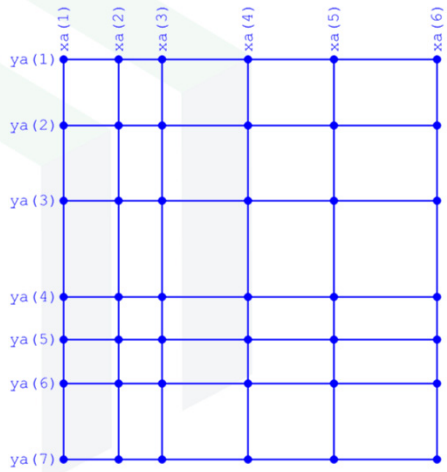
6

Geometry of a Nonuniform Grid

A nonuniform grid is depicted at the right of size 6×7.

It is nonuniform in both x and y directions.

The blue dots indicate the grid positions reported in the axis arrays `xa` and `ya`.



EMPossible

Slide 7

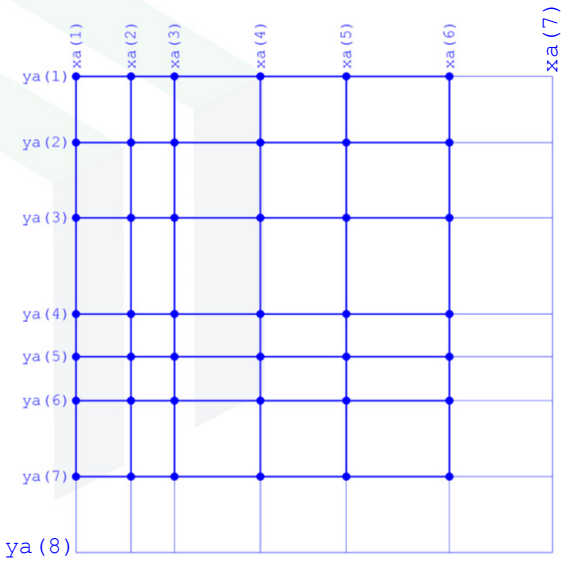
7

Geometry of a Nonuniform Grid

Due to the staggering of the field components, there is extra space on the far side of the grid that should not be forgotten.

An extra number is stored in the axis arrays `xa` and `ya` to locate the position of the far edges of the grid.

`xa` will be of length `Nx + 1`.
`ya` will be of length `Ny + 1`.



EMPossible

Slide 8

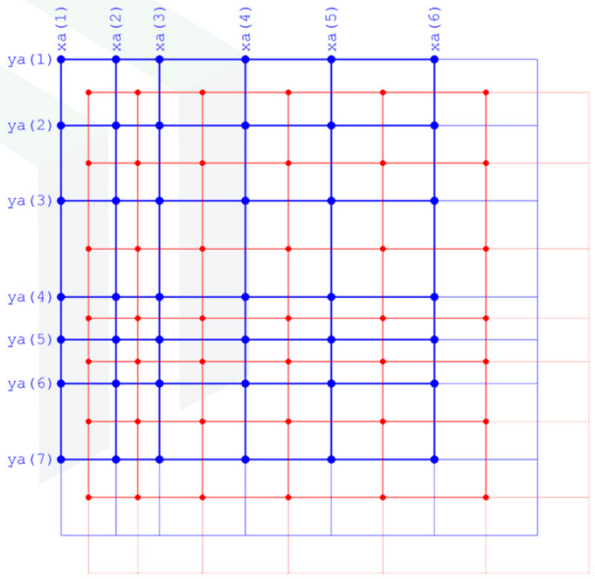
8

Geometry of a Nonuniform Grid

Some people like to visualize an additional dual grid.

In this framework, the dual grid points are placed at the center positions between the points of the primary grid.

This practice will keep finite-difference approximations on the dual grid second-order accurate, while finite-difference approximations on the primary grid will only be first-order accurate.



EMPossible

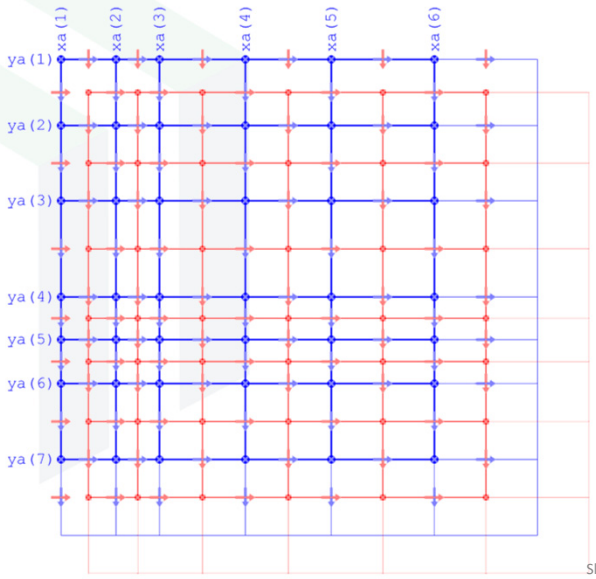
Slide 9

9

Geometry of a Nonuniform Grid

Placing the field components on the grid, the extra space on the right and can be understood more intuitively, and the utility of the offset grid can be more fully appreciated.

However, this is getting quite complicated to see everything!



EMPossible

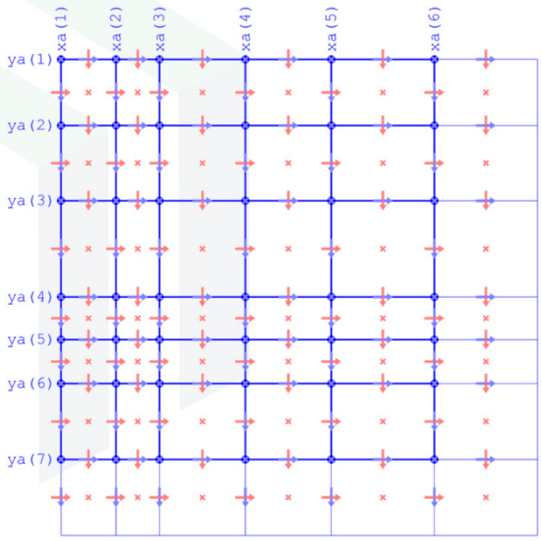
Slide 10

10

Geometry of a Nonuniform Grid

It is simpler to hide the offset grid and see just the primary meshgrid along with the placement of the field components.

It is from this picture that devices will be constructed onto the grid, derivative matrices will be constructed, and the calculated fields will be visualized.



EMPossible

Slide 11

11

Building Devices onto a Nonuniform Grid

Slide 12

12

Steps for Building a Nonuniform Grid

- Define the primary grid. This is where all nonuniformity is incorporated.
- Build the dual grid to be at the midpoints between the points of the primary grid.
- Calculate the 2× grid using information from both the primary and dual grids.

13

Calculating the 1× Grid

Axis Arrays `xa` and `ya`

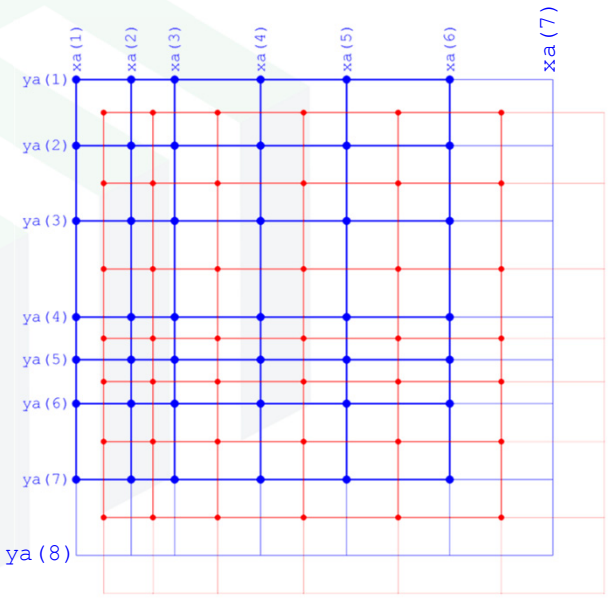
Include an extra point to properly locate the far edge of the grid.

`xa` will be of length `Nx+1`.
`ya` will be of length `Ny+1`.

Meshgrid

Meshgrids are used for calculating functions across the grid so they must be properly sized. Points should be located at the centers of the cells.

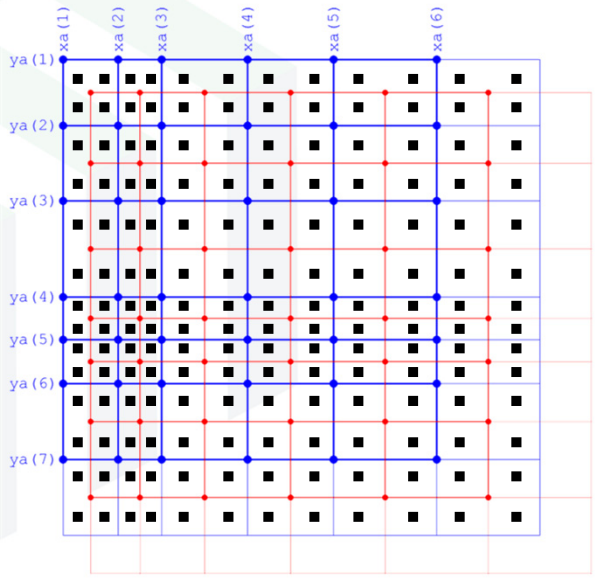
```
% CALCULATE MESHGRID
xac = (xa(1:Nx) + xa(2:Nx+1))/2;
yac = (ya(1:Ny) + ya(2:Ny+1))/2;
[Y,X] = meshgrid(yac,xac);
```



14

Calculating the 2× Grid

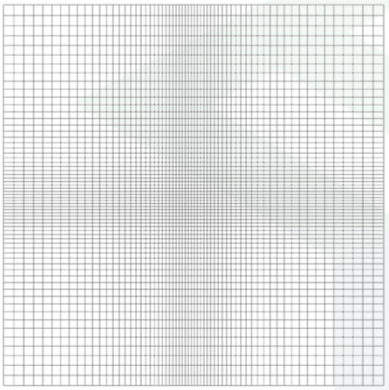
```
% 2X GRID
Nx2 = 2*Nx;
Ny2 = 2*Ny;

xa2 = zeros(1,Nx2);
for nx = 1 : Nx
   xa2(2*nx-1) = xa(nx) + 0.25*(xa(nx+1) - xa(nx));
   xa2(2*nx)   = xa(nx) + 0.75*(xa(nx+1) - xa(nx));
end

ya2 = zeros(1,Ny2);
for ny = 1 : Ny
   ya2(2*ny-1) = ya(ny) + 0.25*(ya(ny+1) - ya(ny));
   ya2(2*ny)   = ya(ny) + 0.75*(ya(ny+1) - ya(ny));
end

[Y2,X2] = meshgrid(ya2,xa2);
```



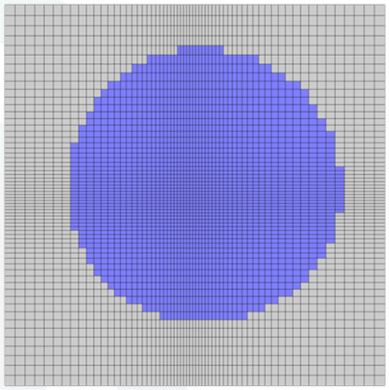
EMPossible

15

Use Meshgrid Techniques



Step 1 – Calculate a nonuniform grid



Step 2 – Use meshgrid techniques to build devices onto the grid

```
RSQ = X.^2 + Y.^2;
ER  = (RSQ <= R^2);
ER  = er1 + (er2 - er1)*ER;
```

EMPossible

16

Derivative Matrices on a Nonuniform Grid

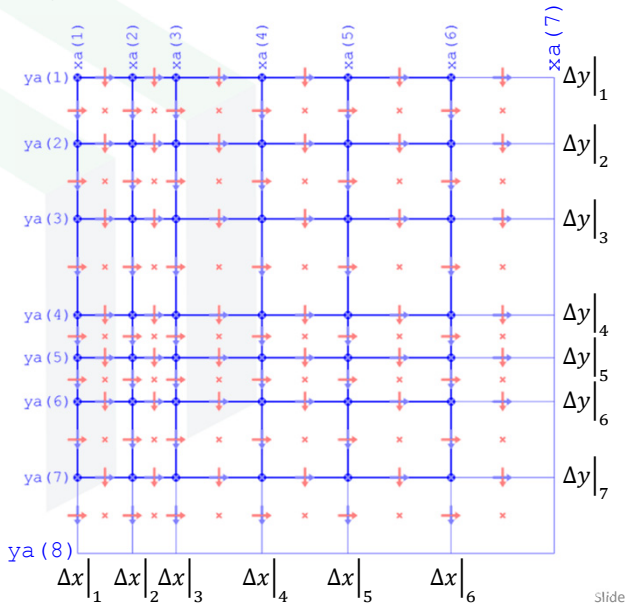
17

Determine $\Delta x|_i$ and $\Delta y|_j$ for the Primary Grid

6×7 grid

$$\Delta x|_i^{\mathrm{e}} = \begin{cases} \left(x|_2 - x|_1 + x|_{N_x+1} - x|_{N_x}\right)/2 & i = 1 \\ \left(x|_{i+1} - x|_{i-1}\right)/2 & i > 1 \end{cases}$$

$$\Delta y|_j^{\mathrm{e}} = \begin{cases} \left(y|_2 - y|_1 + y_{N+1} - y|_N\right)/2 & j = 1 \\ \left(y|_{j+1} - y_{j-1}\right)/2 & j > 1 \end{cases}$$



EMPossible

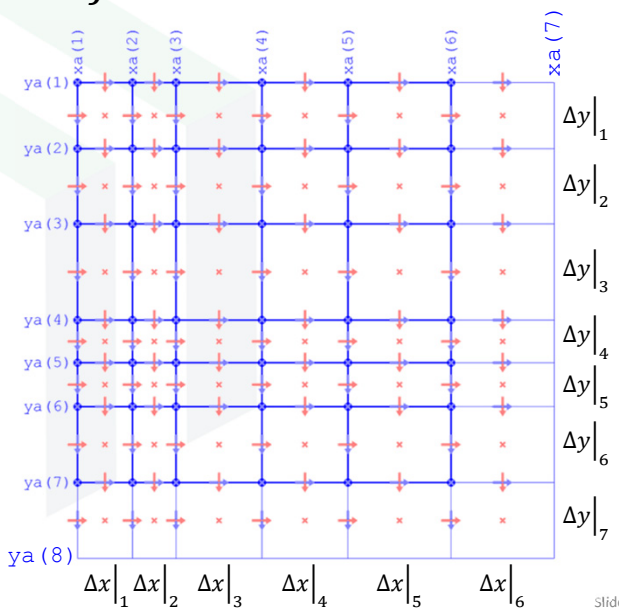
18

Determine $\Delta x|_i$ and $\Delta y|_j$ for the Dual Grid

6×7 grid

$$\Delta x\Big|_i^{\text{h}} = x\Big|_{i+1} - x\Big|_i$$

$$\Delta y\Big|_j^{\text{h}} = y\Big|_{j+1} - y\Big|_j$$



EMPossible

Basic Structure of Derivate Matrix is Unchanged

Uniform Grid
Each row is scaled by the same value of Δx or Δy.

$$\mathbf{D}_x^e = \begin{bmatrix} -1 & 1 & & & & & & \\ & -1 & 1 & & & & & \\ & & -1 & 0 & & & & \\ & & & -1 & 1 & & & \\ & & & & -1 & 1 & & \\ & & & & & -1 & 0 & \\ & & & & & & -1 & 1 \\ & & & & & & & -1 \end{bmatrix} \begin{matrix} \div\Delta x \\ \div\Delta x \\ \div\Delta x \\ \div\Delta x \\ \div\Delta x \\ \div\Delta x \\ \div\Delta x \\ \div\Delta x \end{matrix}$$

$$\mathbf{D}_y^e = \begin{bmatrix} -1 & & 1 & & & & & \\ & -1 & & 1 & & & & \\ & & -1 & & 1 & & & \\ & & & -1 & & 1 & & \\ & & & & -1 & & 1 & \\ & & & & & -1 & & 1 \\ & & & & & & -1 & \\ & & & & & & & -1 \end{bmatrix} \begin{matrix} \div\Delta y \\ \div\Delta y \\ \div\Delta y \\ \div\Delta y \\ \div\Delta y \\ \div\Delta y \\ \div\Delta y \\ \div\Delta y \end{matrix}$$

3×3 grid

Nonuniform Grid
Each row is scaled by a different value of Δx or Δy.

$$\mathbf{D}_x^e = \begin{bmatrix} -1 & 1 & & & & & & \\ & -1 & 1 & & & & & \\ & & -1 & 0 & & & & \\ & & & -1 & 1 & & & \\ & & & & -1 & 1 & & \\ & & & & & -1 & 0 & \\ & & & & & & -1 & 1 \\ & & & & & & & -1 \end{bmatrix} \begin{matrix} \div\Delta x|_1 \\ \div\Delta x|_2 \\ \div\Delta x|_3 \\ \div\Delta x|_1 \\ \div\Delta x|_2 \\ \div\Delta x|_3 \\ \div\Delta x|_1 \\ \div\Delta x|_2 \\ \div\Delta x|_3 \end{matrix}$$

$$\mathbf{D}_y^e = \begin{bmatrix} -1 & & 1 & & & & & \\ & -1 & & 1 & & & & \\ & & -1 & & 1 & & & \\ & & & -1 & & 1 & & \\ & & & & -1 & & 1 & \\ & & & & & -1 & & 1 \\ & & & & & & -1 & \\ & & & & & & & -1 \end{bmatrix} \begin{matrix} \div\Delta y|_1 \\ \div\Delta y|_1 \\ \div\Delta y|_1 \\ \div\Delta y|_2 \\ \div\Delta y|_2 \\ \div\Delta y|_2 \\ \div\Delta y|_3 \\ \div\Delta y|_3 \\ \div\Delta y|_3 \end{matrix}$$

EMPossible

Building $\mathbf{D}_x^{\mathrm{h}}$ and $\mathbf{D}_y^{\mathrm{h}}$

It is no longer accurate to calculate $\mathbf{D}_x^{\mathrm{h}}$ and $\mathbf{D}_y^{\mathrm{h}}$ directly from $\mathbf{D}_x^{\mathrm{e}}$ and $\mathbf{D}_y^{\mathrm{e}}$.

~~$\mathbf{D}_x^{\mathrm{h}} = -(\mathbf{D}_x^{\mathrm{e}})^{\mathrm{H}}$~~ ~~$\mathbf{D}_y^{\mathrm{h}} = -(\mathbf{D}_y^{\mathrm{e}})^{\mathrm{H}}$~~

The derivative matrices must be calculated independently due to the spacing of the nonuniform grid.

EMPossible

Slide 21

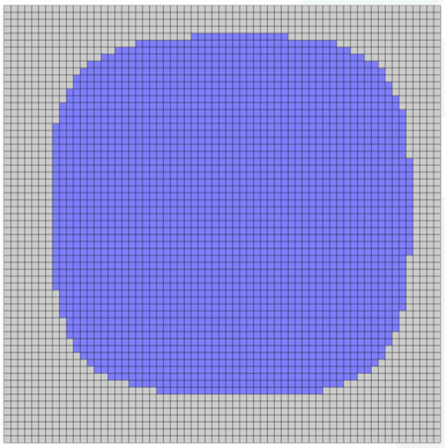
21

Miscellaneous Tips

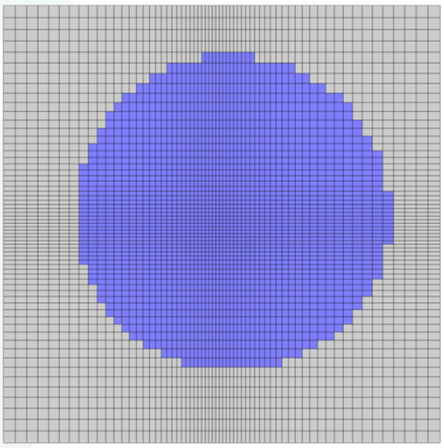
Slide 22

22

Visualizing Data on Nonuniform Grids



```
imagesc(xa,ya,ER.');
```



```
pcolor(X,Y,ER);
set(gca,'Ydir','reverse');
```

EMPossible

Slide 23

23

Analyzing Transmittance & Reflectance

Interpolate the field cross sections onto a uniform grid before calculating FFTs.

```
% INTERPOLATE TO UNIFORM GRID
xau = linspace(X(1,1),X(Nx,1),Nx);
fref = interp1(X(:,1),fref,xau).';
ftrn = interp1(X(:,1),ftrn,xau).';
```

EMPossible

Slide 24

24

Convergence

Due to first-order accuracy for derivatives of electric fields, convergence will be slower where grid nonuniformity is most severe.

Be prepared to use higher resolution where grid is nonuniform.