Electromagnetics:

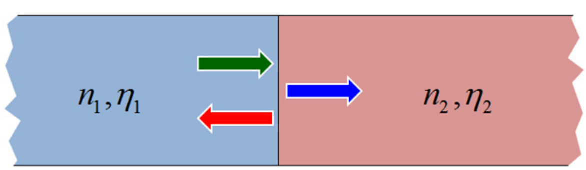
Electromagnetic Field Theory

Anti-Reflection Layer

1

Problem Setup (1 of 2)

Let there be an interface between two materials.



This will produce reflections according to

$$r = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

How can reflection be prevented at this interface?

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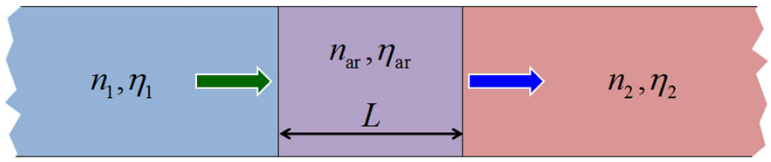
Slide 2

2

Problem Setup (2 of 2)

Insert an intermediate layer that will be called an *anti-reflection layer*.



How can n_{ar}, η_{ar}, and L be chosen to get exactly zero reflections from this interface?

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How to Get $r = 0$ From a Slab (1 of 4)

Recall the overall reflection from a dielectric slab.

$$r = \frac{r_{12} + r_{23} e^{-j2\psi}}{1 + r_{12} r_{23} e^{-j2\psi}}$$

To get $r = 0$, the numerator of this expression must be zero.

$$r_{12} + r_{23} e^{-j2\psi} = 0$$

The reflection coefficients r_{12} and r_{23} arise from the materials in the problem so they cannot be changed. The trick must be in the $e^{-j2\psi}$ term. Solving this term for ψ gives

$$e^{-j2\psi} = -\frac{r_{12}}{r_{23}} \rightarrow \ln\left(e^{-j2\psi}\right) = \ln\left(-\frac{r_{12}}{r_{23}}\right)$$

$$\rightarrow -j\left(2\psi + 2\pi m\right) = \ln\left(\frac{r_{23}}{r_{12}}\right)$$

$$\rightarrow \psi = \frac{1}{j2} \ln\left(\frac{r_{23}}{r_{12}}\right) - \pi m$$

$$m = \text{any integer}$$

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How to Get $r = 0$ From a Slab (2 of 4)

Recall that

$$r_{12} = \frac{\eta_{\text{ar}} - \eta_1}{\eta_{\text{ar}} + \eta_1} \qquad r_{23} = \frac{\eta_2 - \eta_{\text{ar}}}{\eta_2 + \eta_{\text{ar}}}$$

The expression for ψ becomes

$$\psi = \frac{1}{j2}\ln\left(\frac{r_{23}}{r_{12}}\right) - \pi m = \frac{1}{j2}\ln\left(\frac{\dfrac{\eta_2 - \eta_{\text{ar}}}{\eta_2 + \eta_{\text{ar}}}}{\dfrac{\eta_{\text{ar}} - \eta_1}{\eta_{\text{ar}} + \eta_1}}\right) - \pi m = \frac{1}{j2}\ln\left[\frac{(\eta_{\text{ar}} + \eta_1)(\eta_{\text{ar}} - \eta_2)}{(\eta_{\text{ar}} - \eta_1)(\eta_{\text{ar}} + \eta_2)}\right] - \pi m$$

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How to Get $r = 0$ From a Slab (3 of 4)

Recall that $\psi = k_0 n_{\text{ar}} d$. Substitute this into the design equation from the previous slide.

$$k_0 n_{\text{ar}} d = \frac{1}{j2}\ln\left[\frac{(\eta_{\text{ar}} + \eta_1)(\eta_{\text{ar}} - \eta_2)}{(\eta_{\text{ar}} - \eta_1)(\eta_{\text{ar}} + \eta_2)}\right] - \pi m$$

Using $k_0 = 2\pi/\lambda_0$ and solving this for d gives

$$\frac{2\pi}{\lambda_0} n_{\text{ar}} d = \frac{1}{j2}\ln\left[\frac{(\eta_{\text{ar}} + \eta_1)(\eta_{\text{ar}} - \eta_2)}{(\eta_{\text{ar}} - \eta_1)(\eta_{\text{ar}} + \eta_2)}\right] - \pi m$$

$$d = \frac{\lambda_0}{4n_{\text{ar}}}\frac{1}{j\pi}\ln\left[\frac{(\eta_{\text{ar}} + \eta_1)(\eta_{\text{ar}} - \eta_2)}{(\eta_{\text{ar}} - \eta_1)(\eta_{\text{ar}} + \eta_2)}\right] - m\frac{\lambda_0}{2n_{\text{ar}}}$$

This could give a complex number for d.

$$\boxed{d = \text{Re}\left\{\frac{\lambda_0}{4n_{\text{ar}}}\frac{1}{j\pi}\ln\left[\frac{(\eta_{\text{ar}} + \eta_1)(\eta_{\text{ar}} - \eta_2)}{(\eta_{\text{ar}} - \eta_1)(\eta_{\text{ar}} + \eta_2)}\right] - m\frac{\lambda_0}{2n_{\text{ar}}}\right\}}$$

This is the most general design equation and provides more freedom than the simpler one about to be derived.

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How to Get $r = 0$ From a Slab (3 of 4)

The general design equation on the previous slide is complicated. Is it possible to simplify?

To figure out a way to do this, multiply out the expression inside of the natural logarithm function.

$$\frac{(\eta_{\text{ar}} + \eta_1)(\eta_{\text{ar}} - \eta_2)}{(\eta_{\text{ar}} - \eta_1)(\eta_{\text{ar}} + \eta_2)} = \frac{\eta_{\text{ar}}^2 - \eta_{\text{ar}}(\eta_2 - \eta_1) - \eta_1\eta_2}{\eta_{\text{ar}}^2 + \eta_{\text{ar}}(\eta_2 - \eta_1) - \eta_1\eta_2}$$

This simplifies when $\eta_{\text{ar}}^2 = \eta_1\eta_2$. In fact, it reduces to just -1.

Recognizing that $\ln(-1) = j\pi$, the simplified expression for d becomes

$$\boxed{d = \frac{\lambda_0}{4n_{\text{ar}}} - m\frac{\lambda_0}{2n_{\text{ar}}} \qquad \text{for } \eta_{\text{ar}}^2 = \eta_1\eta_2}$$

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Interpretation of Design Equation

When $\eta_{\text{ar}}^2 = \eta_1\eta_2$

$$d = \frac{\lambda_0}{4n_{\text{ar}}} - m\frac{\lambda_0}{2n_{\text{ar}}} \qquad m = \text{any integer}$$

The second term conveys that the length d can be adjusted by any integer multiple of a half-wavelength.

$$m\frac{\lambda_0}{2n_{\text{ar}}} = m\frac{\lambda}{2}$$

Interpret this first terms as a quarter wavelength slab of dielectric.

$$\frac{\lambda_0}{4n_{\text{ar}}} = \frac{\lambda}{4}$$

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Design Procedure

Step 1 – Choose an antireflection material such that

$$\eta_{ar} = \sqrt{\eta_1 \eta_2}$$

There is a bit of freedom here.

However, when only dielectric materials are used (i.e. $\mu_r \approx 1$), only one choice is possible.

$$\varepsilon_{ar} = \sqrt{\varepsilon_1 \varepsilon_2} \quad \text{or} \quad n_{ar} = \sqrt{n_1 n_2}$$

Step 2 – Calculate thickness based on refractive index.

$$d = \frac{\lambda_0}{4n_{ar}} - m\frac{\lambda_0}{2n_{ar}} \qquad m = \text{any integer}$$

$m = 0$ is the most common choice.

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Example

It is desired to maximize the light through a lens. The lens is made of glass with n = 1.52 and resides in air with n = 1.0. Design an anti-reflection coating to maximize transmission at the center of the visible spectrum, λ_0 = 500 nm.

Solution

Step 1: At optical frequencies, materials cannot have a significant magnetic response. Therefore, we will design the anti-reflection layer through the refractive index n_{ar}.

$$n_{ar} = \sqrt{n_1 n_2} = \sqrt{(1.0)(1.52)} \quad \rightarrow \quad \boxed{n_{ar} = 1.2329}$$

Step 2: The thickness of the anti-reflection layer is

$$d = \frac{500 \text{ nm}}{4(1.2329)} - m\frac{500 \text{ nm}}{2(1.2329)} = 101.4 \text{ nm} - m(202.8 \text{ nm})$$

Choose the $m = 0$ solution.

$$\boxed{d = 101.4 \text{ nm}}$$

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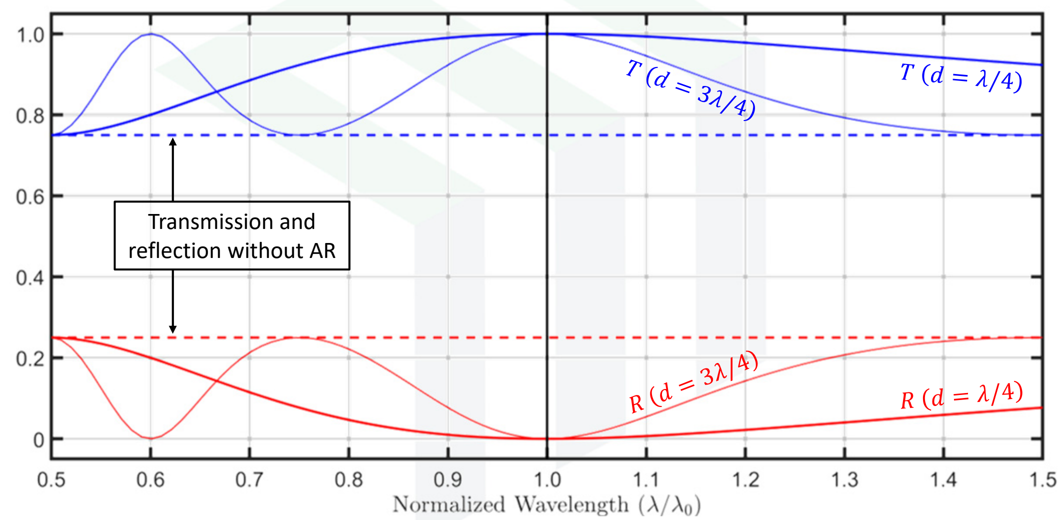
Typical Response of Anti-Reflection Layers



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