Electromagnetics:
Electromagnetic Field Theory

Transmission Line Model

1

Lecture Outline

- Introduction
- $RLGC$ Model of a Transmission Line

Slide 2

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Introduction

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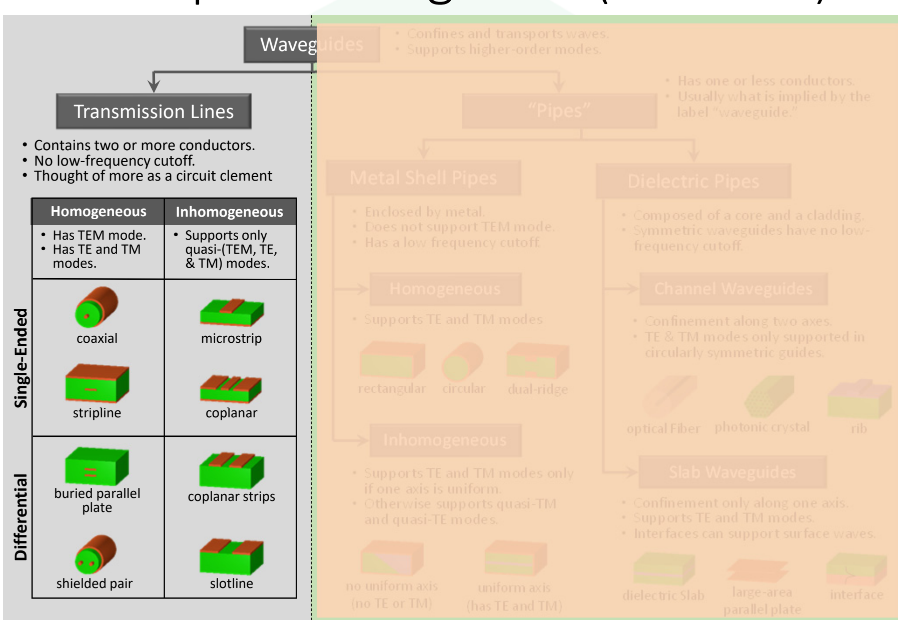
Map of Waveguides (LI Media)



EMPossible

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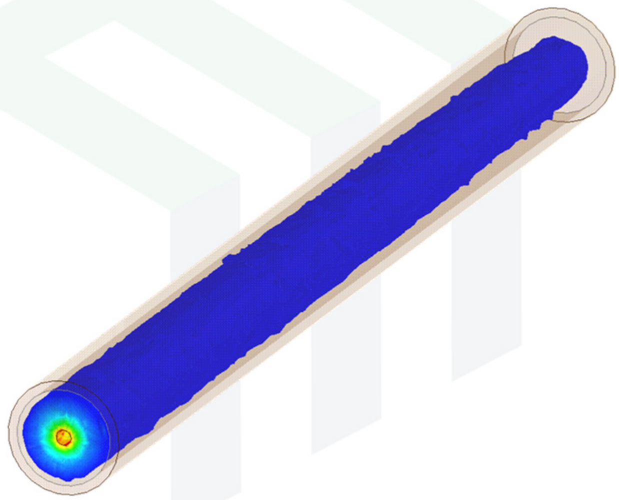
Signals in Transmission Lines: Coax



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Slide 5

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Signals in Transmission Lines: Twisted Pair

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Slide 6

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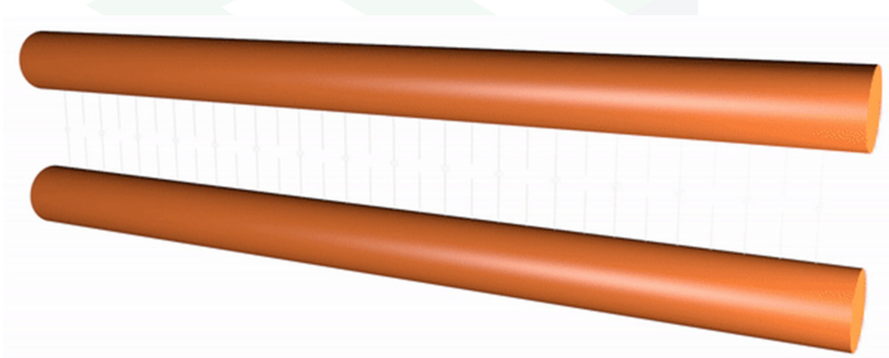
$RLGC$ Model of a Transmission Line

Transmission Line Parameters $RLGC$

It can be useful to think of transmission lines as being composed of millions of tiny little circuit elements that are distributed along the length of the line.



In fact, these circuit element are not discrete, but continuous along the length of the transmission line.

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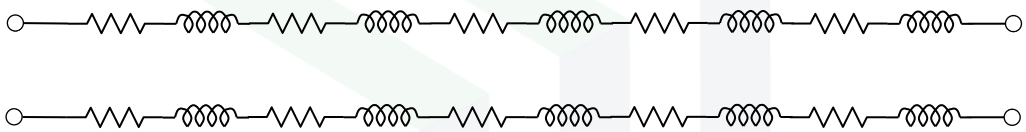
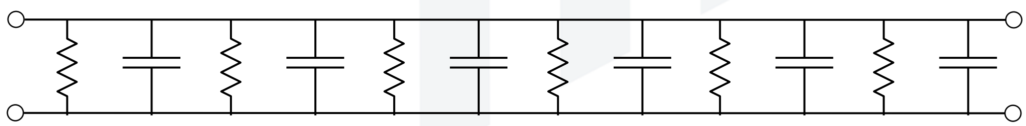
Developing a Circuit Model (1 of 5)

The conductors of the transmission lines are not perfect and will exhibit some level of series resistance. In addition, the lines will generate magnetic fields that store magnetic energy. This makes the lines exhibit some level of series inductance.

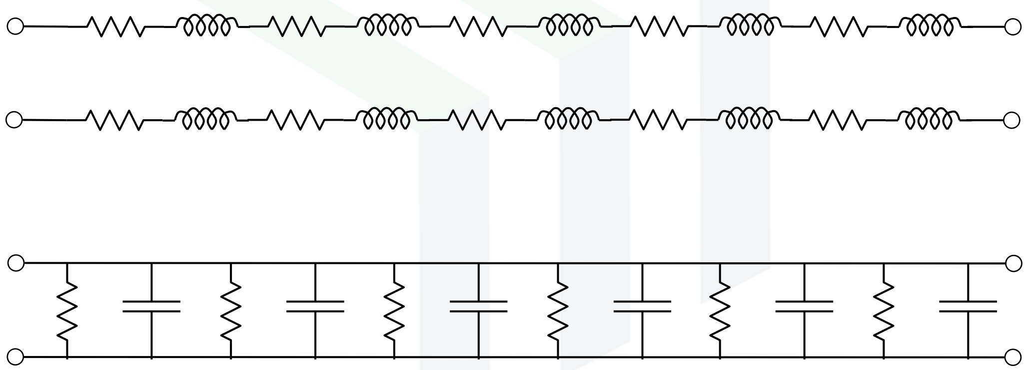


The dielectric between the lines may conduct by some amount. This causes the line to exhibit some level of shunt conductance. In addition, the two lines will generate electric fields between them that store electric energy. This makes the transmission line exhibit some level of shunt capacitance.



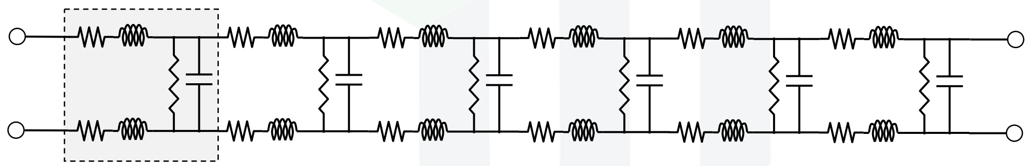
Developing a Circuit Model (2 of 5)

The transmission line will simultaneously exhibit the series resistance, series inductance, shunt conductance, and shunt capacitance.

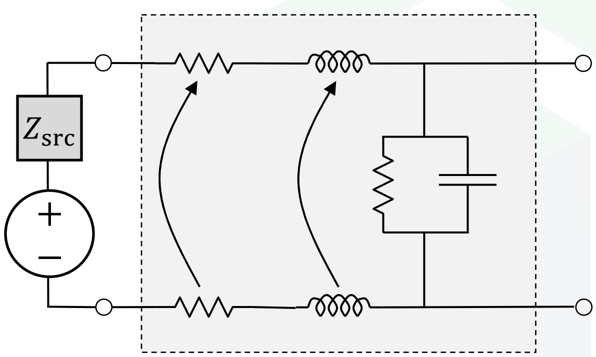


Developing a Circuit Model (3 of 5)

The transmission line will simultaneously exhibit the series resistance, series inductance, shunt conductance, and shunt capacitance.



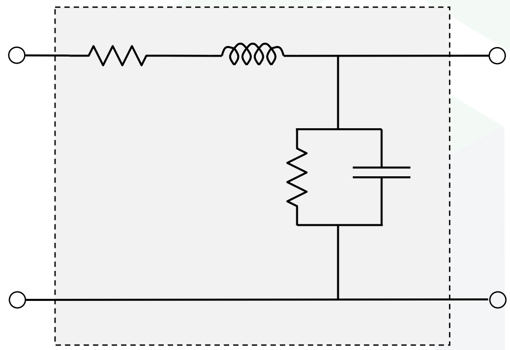
Developing a Circuit Model (4 of 5)



Any circuit driving the transmission line can be reduced to a voltage source and impedance Z_{src}.

When it is only the voltage and current at the terminals of the line that are of interest, the series resistors and inductors can be combined.

Developing a Circuit Model (5 of 5)



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When it is only the voltage and current at the terminals of the line that are of interest, the series resistors and inductors can be combined.

The driving circuit can now be ignored.

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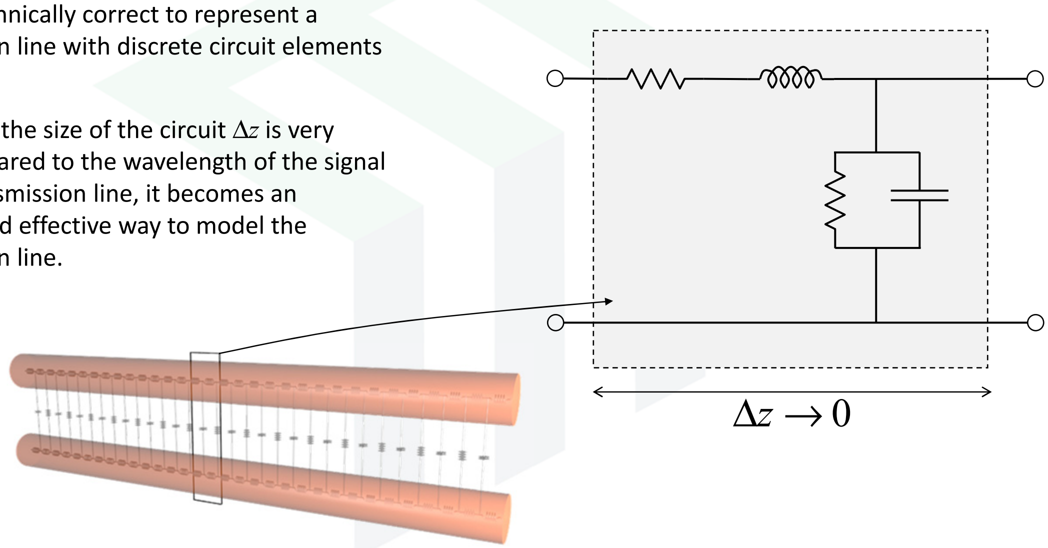
$RLGC$ Circuit Model

It is not technically correct to represent a transmission line with discrete circuit elements like this.

However, if the size of the circuit Δz is very small compared to the wavelength of the signal on the transmission line, it becomes an accurate and effective way to model the transmission line.



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L-Type Equivalent Circuit Model

Distributed Circuit Parameters

R (Ω/m)
Resistance per unit length.
Arises due to resistivity in the conductors.

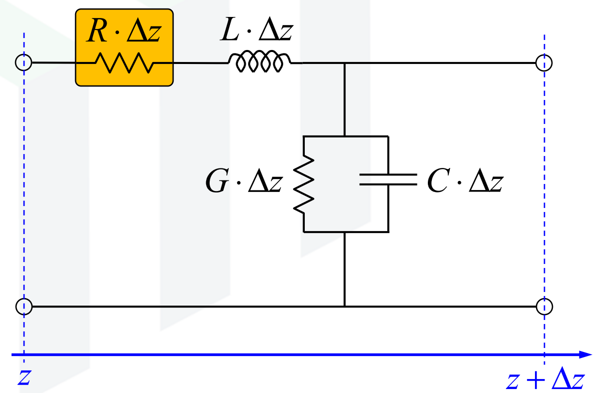
L (H/m)
Inductance per unit length.
Arises due to stored magnetic energy around the line.

G (1/Ω·m)
Conductance per unit length.
Arises due to conductivity in the dielectric separating the conductors.

$$G \neq \frac{1}{R}$$

C (F/m)
Capacitance per unit length.
Arises due to stored electric energy between the conductors.

There are many possible circuit models for transmission lines, but most produce the same equations after analysis.



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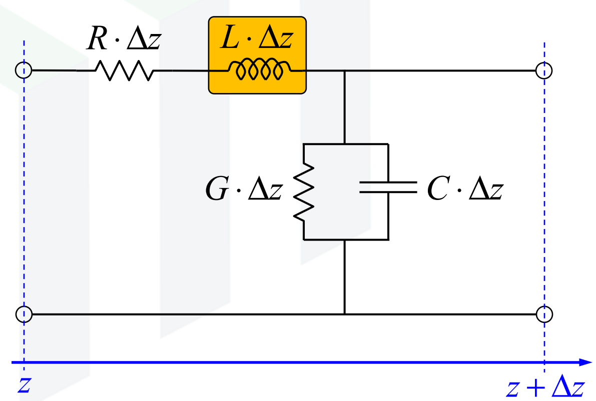

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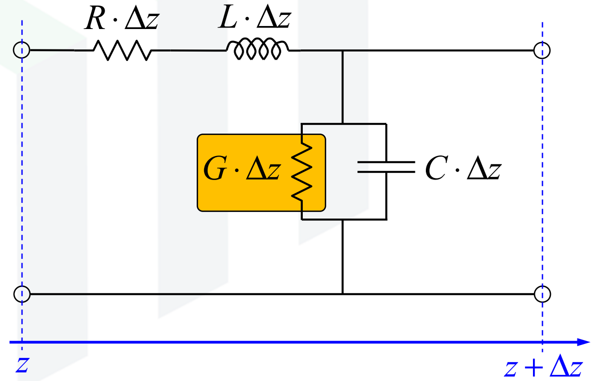

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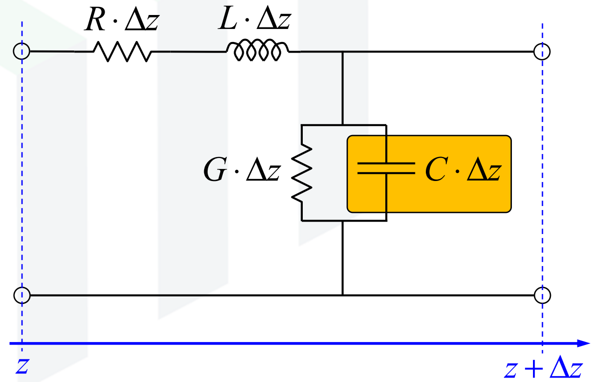
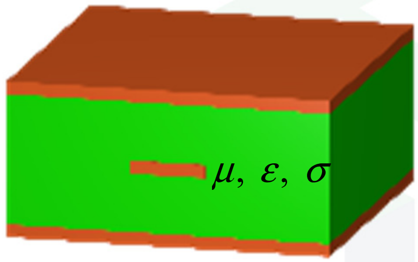
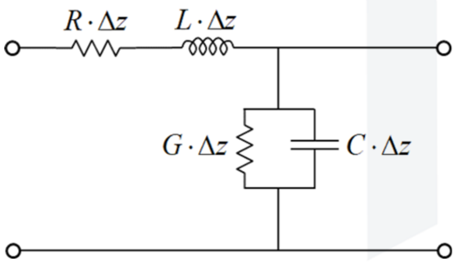


Relation to Electromagnetic Parameters



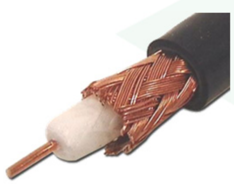
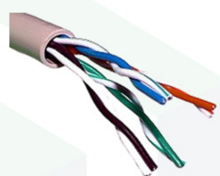
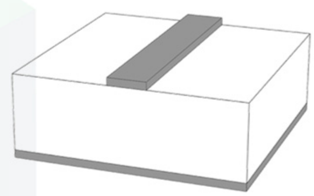


Every transmission line with a homogeneous fill has:

$$LC = \mu\varepsilon$$

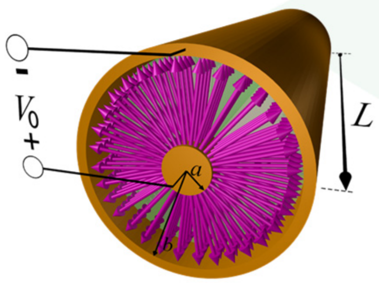
$$\frac{G}{C} = \frac{\sigma}{\varepsilon}$$

Example $RLGC$ Parameters

RG-59 Coax	CAT5 Twisted Pair	Microstrip
		
$R = 36\ \text{m}\Omega/\text{m}$	$R = 176\ \text{m}\Omega/\text{m}$	$R = 150\ \text{m}\Omega/\text{m}$
$L = 430\ \text{nH/m}$	$L = 490\ \text{nH/m}$	$L = 364\ \text{nH/m}$
$G = 10\ \mu\mho/\text{m}$	$G = 2\ \mu\mho/\text{m}$	$G = 3\ \mu\mho/\text{m}$
$C = 69\ \text{pF/m}$	$C = 49\ \text{pF/m}$	$C = 107\ \text{pF/m}$
$Z_0 = 75\ \Omega$	$Z_0 = 100\ \Omega$	$Z_0 = 50\ \Omega$

Surprisingly, almost all transmission lines have parameters very close to these same values.

Determining $RLGC$ Parameters Requires an Electromagnetic Analysis



$$\frac{C}{\ell} = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)}$$

Recall this result from electrostatics

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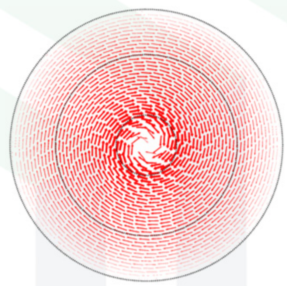

$$\frac{L}{\ell} = \frac{\mu}{2\pi}\left[\frac{1}{4} + \ln\left(\frac{b}{a}\right)\right]$$

Recall this result from magnetostatics

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$$Z_0 = \sqrt{\frac{L/\ell}{C/\ell}}$$

$$= \sqrt{\left(\frac{\eta}{2\pi}\right)^2 \left[\frac{1}{4} + \ln\left(\frac{b}{a}\right)\right] \ln\left(\frac{a}{b}\right)}$$

Now in transmission lines, characteristic impedance Z_0 is meaningful.

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