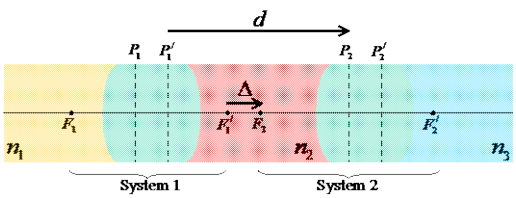
Association of Two Gaussian Systems

General Properties

1) If two rotationally symmetric systems are put in series with a common optical axis, the associated system is also rotationally symmetric.
2) F' is the image of F_1' through system 2.
3) F is the image of F_2 through system 1.

Focal/Focal Combination



$$C_1 = -\frac{n_1}{f_1} = \frac{n_2}{f_1'} \qquad C_2 = -\frac{n_2}{f_2} = \frac{n_3}{f_2'}$$

Note: The two component systems must be characterized with n_2 in place.

$$\overline{F_1F} \cdot \overline{F_2'F'} = ff'$$

If P_2 coincides with P'_1, then $f=f_1$ and $f'=f'_2$.

Procedure
1) Compute C_1, C_2, and C.
2) Compute focal lengths, f and f'.
3) Compute focal points, F_1F and F_2'F'.
4) Compute principle points, P_1P and P_2'P'.

System Power: $$C = C_1 + C_2 - \frac{d}{n_2}C_1C_2 = -\frac{\Delta}{n_2}C_1C_2$$

If C=0, the system is afocal.

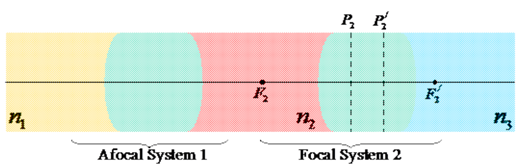
Principle Points: $$\overline{P_1P} = d\frac{n_1}{n_2}\frac{C_2}{C} \qquad \overline{P_2'P'} = -d\frac{n_3}{n_2}\frac{C_1}{C}$$

Focal Points: $$\overline{F_1F} = \underbrace{\frac{f_1^2}{f_1' + f_1 - \Delta}}_{\text{Most General}} = \underbrace{-\frac{f_1^2}{\Delta}}_{n_1=n_2} \qquad \overline{F_2'F'} = \underbrace{\frac{f_2'^2}{\Delta - f_2' - f_2}}_{\text{Most General}} = \underbrace{\frac{f_1^2}{\Delta}}_{n_2=n_3}$$

$$f' = \underbrace{\frac{n_3}{C} = \frac{n_3 f_1' f_2'}{n_2 f_2' + n_3\left(f_2 - \Delta\right)}}_{\text{Most General}} = \underbrace{-\frac{f_1' f_2'}{\Delta}}_{n_1=n_2=n_3}$$

$$f = \underbrace{-\frac{n_1}{C} = \frac{n_1 f_1 f_2}{n_2 f_1 + n_1\left(f_1' + \Delta\right)}}_{\text{Most General}} = \underbrace{\frac{f_1 f_2}{\Delta}}_{n_1=n_2=n_3}$$

Afocal/Focal Combination



$m_1 \equiv$ Magnification
$C_1 = 0$

$$C_2 = -\frac{n_2}{f_2} = \frac{n_3}{f_2'}$$

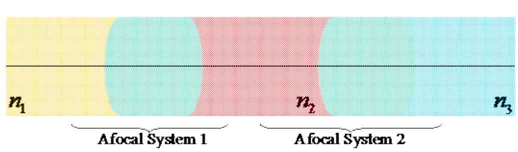
System Power: $C = m_1 C_2$ System will always be focal.

Focal Points: $$f' = -\frac{f_2'}{m_1}$$

Image Location: $$x' = f_2'\left(1 + \frac{f_2'}{f_1'}\right) + x\left(\frac{f_2'}{f_1'}\right)^2 = f_2'\left(1 - m_y\right) + m_x x$$

Magnifications: $$m_y = -\frac{f_2'}{f_1'} \qquad m_\alpha = \frac{f_1'}{f_2'} \qquad m_x = \left(\frac{f_2'}{f_1'}\right)^2$$

Afocal/Afocal Combination



m_1 m_2

Separation of component systems changes position of conjugate locations only.

System Magnification: $m = m_1 m_2$ System will always be afocal.