Electromagnetics:
Electromagnetic Field Theory

Magnetic Torque & Magnetic Dipole Moment

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Outline

- Magnetic torque
- Magnetic dipole moment

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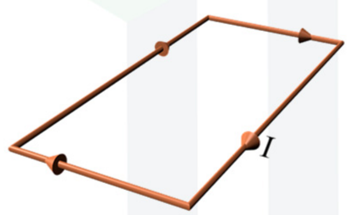
Magnetic Torque

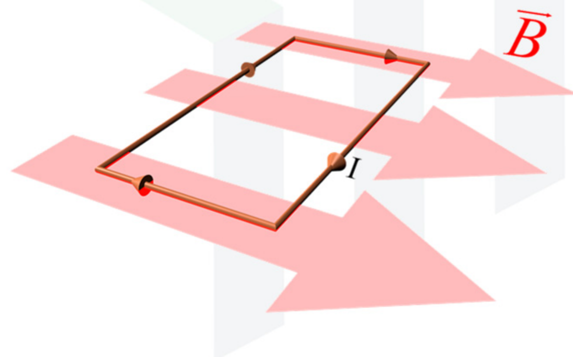
Force on a Wire Loop

Suppose there is a wire loop carrying current I.



Force on a Wire Loop

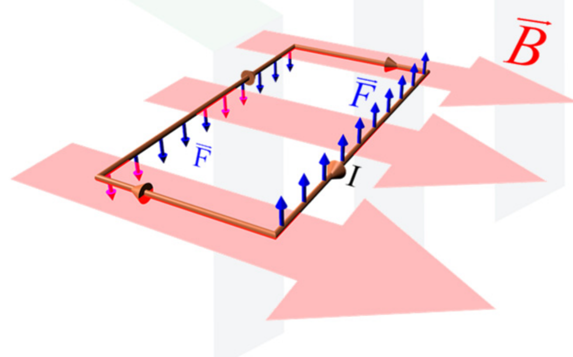
Then apply a magnetic field $\vec{B}$ oriented in the plane of the loop.



Force on a Wire Loop

Each part of the wire loop can experience a force in a different direction.

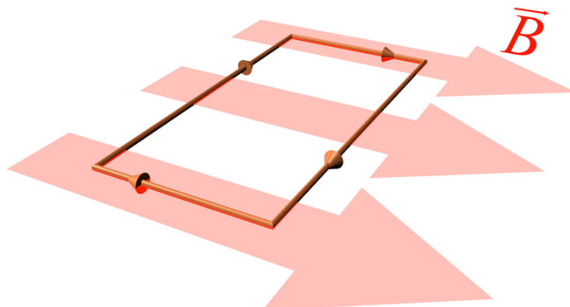
The net force is zero so the loop will not translate its position.



Force on a Wire Loop

However, the forces being different on different parts of the loop will create a torque on the that will make the loop rotate.

The loop will rotate in a way that makes the cross section of the loop perpendicular to $\vec{B}$.



Torque $\vec{T}$ (N·m)

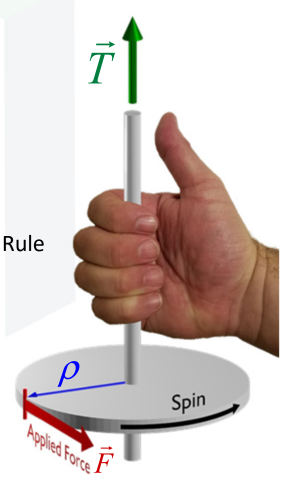
Define torque $\vec{T}$ (or mechanical moment of force) is the cross product of the force $\vec{F}$ and moment arm $\vec{r}$.

$$\underset{\text{N·m}}{\vec{T}} = \underset{\text{meters}}{\vec{r}} \times \underset{\text{Newtons}}{\vec{F}}$$

The vector moment arm $\vec{r}$ is defined as

$$\vec{r} = \rho \hat{a}_n$$

Right-Hand Rule for Torque

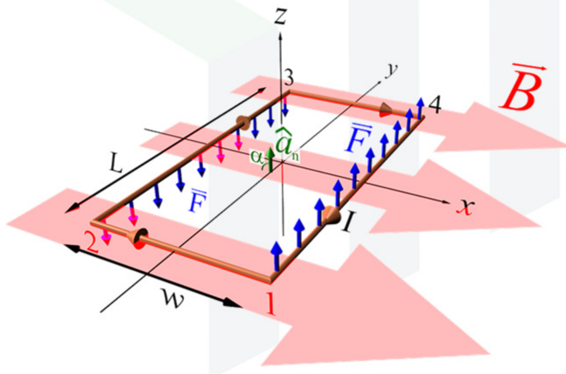


Vector Moment Arm $\vec{r}$ for a Loop

The vector moment arm $\vec{r}$ for a wire loop is defined as

$$\vec{r} = w\hat{a}_n$$

Magnitude of $\vec{r}$ is proportional to how easily the loop can be rotated.
Direction of $\vec{r}$ is perpendicular to the cross section of the loop.
Note that $\vec{r}$ is proportional to twice the radius because force is applied at either side of the axis.



Net Force on Loop in Zero

The net force on the loop is

$$\vec{F} = \oint_L \left(Id\vec{\ell}\right)\times\vec{B}$$

$$\hat{a}_x \times \hat{a}_x = 0$$

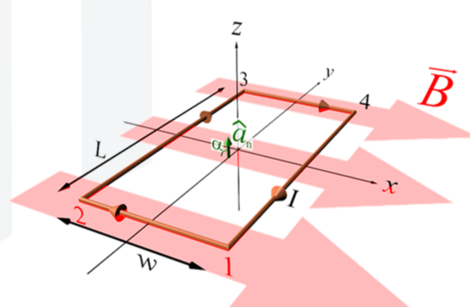
$$= \cancel{\int_1^2 Idx\hat{a}_x \times B\hat{a}_x} + \int_2^3 Idy\hat{a}_y \times B\hat{a}_x + \cancel{\int_3^4 Idx\hat{a}_x \times B\hat{a}_x} + \int_4^1 Idy\hat{a}_y \times B\hat{a}_x$$

$$= \int_2^3 IdyB\left(\hat{a}_y \times \hat{a}_x\right) + \int_4^1 IBdy\left(\hat{a}_y \times \hat{a}_x\right)$$

$$= -IB\hat{a}_z \int_2^3 dy - IB\hat{a}_z \int_4^1 dy$$

$$= -IBL\hat{a}_z + IBL\hat{a}_z$$

$$= 0$$



Net Force on Loop in Zero

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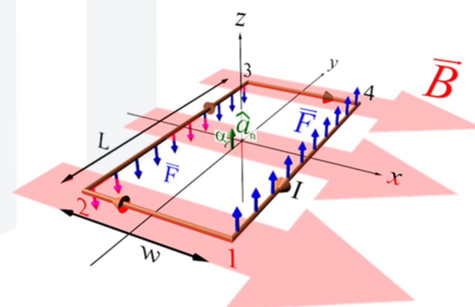
$$= 0$$

Opposite sign on these forces indicates there will be torque.



EMPossible

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Magnetic Dipole Moment

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Magnetic Dipole Moment $\vec{m}$

It is convenient to define the magnetic dipole moment $\vec{m}$ so that torque on a loop can be calculated directly from the magnetic flux $\vec{B}$.

$$\boxed{\vec{T} = \vec{m} \times \vec{B}}$$

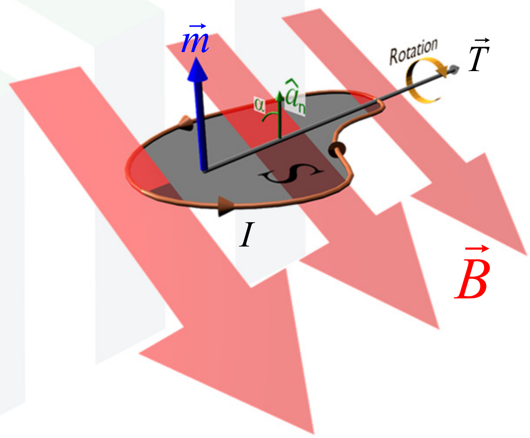
For any planar loop, $\vec{m}$ is

$$\boxed{\vec{m} = IS\hat{a}_{\mathrm{n}} \quad \left(\mathrm{A} \cdot \mathrm{m}^2\right)}$$

This parameter lumps together everything about the loop in order to calculate how the loop will respond to a magnetic field $\vec{B}$.

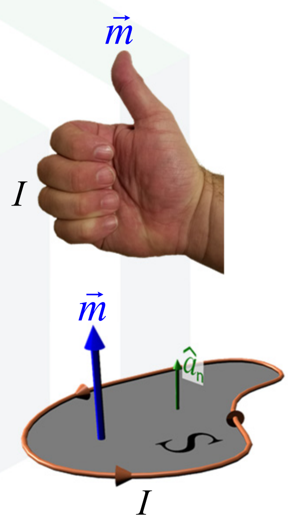
The dependence on alignment angle α is

$$\left|\vec{T}\right| = BIS \sin \alpha$$



Handedness of the Magnetic Dipole Moment

$$\boxed{\vec{m} = IS\hat{a}_{\mathrm{n}} \quad \left(\mathrm{A} \cdot \mathrm{m}^2\right)}$$



Example #5 – Magnetic Moment

Determine the magnetic dipole moment formed by the triangular loop show below.

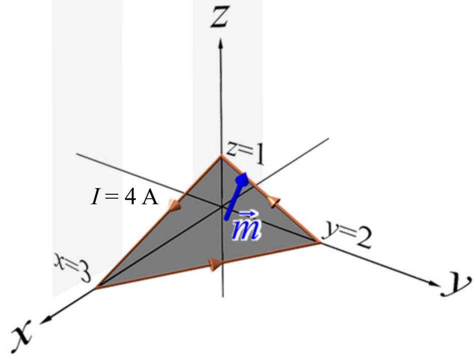
<u>Solution</u>

The magnetic dipole moment $\vec{m}$ is defined as

$$\vec{m} = IS\hat{a}_{\mathrm{n}}$$

The current I is given in the figure to be

$$I = 4\ \mathrm{A}$$



Example #5 – Magnetic Moment

Area S of the loop is calculated using the cross product.

$$S = \frac{1}{2}\left|\vec{b}\times\vec{a}\right| = \frac{1}{2}\left|(-3,2,0)\times(-3,0,1)\right| = \frac{1}{2}\left|(2,3,6)\right| = 3.5$$

$$\vec{a} = (0,0,1)-(3,0,0) = (-3,0,1)$$

$$\vec{b} = (0,2,0)-(3,0,0) = (-3,2,0)$$

Surface normal $\hat{a}_{\mathrm{n}}$ is

$$\hat{a}_{\mathrm{n}} = \frac{\vec{b}\times\vec{a}}{\left|\vec{b}\times\vec{a}\right|} = \frac{(2,3,6)}{\left|(2,3,6)\right|} = \left(\frac{2}{7},\frac{3}{7},\frac{6}{7}\right)$$

Altogether, $\vec{m}$ is

$$\begin{aligned}\vec{m} &= IS\hat{a}_n \\ &= (4\ \mathrm{A})\left(3.5\ \mathrm{m}^2\right)\left(\frac{2}{7},\frac{3}{7},\frac{6}{7}\right) \\ &= (4,6,12)\end{aligned}$$

$$\boxed{\vec{m} = 4\hat{a}_x + 6\hat{a}_y + 12\hat{a}_z\quad \mathrm{A}\cdot\mathrm{m}^2}$$