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Advanced Computation:
Computational Electromagnetics

Fractional Solution FDFD

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Concept of Fractional Linear Systems

Any linear operation can be fractionalized.

$$\frac{d^a f(x)}{dx^a}$$ Derivative of order a.
Order a does not have to be integer.

$$F^a\{f(x)\}$$ Fourier transform of order a.
Order a does not have to be integer.

The ordinary FDFD method converts Maxwell's equations into a matrix equation.

$$\mathbf{A}\mathbf{f} = \mathbf{b}$$ The matrix $\mathbf{A}$ is a linear operation so it can be fractionalized.

For example, the problem can be written in terms of half $\mathbf{A}$.

$$\mathbf{A}^{1/2}\mathbf{A}^{1/2}\mathbf{f} = \mathbf{b}$$ In this case, $\mathbf{A}^{1/2}$ is a matrix square root.

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Solution via Half $\mathbf{A}$

Starting with $\mathbf{A}^{1/2}\mathbf{A}^{1/2}\mathbf{f} = \mathbf{b}$, let $\mathbf{f}^{1/2} = \mathbf{A}^{1/2}\mathbf{f}$. The matrix equation becomes

$$\mathbf{A}^{1/2}\mathbf{f}^{1/2} = \mathbf{b}$$

This is solved for the half solution $\mathbf{f}^{1/2}$ according to

$$\mathbf{f}^{1/2} = \left(\mathbf{A}^{1/2}\right)^{-1}\mathbf{b}$$

The full solution is then

$$\mathbf{f} = \left(\mathbf{A}^{1/2}\right)^{-1}\mathbf{f}^{1/2}$$

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Generalization to Order $1/N$

Starting with

$$\left(\mathbf{A}^{1/N}\mathbf{A}^{1/N}\cdots\mathbf{A}^{1/N}\right)\mathbf{f} = \mathbf{b}$$

This is solved in N steps as

$$\mathbf{f}^{1/N} = \left(\mathbf{A}^{1/N}\right)^{-1}\mathbf{f}^0 \qquad \mathbf{f}^0 = \mathbf{b}$$
$$\mathbf{f}^{2/N} = \left(\mathbf{A}^{1/N}\right)^{-1}\mathbf{f}^{1/N}$$
$$\mathbf{f}^{3/N} = \left(\mathbf{A}^{1/N}\right)^{-1}\mathbf{f}^{2/N}$$
$$\vdots$$
$$\mathbf{f}^1 = \left(\mathbf{A}^{1/N}\right)^{-1}\mathbf{f}^{(N-1)/N} \qquad \mathbf{f} = \mathbf{f}^1 = \mathbf{f}^{N/N}$$

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Example – Scattering from a Cylinder

Solution using order 1/2

Solution using order 1/3

Solution using order 1/6

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